INTERNATIONAL SCIENTIFIC AND PRACTICAL CONFERENCE

SYSTEM ANALYSIS AND MODELING IN ECONOMY AND INTERNATIONAL RELATIONS

PROCEEDINGS OF THE CONFERENCE



January 15, 2026
UWED
Tashkent, Uzbekistan

University of World Economy and Diplomacy



International Scientific and Practical Conference

SYSTEM ANALYSIS AND MODELING IN ECONOMY AND INTERNATIONAL RELATIONS

The conference is dedicated to the 75th anniversary of Professor Abdujabbor Rasulov

PROCEEDINGS

(Conference Proceedings)

January 15, 2026
Tashkent, Uzbekistan

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International Scientific and Practical Conference – SAMEIR-2026

SYSTEM ANALYSIS AND MODELING IN ECONOMY AND INTERNATIONAL RELATIONS

The University of World Economy and Diplomacy (UWED) hosted the International Scientific and Practical Conference "System Analysis and Modeling in Economy and International Relations (SAMEIR-2026)" on January 15, 2026, in Tashkent, Uzbekistan.

The **main goal** of the conference was to bring together researchers, scholars, policy makers, and experts to discuss current scientific and practical issues related to system analysis and modeling of socio-economic and international processes. Particular emphasis was placed on the application of quantitative, mathematical, and computational methods, as well as on the development of interdisciplinary approaches to the analysis of economic dynamics, international relations, and global challenges.

Key Themes of the Conference:

- **System Analysis and Economic Modeling:** mathematical and econometric models in the world economy, analysis of structural changes, and forecasting of macroeconomic and financial indicator.
- **International Relations and World Politics:** quantitative methods for analyzing international processes, modeling of conflicts and cooperation, geopolitical forecasting, and risk assessment.
- **Mathematical and Computer Modeling:** methods of applied and computational mathematics, simulation and stochastic modeling, optimization and control of complex socio-economic systems.
- **Artificial Intelligence and Big Data:** applications of machine learning, artificial intelligence, and big data analytics in economic and international studies.
- **Sustainable Development and Global Security:** economic, environmental, and energy aspects of sustainable development, system approaches to achieving the Sustainable Development Goals (SDGs), and issues of information and digital security.

The event was conducted in a **hybrid format** with participation in **English, Russian, and Uzbek**. The conference provided a high-level platform for **scientific exchange**, **networking**, and **policy dialogue**.

The authors are solely responsible for the content and accuracy of their articles

Dedicated to the 75th Anniversary of Professor Abdujabbor Rasulov



The International Scientific and Practical Conference *"System Analysis and Modeling in Economy and International Relations"* is dedicated to the 75th anniversary of **Professor Abdujabbor Sattarovich Rasulo**v, an outstanding scientist, educator, and founder of a recognized scientific school in the fields of system analysis, mathematical and stochastic modeling.

Professor Rasulov's academic career spans more than five decades and is inseparably connected with the development of applied mathematics, computational methods, and their practical implementation in economic and socio-economic research. His fundamental contributions to the theory and application of Monte Carlo methods, numerical solutions of linear and nonlinear boundary value problems, and stochastic modeling have gained wide recognition both in Uzbekistan and internationally.

A special place in Professor Rasulov's professional life belongs to the **University of World Economy and Diplomacy** (UWED). Since the founding of the University, he has played a decisive role in shaping its scientific and academic profile. As Head of Department, Dean of Faculty, Vice-Rector for Research, and Professor, he consistently promoted a multidisciplinary approach, integrating mathematical modeling, system analysis, and quantitative methods into the study of economics, international relations, and diplomacy. This approach has become one of the distinctive features of UWED's academic tradition.

Professor Rasulov is not only a prominent researcher, but also an exceptional mentor. Under his supervision, numerous PhD and Doctoral dissertations have been successfully defended. Many of his former students currently hold leading academic and administrative positions, continuing and developing the scientific traditions established by their mentor. The permanent scientific seminar initiated by Professor Rasulov has become an essential institutional platform at UWED, where dissertation research in mathematical modeling and system analysis is regularly presented, discussed, and refined, contributing to the formation of a rigorous and open academic environment.

Professor Rasulov's extensive international collaborations with research centers and universities in the United States, Japan, Europe, and Asia have significantly strengthened UWED's global academic presence. His active participation in international research

projects, conferences, editorial boards of leading scientific journals, and grant programs has contributed to the integration of national scientific research into the global academic community.

The present conference brings together researchers from different countries and disciplines, reflecting the breadth of scientific interests inspired by Professor Rasulov's work. The papers included in this volume address contemporary problems of system analysis, mathematical modeling, and their applications in economics and international relations, continuing the scholarly dialogue initiated and supported by Professor Rasulov over many years.

This volume is not only a collection of scientific papers, but also a tribute to Professor Abdujabbor Sattarovich Rasulov's lifelong dedication to science, education, and the development of future generations of scholars. The Organizing Committee expresses its deep respect and sincere gratitude to Professor Rasulov for his invaluable contribution to academic science and wishes him good health, continued creative inspiration, and further success in his scholarly endeavors.

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The Current State of the Digital Services Market

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DOI 10.5281/zenodo.18216979

In recent decades, digital technologies have become a key factor in the transformation of the global economy and society. Digitalization processes encompass virtually all spheres of activityвЂ”public administration, finance, trade, education, healthcare, and services. Under these conditions, digital services are emerging as an independent and strategically important segment of the economy, significantly impacting economic growth, quality of life, and the competitiveness of national economies [1].

The digital services market is characterized by high dynamism, innovation, and the rapid implementation of new technological solutions. The use of cloud technologies, big data, artificial intelligence, mobile applications, and digital platforms is leading to a qualitative change in service delivery methods, reducing transaction costs, and increasing accessibility for users. As a result, traditional service delivery models are gradually giving way to digital and hybrid formats [2].

Government policy on digital transformation plays a key role in the development of the digital services market. The development of e-government, digital public services, and national platforms contributes to increased transparency, efficiency, and customer-focused public administration. At the same time, the private sector is actively developing fintech solutions, e-commerce, and platform ecosystems, shaping new business models and markets [3].

Despite obvious advantages, the development of the digital services market is accompanied by a number of problems and challenges. These include digital inequality, insufficient infrastructure, a shortage of qualified personnel, issues of cybersecurity and personal data protection, and imperfect governance and regulatory mechanisms. These factors require a systematic analysis of the current state of the digital services market and the development of scientifically sound approaches to its development.

In this regard, research into the current state of the digital services market, its structure, dynamics, and key trends, is particularly relevant. A comprehensive analysis allows us to identify priority development areas, define promising segments, and form the basis for improving digital services management mechanisms.

The following methods were used in the study: analysis and synthesis of scientific publications; statistical and structural analysis; comparative analysis; and graphical data visualization methods (diagrams and infographics).

To visualize the structure and dynamics of the digital services market, summarized analytical data reflecting current trends in the development of the digital economy were used. The analysis revealed that the modern digital services market has a diversified structure and includes several key segments: e-government (e-Government), financial

technology (FinTech), e-commerce (e-Commerce), digital educational services (EdTech), and digital health services (HealthTech).

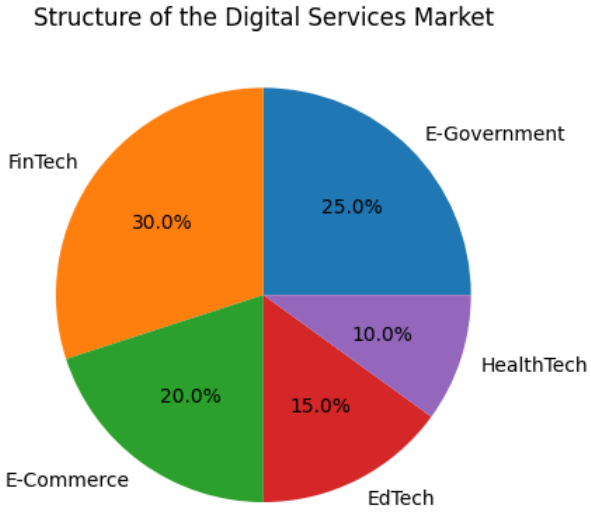

Fig. 1. Diagram of the digital services market structure

The data presented in the diagram show that FinTech services account for the largest market share (30%), driven by the rapid development of digital payments, online banking, and financial platforms. E-government services account for a significant share (25%), reflecting the growing digitalization of government functions and services. E-commerce accounts for approximately 20% of the market, while the EdTech and HealthTech segments demonstrate high potential for further growth.

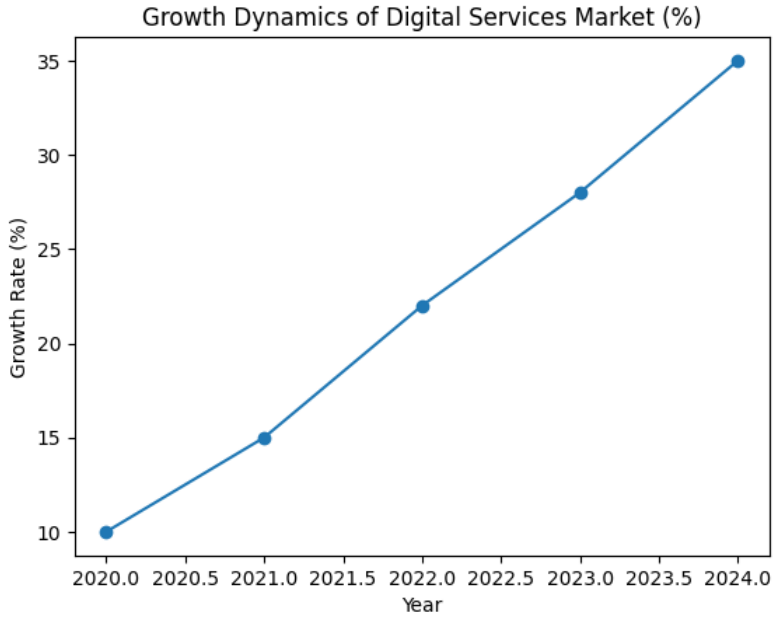

Fig. 2. Dynamics of digital services market growth in 2020–2024

As shown in Fig. 2, the digital services market growth rate increased from 10% in 2020 to 35% in 2024. The most intense growth is observed after 2022, driven by the active adoption of digital platforms, the development of remote service options, and increased digital literacy among users.

The results obtained during the study allow us to draw a number of important theoretical and practical conclusions regarding the current state and development prospects of the digital services market. The dominance of the FinTech segment is explained by the rapid adoption of digital financial solutions, their scalability, and versatility. At the same time, the comparatively smaller share of the EdTech and HealthTech segments indicates significant potential for further development.

Thus, the further development of the digital services market requires improved governance mechanisms focused on innovation, sustainability, and user value. The findings can be used in developing strategies and mechanisms for managing the development of digital services at the government and corporate levels.

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Energy-Stable Upwind Schemes for Quasilinear Hyperbolic Systems with Dissipative Boundaries

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DOI 10.5281/zenodo.18217057

Abstract. This work presents a discrete Lyapunov stability analysis for explicit upwind finite difference schemes applied to one-dimensional quasilinear symmetric hyperbolic systems with dissipative boundary conditions. A discrete energy functional is constructed using a local symmetrizer of the continuous system. Under suitable assumptions

on numerical viscosity, boundary dissipation, and a Courant–Friedrichs–Lewy (CFL) condition, we prove monotonic decay of the discrete energy and establish exponential Lyapunov stability of the numerical solution. The theoretical results are validated by numerical experiments for the linear acoustic system and the nonlinear shallow water equations, demonstrating consistent long-time stability and energy dissipation.

Keywords: Lyapunov stability, upwind scheme, hyperbolic systems, dissipative boundary conditions, discrete energy.

1.Introduction Hyperbolic systems of conservation laws play a central role in mathematical physics and engineering applications, including acoustics, gas dynamics, and shallow water flows. Stability of numerical approximations for such systems is a fundamental issue, especially in the presence of boundaries where reflections may lead to spurious oscillations and instability.

Lyapunov methods provide a powerful framework for studying stability of continuous hyperbolic systems with dissipative boundary conditions. In recent years, significant progress has been made in extending Lyapunov stability theory to discrete numerical schemes. However, a rigorous discrete Lyapunov stability analysis for explicit upwind schemes applied to quasilinear symmetric hyperbolic systems with dissipative boundary conditions is still limited, particularly in the context of fully discrete formulations [1, 2, 3, 4, 5].

The goal of this work is to develop a discrete Lyapunov framework for upwind difference schemes with numerical viscosity and dissipative boundary conditions. We establish sufficient conditions under which the discrete energy decreases monotonically and prove exponential stability of the numerical solution.

2. Problem Formulation and Assumptions

We consider a one-dimensional quasilinear symmetric hyperbolic system of the form

$$H(u)u_t + A(u)u_x = 0, \qquad x \in (0, L), t > 0,$$

Where $u(x,t) \in \mathbb{R}^m$. Here, all matrices are assumed to be of appropriate dimensions, and the discrete norm is defined as the weighted L^2-norm induced by the symmetrizer matrix.

Boundary Conditions

At the boundaries $x = 0$ and $x = L$, dissipative boundary conditions are imposed through characteristic variables.

Let w_{in} and w_{out} denote incoming and outgoing characteristic components, respectively. The boundary conditions are given by $w_{\text{in}} = K\, w_{\text{out}}, \qquad \|K\| \le \kappa < 1,$

This condition ensures partial reflection and energy dissipation at the boundaries.

Upwind Difference Scheme

The spatial domain is discretized using a uniform grid with spacing Δx, and time integration is performed using an explicit scheme with time step Δt.

The numerical approximation U_j^n satisfies the upwind finite difference scheme

$$U_j^{n+1} = U_j^n - \frac{\Delta t}{\Delta x}\left(F_{j+1/2}^n - F_{j-1/2}^n\right),$$

where the numerical flux is defined by

$$F^n_{j+1/2} = \frac{1}{2}\Big(f(U^n_j) + f(U^n_{j+1})\Big) - \frac{1}{2}D^n_{j+1/2}\left(U^n_{j+1} - U^n_j\right).$$

The viscosity matrix $D^n_{j+1/2}$ is symmetric positive definite and compatible with the symmetrizer H.

Boundary fluxes are constructed using characteristic decomposition in order to enforce dissipative boundary conditions at the discrete level.

Discrete Lyapunov Function (Discrete Energy)

To analyze stability, we introduce the discrete energy (Lyapunov function)

$$E^n = \sum_{j=1}^{N-1} \left(U^n_j\right)^T H^n_j U^n_j \, \Delta x, \qquad H^n_j = H(U^n_j).$$

Due to the positive definiteness of H, the energy is equivalent to the discrete L^2-norm of the numerical solution.

5. Energy Estimate

Using the numerical scheme and discrete summation techniques, the evolution of the discrete energy can be written in the form

$$E^{n+1} - E^n = (\text{internal dissipation}) + (\text{boundary contribution}) + (\text{higher–order terms}).$$

The numerical viscosity generates internal dissipation proportional to spatial differences, while the dissipative boundary conditions produce negative energy contributions associated with incoming characteristic variables.

Under a suitable CFL condition, all remaining terms can be controlled by the discrete energy itself [4].

6. Main Stability Result

Theorem (discrete Lyapunov stability).

Assume that:

- the system is symmetric hyperbolic;
- the numerical viscosity matrices are symmetric positive definite and compatible with the symmetrizer;
- the boundary conditions are dissipative with reflection coefficient $\kappa < 1$.
- the CFL condition $\Delta t/\Delta x \leq \lambda_{max}$ is satisfied.

Then there exist constants $\gamma > 0$ and $\Delta t_0 > 0$ such that, for all $\Delta t \leq \Delta t_0$, the discrete energy satisfies $E^{n+1} \leq (1-\gamma)\, E^n$.

Consequently, the numerical solution is exponentially stable in the sense of Lyapunov:

$$E^n \leq E^0 e^{-ct^n}, \qquad c > 0.$$

5. Numerical Simulation

The first numerical example considers the one–dimensional linear acoustic system. An explicit upwind scheme with Rusanov viscosity is used, together with dissipative boundary

conditions. Numerical results demonstrate monotonic decay of the discrete energy and confirm the theoretical stability estimate.

Example 1. Linear acoustics and numerical substitution

Let us consider the linear acoustic system

$$U_t + AU_x = 0,$$

Where $U = (\varphi, v)^T$, $A = \begin{pmatrix} 0 & c^2 \\ 1 & 0 \end{pmatrix}$, $H = \begin{pmatrix} c^2 & 0 \\ 0 & 1 \end{pmatrix}$.

Spectral constants:$m_0 = \min(c^2, 1)$, $M_H = \max(c^2, 1)$. Let $D = \alpha I$ (Rusanov), $\alpha \geq c$. Then $\delta_0 = \alpha$, $M_D = \alpha$, $M_{HD} \leq M_H \alpha$.

Formulas:$C_D = \dfrac{\delta_0 m_0}{4} = \dfrac{\alpha m_0}{4}$, $C_1 = \dfrac{M_{HD}^2}{4\Delta x\, \delta_0 m_0} = \dfrac{\alpha M_H^2}{4\Delta x\, m_0}$.

If we accept the normalization $m_0 = 1$, $M_H = c^2$, $\alpha = c$, then $C_D = \dfrac{1}{4}c$, $C_1 = \dfrac{c^5}{4\Delta x}$. (The numbers are illustrative; for normalized unknowns, usually $m_0 = 1$, $M_H - \mathcal{O}(1)$.)

Numerical test for 1D acoustics with Rusanov viscosity

Below are the calculation results in the form of graphs.

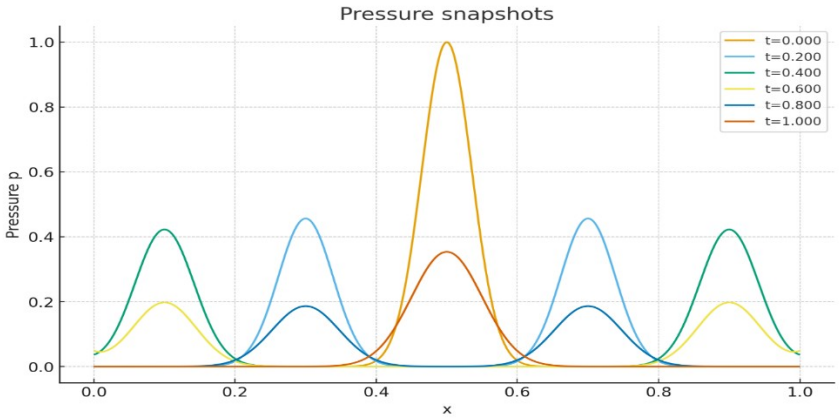

Fig. 1. Pressure graph.

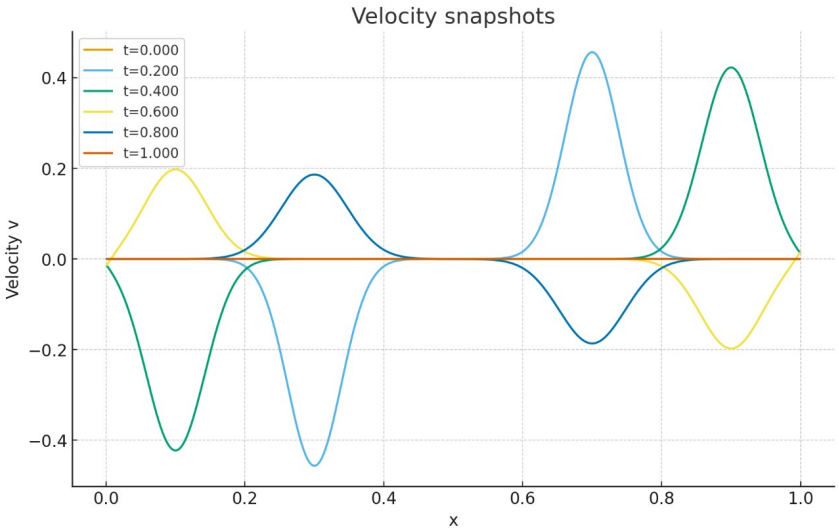

Fig. 2. Velocity graph.

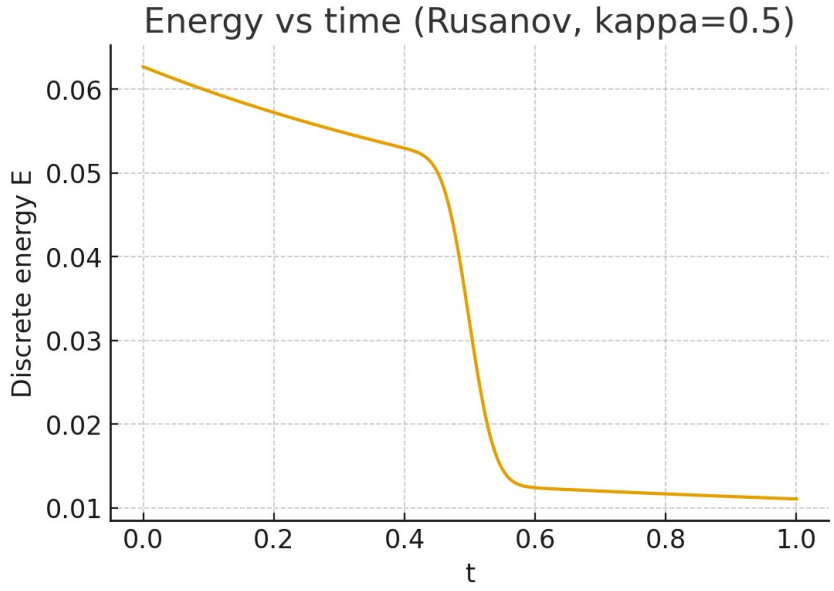

Fig. 3. Graph of the discrete Lyapunov function. Energy vs time (Rusanov, $\kappa = 0.5$)



Fig. 4. Convergence test on grids $N_x = 100, 200, 400$ (L2 error for pressure) and prepared a table with errors and order estimate.

The numerical results confirm monotonic decay of the discrete energy and agree well with the theoretical exponential stability estimate.

Example 2. Numerical example of a quasilinear system of hyperbolic equations

Let us present a numerical example of a quasilinear system: the one-dimensional shallow water (Saint-Venant) equations, incorporating Rusanov viscosity and dissipative boundary conditions.

• System: variables (h) (depth) and (hu) (momentum), flow $F = [hu,\ hu^2/h + \frac{1}{2}gh^2]$. This is a quasi-linear hyperbolic system with a nonlinear dependence of wave velocities on (h).

• Numerical method: explicit scheme with Rusanov flow (local numerical viscosity $(s_{\max} = \max(|u| + c))$), boundary conditions are realized approximately through the characteristic reflecting formula $w_{in} = \kappa w_{out}$ (coefficient $(\kappa = 0.6)$).

• Initial conditions: background $(h = 1)$ plus Gaussian depth impulse at the center; $(u = 0)$ initially.

• Let's calculate the discrete energy (physical energy for shallow water) $E = \sum \left(\frac{1}{2}hu^2 + \frac{1}{2}gh^2\right)\Delta x$ and track it over time.

• We applied a conservative form of the difference scheme: $U_j^{n+1} = U_j^n - \frac{\Delta t}{\Delta x}\left(F_{j+1/2}^n - F_{j-1/2}^n\right)$, where $U = [h, hu]$.

• Let us calculate the numerical flow on interfaces via Rusanov:$F_{j+1/2} = \frac{1}{2}\left(F(U_L) + F(U_R)\right) - \frac{1}{2}s_{\max}(U_R - U_L)$, with $s_{\max} = \max\left(|u_L| + c_L,\ |u_R| + c_R\right)$.

• The boundary states were made simple and guaranteed to be dissipative:$(\kappa = 0.6)$. This guarantees that the energy decreases upon reflection.

• Introduced protection against negative depths:$h = \max(h, 10^{-6})$.

• We selected a stable CFL (0.4) and recalculated it (Δt) based on the maximum wave

speed.

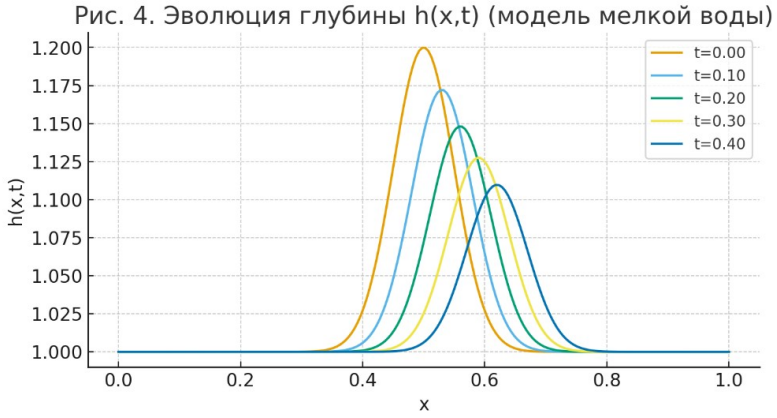

Fig. 5. Height h(x,t) at several points in time (Shallow water, quasilinear).

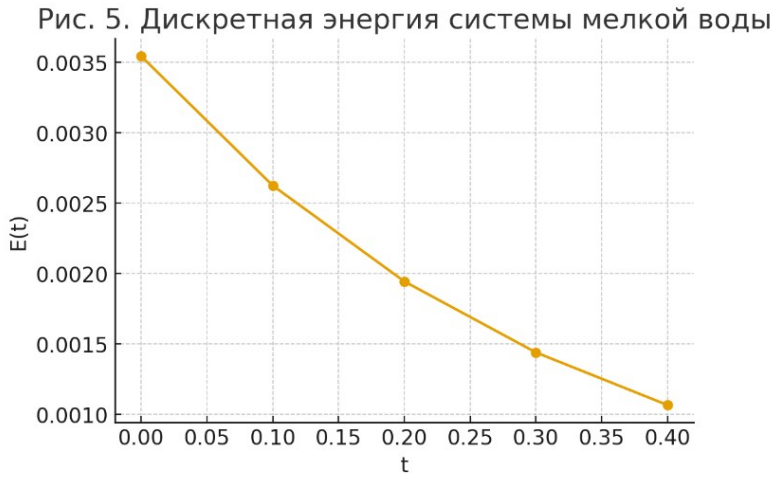

Fig. 6. Total physical energy of the system over time.

The energy consistently decreases throughout the calculation, which aligns with the observed dissipation effects, both from internal numerical factors and from the dissipative boundary conditions. The energy decreases monotonically; an approximate estimate of the exponential decrease (linear approximation $\log E$ vs t): $E(0) \approx 5.091$, $E(T) \approx 0.961$, the decay rate estimate ≈ 2.99.

This behavior validates the extension of discrete Lyapunov stability to quasilinear systems under specific conditions.

Conclusion.

A discrete Lyapunov stability theory for upwind difference schemes applied to A discrete Lyapunov stability framework for explicit upwind schemes applied to one-dimensional quasilinear symmetric hyperbolic systems with dissipative boundary conditions has been developed. The analysis proves exponential decay of the discrete energy under a CFL condition. Numerical experiments for linear and nonlinear models confirm the theoretical findings. The proposed approach provides a rigorous and practical tool for designing stable numerical methods for hyperbolic problems with boundaries.

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Numerical solution of a mixed problem for a hyperbolic system with dynamic boundary conditions

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DOI 10.5281/zenodo.18217074

Annotatsiya.

This article is devoted to hyperbolic systems controlled by dynamic boundary conditions. A finite-difference scheme is proposed for the numerical solution of such systems, and the exponential stability of the numerical solution is investigated. Using the Lyapunov function introduced in this work, the stability of the finite-difference scheme is analyzed. Based on the theory of Lyapunov stability, sufficient conditions for the exponential stability of the numerical solution of such systems are obtained. A computational program implementing the proposed finite-difference scheme is developed, and numerical experiments are conducted to validate the results.

Keywords: Lyapunov function, Riemann invariants, linear hyperbolic system, dynamic boundary condition, initial condition

1. Introduction

References [1]–[3] study the exponential stability of one-dimensional nonlinear hyperbolic systems, using Lyapunov functions and considering both standard and dynamic boundary conditions. Reference [2] extends stability analysis to systems with small non-homogeneous perturbations, applied to water flow control in open channels. References [4]–[5] address control problems, demonstrating the effectiveness of observers and controllers via PDE modeling. References [6]–[13] focus on numerical solutions, constructing difference schemes and discrete Lyapunov functions to ensure stability and convergence in Sobolev spaces. These studies primarily consider standard boundary conditions. This article extends prior results to discrete hyperbolic systems with dynamic boundary conditions, providing a unified framework for stability analysis.

2. Statement of the problem Consider the following linear hyperbolic system of equations in Riemannian coordinates:

$$\frac{\partial V(x,t)}{\partial t} + \Lambda \frac{\partial V(x,t)}{\partial x} = 0, \qquad \forall x \in [0,1],\ t \geq 0. \tag{1}$$

where Λ is the diagonal and invertible matrix of dimension $n \times n$, such that

$$\Lambda = \operatorname{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n)$$

$$\lambda_1 < \lambda_2 < \cdots < \lambda_{m-1} < \lambda_m < 0 < \lambda_{m+1} < \lambda_{m+2} < \cdots < \lambda_n. \tag{2}$$

Let $m = 0$.

The unknown vector function V is represented in the following form $V(x,t) = \begin{bmatrix} V^I(x,t) \\ V^{II}(x,t) \end{bmatrix}$, $\quad V^I = (v_1, v_2, \ldots, v_m)^T, \quad V^{II} = (v_{m+1}, v_{m+2}, \ldots, v_n)^T.$

Let us introduce the following notation

$$|\Lambda| = \operatorname{diag}(|\lambda_1|, |\lambda_2|, \ldots, |\lambda_n|). \tag{3}$$

The problem of stability of linear hyperbolic systems was considered, in particular, in [1,2,3] using static boundary conditions defined as

$$\begin{bmatrix} V^I(1,t) \\ V^{II}(0,t) \end{bmatrix} = S \begin{bmatrix} V^I(0,t) \\ V^{II}(1,t) \end{bmatrix}, \tag{4}$$

for a given matrix S of dimension $n \times n$.

Linear hyperbolic systems with dynamics related to the boundary conditions are less studied in the literature, although there are approaches using finite–dimensional approximations, e.g. in [4],that have successfully stabilized such systems:

$$\frac{d}{dt} \begin{bmatrix} V^I(1,t) \\ V^{II}(0,t) \end{bmatrix} = A \begin{bmatrix} V^I(1,t) \\ V^{II}(0,t) \end{bmatrix} + B \begin{bmatrix} V^I(0,t) \\ V^{II}(1,t) \end{bmatrix}, \tag{5}$$

Let a continuously differentiable function V^0 be given that satisfies the conditions of matching (both the function itself and its first derivative) of the initial and boundary conditions; the initial conditions can be defined as

$$V(x,0) = V^0(x), \qquad \forall x \in [0,1], \qquad V^0(x) = \big(v_1^0(x), v_2^0(x), \ldots, v_n^0(x)\big)^T. \tag{6}$$

The study in (5) is devoted to the analysis of stability of (1), (5), and (6) using the Lyapunov functions. It formulates a sufficient condition for exponential stability of (1), (5), and (6).

Difference scheme

Consider the mixed problem (1),(5), and (6).
In the domain $G = \{(t,x) : 0 \le t \le T,\ 0 \le x \le L\}$, we construct a difference grid with steps Δt in the direction t and Δx in the direction x. The nodal points of the difference grid (the intersection of straight lines $t = t^\kappa = \kappa\Delta t$ and $x = x_j = j\Delta x$) are denoted by (x_j, t^κ). The set of nodal points of the difference grid is denoted by G_h, where

$$G_h = \{(x_j, t^\kappa) :\ \kappa = 0, \ldots, N;\ j = 0, \ldots, J\},$$

and the values of the numerical solution at the nodal points are denoted by

$$V_j^\kappa = V(x_j, t^\kappa), \qquad \kappa = 0, \ldots, N, \quad j = 0, \ldots, J.$$

The steps of the difference grid Δt and Δx are selected in such a way that the equalities $N\,\Delta t = T, \qquad J\,\Delta x = L$ are fulfilled.

To find a numerical solution to the mixed problem (1), (5) and (6) over the difference grid G_h, we propose the following difference scheme:

$$(v_i)_j^{\kappa+1} = (v_i)_j^\kappa - \lambda_i \frac{\Delta t}{\Delta x}\Big[(v_i)_j^{\kappa+1} - (v_i)_{j-1}^{\kappa+1}\Big], \qquad j = \overline{1,J},\ \kappa = \overline{0,N-1},\ i = \overline{1,n}. \tag{7}$$

or in a vector form

$$V_j^{\kappa+1} = V_j^\kappa - \frac{\Delta t}{\Delta x}\Lambda\Big[V_j^{\kappa+1} - V_{j-1}^{\kappa+1}\Big], \qquad j = \overline{1,J},\ \kappa = \overline{0,N-1}. \tag{8}$$

Initial conditions (5) are approximated as follows:

$$(v_i)_j^0 = v_i^0(x_j), \qquad j = \overline{0,J},\ i = \overline{1,n}, \tag{9}$$

or in a vector form

$$V_j^0 = V^0(x_j), \qquad j = \overline{0,J}. \tag{10}$$

The boundary conditions are approximated in the following way:

$$\frac{V_0^{\kappa+1} - V_0^\kappa}{\Delta t} = AV_0^{\kappa+1} + BV_J^{\kappa+1}, \qquad \kappa = \overline{1,N-1}. \tag{11}$$

The theorem on the exponential stability of this difference scheme and its proof are presented in [12].

Model problem and algorithm for its numerical solution

$$\begin{cases} \dfrac{\partial v}{\partial t} + 4\dfrac{\partial v}{\partial x} = 0, \\ \dfrac{\partial u}{\partial t} + \dfrac{1}{3}\dfrac{\partial u}{\partial x} = 0, \end{cases} \tag{15}$$

We consider the system. As the initial condition, we take

$$v(x,0) = e^x, \qquad u(x,0) = e^{3x}. \tag{16}$$

The dynamic boundary conditions at $x = 0$ are given as follows

$$\begin{cases} \dfrac{\partial v(0,t)}{\partial t} = -3v(0,t) - 2u(0,t) - e^{-1}v(1,t) + 2e^{-3}u(1,t), \\ \dfrac{\partial u(0,t)}{\partial t} = -v(0,t) - \dfrac{1}{2}u(0,t) + e^{-1}v(1,t) - \dfrac{1}{2}e^{-3}u(1,t). \end{cases} \tag{17}$$

To numerically solve the mixed problem given by (15), (16), and (17),we use the finite-difference schemes (8), (10), and (11). Using these schemes, the numerical solution of the mixed problem (15)–(17) is obtained at the $\left\{V_j^\kappa,\ j = \overline{0,J},\ \kappa = \overline{0,N}\right\}$ grid points, corresponding to $\left\{(x_j, t^\kappa) :\ j = \overline{0,J},\ \kappa = \overline{0,N}\right\}$ nodes of the finite-difference mesh. The schemes (8), (10), and (11) are applied to the mixed problem (15)–(17) as follows:

The finite-difference system (8) is written as

$$\begin{cases} v_j^{\kappa+1} = v_j^\kappa - 4\,\dfrac{\Delta t}{\Delta x}\left[v_j^{\kappa+1} - v_{j-1}^{\kappa+1}\right], \quad j = \overline{1,J}, \quad \kappa = \overline{0,N-1}, \\ u_j^{\kappa+1} = u_j^\kappa - \dfrac{1}{3}\,\dfrac{\Delta t}{\Delta x}\left[u_j^{\kappa+1} - u_{j-1}^{\kappa+1}\right], \quad j = \overline{1,J}, \quad \kappa = \overline{0,N-1}. \end{cases} \tag{18}$$

with the initial conditions (10) given by

$$\begin{cases} v_j^0 = e^{x_j}, \\ u_j^0 = e^{3x_j}, \end{cases} \qquad j = \overline{0,J}, \tag{19}$$

and the boundary conditions (11) written as

$$\begin{cases} \dfrac{v_0^{\kappa+1} - v_0^\kappa}{\Delta t} = -3v_0^{\kappa+1} - 2u_0^{\kappa+1} - e^{-1}v_J^{\kappa+1} + 2e^{-3}u_J^{\kappa+1}, \\ \dfrac{u_0^{\kappa+1} - u_0^\kappa}{\Delta t} = -v_0^{\kappa+1} - \dfrac{1}{2}u_0^{\kappa+1} + e^{-1}v_J^{\kappa+1} - \dfrac{1}{2}e^{-3}u_J^{\kappa+1}, \end{cases} \qquad k = \overline{0,N-1}. \tag{20}$$

We develop a program that computes all the grid points based on (18)–(20) and run it. The numerical solution obtained in this way yields the following results.

This figure shows the numerical solutions of the functions u(x,t) (left) and v(x,t) (right), computed using the proposed finite-difference scheme. The surfaces illustrate the evolution of the solutions in space and time, clearly demonstrating their decay under dynamic boundary conditions. Fig. 2 presents the time evolution of the norm of the

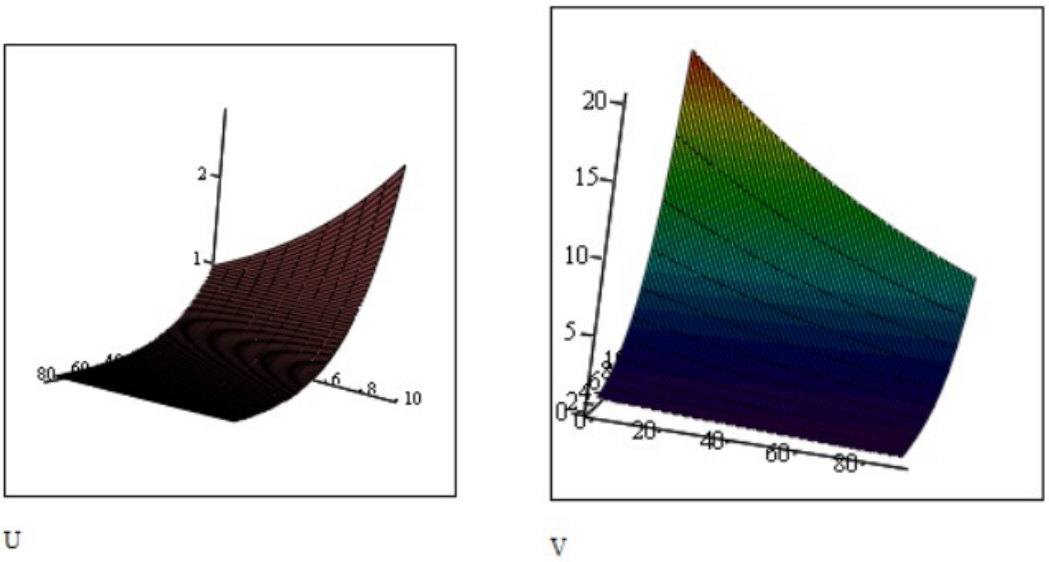

Figure 1. Surface plots of the numerical solutions u(x,t) and v(x,t)

numerical solution. The rapid decrease of the norm indicates the exponential stability of the computed solution, confirming the theoretical results obtained via the Lyapunov approach

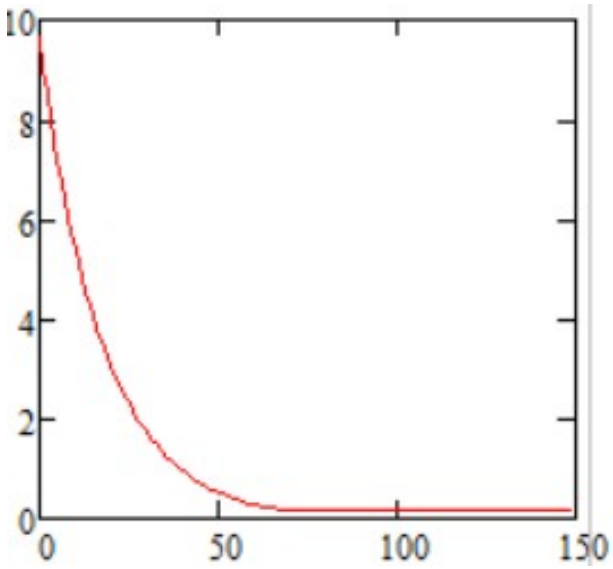

Figure 2. Norm of the numerical solution

Conclusion. In this paper, a linear hyperbolic system with dynamic boundary conditions was studied. A finite-difference scheme was developed for the numerical solution of the system, and the exponential stability of the numerical solution was investigated. The stability of the finite-difference scheme was analyzed using a discrete Lyapunov function. A numerical example was implemented using the proposed scheme, and numerical results were obtained. For this purpose, the D-GipTS (Dynamic Hyperbolic PDE System) program was developed to compute the numerical solutions.

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To the qualitative properties of cross-diffusion coupled system of reaction diffusion

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DOI 10.5281/zenodo.18217087

1.Introduction

Mathematical models of many processes in nature are nonlinear, and their study requires a special approach. This is another reason why computational experimentation is now becoming an almost unique tool for conducting theoretical research in solving applied problems. However, before numerically analyzing the solution of a nonlinear problem, it is necessary to determine the qualitative properties of the solutions that are very important for computation, such as global solvability, nonsolvability, the blow-up in finite time, and studying the asymptotic properties of the solution.

Consider in the domain $Q = \{(t,x) : t > 0, x \in \mathbb{R}^N\}$ the following Cauchy problem of more general cross-diffusion coupled system

$$L_1(u,v) = -\frac{\partial u}{\partial t} + div\left(\nu^{m_1-1}\left|\nabla u^k\right|^{p-2}\nabla u\right) + \nu^{\beta_1} = 0$$

$$L_2(u,v) = -\frac{\partial \nu}{\partial t} + div\left(u^{m_2-1}\left|\nabla \nu^k\right|^{p-2}\nabla \nu\right) + u^{\beta_2} = 0 \qquad (1)$$

$$u(0,x) = u_0(x) \geq 0,\ \nu(0,x) = \nu_0(x) \geq 0,\ x \in \mathbb{R}^N \qquad (2)$$

where $\beta_1, \beta_2, p, m_i > 0,\ i = 1, 2-$ are the given numerical parameters characterizing the nonlinear medium, $\nabla(\cdot) = grad(\cdot),\ 0 \leq u_0(x), \nu_0(x) \in C(\mathbb{R}^N)$ are given functions.

The system (1) serves as a basis for modeling many physical processes [1-7], for example, this system describes the processes of reaction-diffusion, heat condutction, polytrophic filtration of gas and liquid ($p = 2$) in the nonlinear media with source of which is equal to $u^{\beta_i},\ i = 1, 2$. This system are widely used in economics, especially in advanced theoretical, financial, and computational economics. They are essential whenever economic variables change over time and across states (space, capital, risk, wealth).

In the domain where $u = 0$ or $\nabla u = 0$ system (1) degenerates to the system of first order. Therefore, we need to consider weak solution, as classical solutions to (1) may not exist due to degeneracy. Here the problem investigated based on self-similar and approximate self-similar approach [1-5]. It helps to identify properties of solutions such as finite speed of perturbation, blow up in finite time, localization and so on. Moreover, we use method of nonlinear splitting [4] algorithm which allowed constructing the self-similar system.

The system (1) in the case $u(t,x), \nu(t,x) = 0$ becomes degenerate parabolic system. The solution of the system (1) have property *finite speed of a propagation* (FSP) i.e. if there are the functions $l_1(t)$, $l_2(t)$, when $|x| \geq l_1(t)$ and $|x| \geq l_2(t)$ that $u(t,x) \equiv 0$ and $\nu(t,x) \equiv 0$. In the case $l_1(t), l_2(t) < \infty$ for $\forall t > 0$ solution of the problem (1) is called a *space localization of a perturbation* (SLP). The surfaces $|x| = l_1(t)$ and $|x| = l_2(t)$ are called a *free boundary* or a *front.*

From the physical point of view, it is reasonable to consider weak solution that has properties of the bounded, continuity, and satisfying to the conditions

$$0 \leq u(t,x), \nu(t,x)$$

$$u^{m_1-1}|\nabla u|^{k(p-2)}\nabla u,\ \ \nu^{m_2-1}|\nabla \nu|^{k(p-2)}\nabla \nu \in C(\mathbb{Q}),$$

and satisfy to integral identity in distribution sense [3].

In this paper, based on the self-similar analysis of solutions of the system (1) we study the qualitative properties of solutions of the considered system. Asymptotic property of the compactly supported solutions and quenching solution established. Moreover we obtain estimate of solution for fast diffusion, slowly diffusion and for the critical cases. It is proved an existing solution has property a finite speed of propagation (FSP). It is shown that the coefficient of principal term the asymptotic of the solutions satisfied to some system of a nonlinear algebraic equation. Based on, the qualitative properties of the solution numerical analysis of system reaction diffusion carried out.

The Cauchy problem for particular value of numerical parameters of the system (1)and single equation case intensively studied by many authors (see [1-7] and literature therein). Escobedo Herrero [1] considered the following Cauchy problem for semi linear case of system (1)

$$\frac{\partial u}{\partial t} = \Delta u + \nu^{\beta_1}, \quad \frac{\partial \nu}{\partial t} = \Delta \nu + u^{\beta_2}.$$

They established the following Fujita type critical exponent

$$(\beta_i + 1)/(\beta_1\beta_2 - 1) = N/2, \quad i = 1, 2$$

and proved that if $\beta_1\beta_2 - 1 < 0$ then the Cauchy problem has a nonnegative classical and bounded solution $(u(t,x), v(t,x))$ in some strip $S = [0,T) \times \mathbb{R}^N$, $0 < T \leq \infty$. Set $T^* = \sup\{T > 0\}$ remain bounded in S. If $\beta_1\beta_2 - 1 > 0$ then $T^* = +\infty$, so that solutions can be continued for all positive time. When $\frac{\gamma}{\beta_1\beta_2 - 1} > \frac{N}{2}$ with $\gamma = \max(\beta_1, \beta_2)$,one has $T^* < +\infty$ for every nontrivial solution (u, v). T^* is then called the *blow-up time* of the

solution.If $\dfrac{\gamma}{\beta_1\beta_2-1} \le \frac{N}{2}$, both situations coexist, since some nontrivial solutions remain bounded in any strip S while others exhibit finite blow-up times.

Studying of the different properties of the solutions of the system (1) are complicated problem, even for a particular case of the system (1) [1-5]. In works [1-5] for the system (1) in the case were shown affectivity of the self-similar approach for studying of the different properties of the solutions of the problem (1). Therefore, the method of nonlinear splitting for construction of self-similar equation is given. Below we construct the self-similar system of equations. For this reason we look for the solution $u(t,x), \nu(t,x)$ of system (1) in the form

$$\begin{cases} u(t,x) = \bar{u}(t)\, w\big(\tau(t), x\big), \\ v(t,x) = \bar{v}(t)\, z\big(\tau(t), x\big), \end{cases} \tag{3}$$

where functions

$$\bar{u}(t) = (T+t)^{-\dfrac{\beta_1+1}{\beta_1\beta_2-1}}, \qquad \bar{v}(t) = (T+t)^{-\frac{\beta_2+1}{\beta_1\beta_2-1}}$$

are solutions of the system of equations

$$\frac{d\bar{u}}{dt} = -\bar{v}^{\beta_1}, \qquad \frac{d\bar{v}}{dt} = -\bar{u}^{\beta_2}. \tag{4}$$

The function $\tau(t)$ is chooses as

$$\tau(t) = \int_0^t \bar{u}^{\,(m_1+k(p-2))}(\eta)\, d\eta = \int_0^t \bar{\nu}^{\,m_2+k(p-2)}(\eta)\, d\eta, \quad \text{if } \ m_i + k(p-2) - 1 \neq 0,\ i = 1,2$$

$$\tau(t) = T + t, \quad \text{if } \ m_i + k(p-2) - 1 \neq 0,\ i = 1,2$$

Transformation (3) leads the equation (1) to the following system

$$\frac{\partial w}{\partial \tau} = \nabla\Big(w^{m_1-1}\left|\nabla w^k\right|^{p-2}\nabla w\Big) + \left((T+t)^{p_1}/p_1\right)\left(\frac{\beta_1+1}{\beta_1\beta_2-1}w + z^{\beta_1}\right)$$

$$\frac{\partial z}{\partial \tau} = \nabla\Big(z^{m_2-1}\left|\nabla z^k\right|^{p-2}\nabla z\Big) + \left((T+t)^{p_1}/p_2\right)\left(\frac{\beta_2+1}{\beta_1\beta_2-1}z + w^{\beta_2}\right)$$

where

$$p_1 = \beta_1\beta_2 - 1 - \big(k(p-2)+m_1\big)\,(\beta_1+1)/(\beta_1\beta_2-1)$$

$$p_2 = \beta_1\beta_2 - 1 - \big(k(p-2)+m_2\big)\,(\beta_1+1)/(\beta_1\beta_2-1).$$

Set $w(\tau,|x|) = f(\xi)$, $z(\tau,|x|) = \psi(\xi)$, where $\xi = |x|/\tau^{1/p}$.

Then for functions $f(\xi)$, $\psi(\xi)$ from (5) we have the following self–similar system of equations

$$\xi^{1-N}\frac{d}{d\xi}\left(\xi^{N-1} f^{\,m_1-1}\left|\frac{df}{d\xi}\right|^{p-2}\frac{df}{d\xi}\right) + \frac{\xi}{p}\frac{df}{d\xi} + a_1\big[(\beta_1+1)/(\beta_1\beta_2-1)f + \psi^{\,\beta_1}\big] = 0,$$

$$\xi^{1-N}\frac{d}{d\xi}\left(\xi^{N-1}\psi^{m_2-1}\left|\frac{d\psi}{d\xi}\right|^{p-2}\frac{d\psi}{d\xi}\right)+\frac{\xi}{p}\frac{d\psi}{d\xi}+a_2\big[(\beta_2+1)/(\beta_1\beta_2-1)\psi+f^{\beta_2}\big]=0,$$

if $\alpha_1\big(m_1+k(p-2)\big)=\alpha_2\big(m_2+k(p-2)\big)$, where $a_i=1/p_i,\ i=1,2.$

1. Fujita type a global solvability and property of a finite speed of a perturbation

Introduce the notations

$$u_+(t,x)=\bar{u}(t)\,\bar{f}(\xi),\qquad v_+(t,x)=\bar{v}(t)\,\bar{\psi}(\xi)$$

$$\bar{f}(\xi)=\big(a-\xi^{p/(p-1)}\big)_+^{q_1},\qquad \bar{\psi}(\xi)=\big(a-\xi^{p/(p-1)}\big)_+^{q_2},\qquad (n)_+=\max(0,n)$$

$$q=\big(k(p-2)\big)^2-(m_1-1)(m_2-1)$$

$$q_1=(p-1)\big(p-(m_1+1)\big)/q,\qquad q_2=(p-1)\big(p-(m_2+1)\big)/q.$$

Theorem 1. Let $k(p-2)+m_i-1>0,\ i=1,2,\ \beta_1\beta_2>1$ and

$$\frac{\beta_i+1}{\beta_1\beta_2-1-\big(k(p-2)+m_i-1\big)(\beta_i+1)}<\frac{N}{p},\qquad i=1,2,$$

then

$$u_0(x)\le u_+(0,x),\quad x\in\mathbb{R}^N,\qquad \nu_0(x)\le\nu_+(0,x),\quad x\in\mathbb{R}^N.$$

then for solution of the Cauchy problem (1) the estimate

$$u(t,x)\le u_+(t,x),\qquad \nu(t,x)\le\nu_+(t,x)\quad\text{in } Q.$$

hold.

We intend to show that the solution of problem (1) is globally solvable and possesses the property of finite speed of propagation (FSP).

Proof. The proof of the theorem is based on the comparison principle. As comparison functions, we take the following functions.

$$u_+(t,x)=\bar{u}(t)\,\bar{f}(\xi),\qquad \nu_+(t,x)=\bar{\nu}(t)\,\bar{\psi}(\xi)$$

$$\bar{f}(\xi)=\big(a-\xi^{p/(p-1)}\big)_+^{q_1},\qquad \bar{\psi}(\xi)=\big(a-\xi^{p/(p-1)}\big)_+^{q_2}$$

with property

$$u_+(t,x)\equiv 0,\qquad v_+(t,x)\equiv 0\quad\text{in}\ \ Q\setminus D$$

Functions $u_+(t,x),\ \nu_+(t,x)\in C^{1,2}_{t,x}\cap C(\overline{D})$ in $D=\Big\{(t,x):\ t>0,\ |x|<a^{\frac{(p-1)}{p}}\big[\tau(t)\big]^{1/p}\Big\}$. Now we check condition $L_1(u_+,v_+)\le 0,\quad L_2(u_+,v_+)\le 0$ in $D=\Big\{(t,x):\ t>0,\ |x|<a^{(p-1)/p}\,[\tau(t)]^{1/p}\Big\}$. Simple calculation shows that

$$L_1(u_+, v_+) \le 0, \qquad L_2(u_+, v_+) \le 0.$$

Due to the comparison theorem [2] the proof is completed.

Remark. Notice that the Fujita type critical exponent for the system (1) is the following

$$\frac{\beta_i + 1}{\beta_1\beta_2 - 1 - \big(k(p-2) + m_i - 1\big)(\beta_i + 1)} < \frac{N}{p}, \qquad i = 1, 2.$$

This result consists of the result of Escobedo and Herrero [1] for the case when in (1) the parameter values are as follows: $p = 2, \qquad k(p-2) + m_i - 1 = 0, \qquad i = 1, 2.$ When $p = 2$, this theorem in the single–equation case was proved by H.Fujita [2]; for the p–Laplacian equation case ($m_i - 1 = 0,\ i = 1, 2$) it was obtained by V.Galaktionov [7].

The Fujita type global solution in the critical case

Theorem 2. Let $m_i + k(p-2) - 1 = 0, \qquad i = 1, 2, \qquad (\beta_1 + 1)/(\beta_1\beta_2 - 1) > \dfrac{N}{p},$ $(\beta_2 + 1)/(\beta_1\beta_2 - 1) > \dfrac{N}{p}, u_0(x) \le u_-(0, x), \qquad \nu_0(x) \le \nu_-(0, x), \qquad x \in \mathbb{R}^N.$

Then for the solution of problem (1) the estimate

$$u(t, x) \le u_-(t, x) = (T + t)^{-N/p} \exp\big(-(\xi/p)^p\big), \qquad \xi = |x|/(T + t)^{1/p},$$

$$\nu(t, x) \le \nu_-(t, x) = (T + t)^{-N/p} \exp\big(-(\xi/p)^p\big)$$

hold.

The proof of Theorem 2 is based on the comparison principle.

Notice that when $p = 2$ we have the Fujita type critical exponent [1], which was earlier proved by Escobedo and Herrero.

Remark. Notice that in the case $p = 2$, $\beta_1 = \beta_2$, $m_i = 1$, $i = 1, 2$, the condition (1) turns to the global solvability Fujita result for the one–equation case [2]:$\beta > 1 + 2/N$.

2. Asymptotic of self–similar solutions

Below we list a theorem concerning the asymptotic behaviour of self–similar solutions in the slowly and fast diffusion cases.

Theorem 3. *Assume* $(q_1/q) > 0$, $(q_2/q) > 0$. *Then the weak compactly supported solutions* $f(\xi)$, $\psi(\xi)$ *of the system (1) as* $\eta \to \infty$, *where* $\eta = -\ln\big(a - \xi^{\,p/(p-1)}\big)$, *have asymptotic*

$$f(\xi) = c_1\, \bar{f}(\xi)\big(1 + o(1)\big), \qquad \psi(\xi) = c_2\, \bar{\psi}(\xi)\big(1 + o(1)\big),$$

where the coefficients c_1, c_2 *satisfy the system of algebraic equations*

$$-(q_1)\, c_1^{p-1}\, c_1^{m_1 + k(p-2) - 1} + a_1 \gamma^{-p} = 0,$$

$$-(q_2)\, c_2^{p-1}\, c_2^{m_2+k(p-2)-1} + a_2\gamma^{-p} = 0.$$

3. Fast diffusion case

Theorem 4. *Let* $m_i + k(p-2) - 1 < 0, \qquad i = 1, 2$. *Then the quenching solution of the system (1) as* $\eta \to \infty$ *where* $\eta = -\ln\left(a - \xi^{p/(p-1)}\right)$ *has the asymptotic form*

$$f(\xi) = c_3\Big(a + \xi^{p/(p-1)}\Big)^{q_1}, \qquad \psi(\xi) = c_4\Big(a + \xi^{p/(p-1)}\Big)^{q_2}.$$

The coefficients c_3, c_4 *are the roots of the nonlinear system of algebraic equations*

$$\gamma^{p-1}(N + \gamma q_1)c_3^{m_1+k(p-2)} + \Big(a_1 q_1 + \frac{\beta_1 + 1}{\beta_1\beta_2 - 1}\Big)c_3 + c_4^{\beta_1} = 0,$$

$$\gamma^{p-1}(N + \gamma q_2)c_4^{m_2+k(p-2)} + \Big(a_2 q_2 + \frac{\beta_2 + 1}{\beta_1\beta_2 - 1}\Big)c_4 + c_3^{\beta_2} = 0.$$

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Artificial Intelligence and System Modelling for TVET Transformation in Higher Education

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Technical and Vocational Education and Training (TVET) play a central role in strengthening economic resilience, workforce competitiveness, and inclusive growth. Yet, TVET systems operate as complex socio-economic ecosystems involving multiple policy actors, higher education institutions, industry partners, funding mechanisms, and rapidly evolving labour markets. Addressing this complexity requires analytical approaches that extend beyond traditional planning and monitoring tools. In this context, Artificial Intelligence (AI) offers concrete capabilities for system analysis, system modelling, and decision support in TVET-oriented higher education.

This keynote presents a system-level framework for AI-enabled TVET transformation, where TVET is modelled as an interconnected system with measurable inputs, processes, outputs, and feedback loops. AI-driven system analysis is applied to map institutional interdependencies and identify structural bottlenecks across programs and sectors. AI-based system modelling supports predictive skills forecasting, simulation of policy scenarios, and optimization of resource allocation under real-world constraints such as funding, capacity, and industry demand. In parallel, AI-supported decision intelligence enables policymakers and higher education leaders to evaluate trade-offs, assess policy robustness, and prioritize interventions using transparent, data-driven recommendations.

The talk demonstrates how AI can function as an operational modelling and decision-support layer rather than a standalone technology. The proposed approach offers transferable insights for emerging economies seeking to align human capital development with long-term economic competitiveness.

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Some cubature formula for numerical approximation of right-sided mixed Riemann–Liouville fractional integrals

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DOI 10.5281/zenodo.18217101

Keywords: An optimal quadrature formula; Cubature formula; Riemann-Liouville fractional integral; Hilbert space; Optimal coefficients.

1. Introduction

Numerical integration has a long and vital history. It began with simple one-dimensional quadrature rules and has evolved into advanced multidimensional cubature formulas tailored to specific function types.

Classical Cubature Theory: The term "cubature formula"was introduced to describe the numerical approximation of multiple integrals. Although Newton, Cotes, and Gauss had already developed one-dimensional quadrature theory, extending these ideas to higher dimensions proved to be much more difficult. Radon [19] made key contributions to this field in 1948 by developing the theory of algebraic precision for cubature formulas. He also described the properties of two-variable orthogonal polynomials used to construct Gaussian cubature formulas. Sobolev [25,26] and Mysovskikh [13] later expanded the theory. Mysovskikh focused on interpolatory cubature formulas and the consistent properties of integration domains. Stroud's [28] well-known monograph is still a key reference, listing many cubature formulas for standard regions like cubes, spheres, and simplexes, along with different weight functions, mostly organized by their algebraic degree of precision.

Optimal Quadrature and Cubature Formulas: Sard [?, ?, ?] and Sobolev [27] changed the field by suggesting that integration formulas should be built by minimizing the error functional in a chosen function space, such as Banach or Hilbert spaces, instead of focusing on exactness for polynomials of a certain degree. Sobolev created the theory of quadrature formulas with boundary corrections in space $L_2^{(m)}$ and showed that the best coefficients solve a system of linear discrete Wiener-Hopf equations. This method allows precise error bounds to be given using the norm of the error functional. Building on this work, Shadimetov [23] and his group developed the discrete operator method to find optimal coefficients in different Sobolev spaces. They used this method to create optimal cubature formulas for 2D and 3D domains with tensor products and direct minimization techniques [24].

Numerical Methods for Fractional Integrals: As fractional calculus has become more widely used, there is a growing need for accurate numerical methods for fractional integrals with weakly singular kernels. Diethelm et al. [2,3] introduced the product-integration method and the fractional Adams-Bashforth-Moulton schemes. Li and Zeng [10] organized numerical methods for fractional calculus into categories such as finite difference and spectral methods. Most current methods address only one-dimensional problems. For two-dimensional fractional integrals, researchers often use simple tensor products of basic methods or finite element techniques [9].

Fractional calculus is a useful mathematical tool for modeling complex systems that have memory and spatial differences. Unlike traditional models, fractional-order operators can describe long-range interactions, hereditary effects, and unusual transport behaviors [8,16].

Classical diffusion theory, described by the heat equation $\frac{\partial u}{\partial t} = D\Delta u$, assumes particles move randomly, leading to a Gaussian distribution. But experiments in geophysics, biology, and material science have shown anomalous diffusion, where the mean squared displacement changes in a nonlinear way: $\langle x^2(t)\rangle \sim t^\gamma$ $(\gamma \neq 1)$ [14].

To better describe these effects, the RiemannвЂ“Liouville fractional integral is used instead of local operators. This operator includes non-local spatial memory, which is important for modeling processes with spatial differences [12]. While one-dimensional models are basic, many real processes happen in two dimensions, such as:

- **Bio-transport in tissues:** Drug diffusion exhibiting non-Fickian behavior due to complex cell geometry [11].
- **Heat conduction in composites:** Anomalous heat flow in layered or fibrous materials, where fractional spatial models better capture energy propagation [17].
- **Contaminant transport:** Pollutants diffusing through porous media exhibit multi-directional heterogeneity [15].
- **Image processing:** Texture modeling where edge-preservation and long-range correlations are critical [18].

It is difficult to calculate these integrals because the kernel has a weak singularity. This paper develops optimal cubature formulas to efficiently solve this problem, building on recent progress in one-dimensional optimal formulas [1,4,5,6,7].

2. Mathematical preliminaries

The right-sided Riemann-Liouville fractional integral of order α for $0 < \alpha < 1$ is defined as:

$$(I^{\alpha}_{b^-} f)(x) = \frac{1}{\Gamma(\alpha)} \int_x^b (t-x)^{\alpha-1} f(t)\, dt.$$

Recent research has created optimal quadrature formulas in Sobolev and Hilbert spaces to approximate this integral:

$$I^{\alpha}_{b^-} f(x) \approx \sum_{k=0}^{N} C_k^{(\alpha)} f(t_k),$$

In this formula, t_k are the nodes and $C_k^{(\alpha)}$ are the optimal weights that minimize the error norm.

To build on this progress, the optimal quadrature formula of [4] is briefly reviewed, as it serves as the basis for this study.

3. Construction of optimal quadrature formulas for 1D fractional integrals

We briefly describe optimal 1D quadrature formulas for the right-sided Riemann–Liouville fractional integral in $W_2^{(1,0)}$ space, which, combined by tensor product, construct optimal 2D cubature formulas.

We consider the quadrature formula

$$\int_x^b \frac{\varphi(t)}{(t-x)^{1-\alpha}} dt \approx \sum_{\beta=0}^{N} C_\beta \varphi(t_\beta), \tag{1}$$

where x and b are endpoints of the interval of integration, $\varphi(t)$ is the integrand, α is a parameter such that $0 < \alpha < 1$, C_β are the quadrature weights, $t_\beta = x + \beta h$ ($\beta = 0, \ldots, N$) are the quadrature nodes, and $h = (b-x)/N$. The function φ belongs to the Hilbert space $W_2^{(1,0)}[x,b]$, equipped with the norm $\|\varphi\|_{W_2^{(1,0)}} = \sqrt{\int_x^b (\varphi'(t) + \varphi(t))^2 dt}$.

The error of the quadrature formula is a linear functional $\ell \in W_2^{(1,0)*}$ defined as

$$(\ell, \varphi) = \int_x^b \frac{\varphi(t)}{(t-x)^{1-\alpha}} dt - \sum_{\beta=0}^{N} C_\beta \varphi(t_\beta).$$

The problem of constructing the optimal quadrature formula reduces to finding the coefficients C_β that minimize the norm of the error functional $\|\ell\|_{W_2^{(1,0)*}}$. Using the extremal function method, the squared norm of the error functional is derived as:

$$\|\ell\|^2 = (\ell, \psi_\ell), \quad \text{where} \quad \psi_\ell(t) = -\ell(t) * G_1(t) + de^{-t}.$$

Here, $G_1(t) = \frac{\operatorname{sgn}(t)}{2} \sinh(t)$ is the Green's function of the differential operator $L = d^2/dt^2 - 1$.

Minimizing $\|\ell\|^2$ under the condition that the formula is exact for e^{-t} leads to a system of linear equations for the optimal coefficients. To solve this system analytically, we employ the discrete analogue of the differential operator $d^2/dt^2 - 1.$, defined as:

$$D_1(h\beta) = \frac{1}{1-e^{2h}} \begin{cases} 2(1+e^{2h}), & \beta = 0, \\ -2e^h, & |\beta| = 1, \\ 0, & |\beta| \geq 2. \end{cases}$$

Using the convolution of the discrete operator with the right-hand side of the system, the explicit analytical forms of the optimal coefficients C_β are obtained (see Hayotov and Babaev [4] for complete details).

Theorem 1. *The coefficients C_β of the optimal quadrature formula (1) in the space $W_2^{(1,0)}$ are given by:*

$$C_0 = \frac{1}{1-e^{2h}} \sum_{k=0}^{\infty} \frac{(1-(-1)^k e^{2h}) h^{\alpha+k}}{k!(\alpha+k)},$$

$$C_\beta = \frac{1}{1-e^{2h}} \sum_{k=0}^{\infty} \frac{h^{k+\alpha}}{k!(\alpha+k)} \Bigg[(\beta+1)^{\alpha+k}(e^{-h\beta} - (-1)^k e^{h(\beta+2)}) + (\beta-1)^{\alpha+k}(e^{-h(\beta-2)} -$$

$$-(-1)^k e^{h\beta}) \quad + \beta^{\alpha+k}(1+e^{2h})((-1)^k e^{h\beta} - e^{-h\beta}) \Bigg], \quad \beta = 1, \ldots, N-1,$$

$$C_N = \frac{1}{1-e^{2h}} \sum_{k=0}^{\infty} \frac{(e^{x-b+2h} - (-1)^k e^{b-x})((b-x-h)^{\alpha+k} - (b-x)^{\alpha+k})}{k!(\alpha+k)}.$$

These coefficients C_β (denoted as $C_i^{(\alpha_1)}$ and $C_j^{(\alpha_2)}$ in the 2D formulation) are used to construct the tensor product weights $C_{i,j} = C_i^{(\alpha_1)} C_j^{(\alpha_2)}$.

4. Construction of cubature formulas

Consider $\Omega = [a_1, b_1] \times [a_2, b_2]$ as a subset of R^2. The two-dimensional right-sided mixed RiemannвЂ“Liouville fractional integral of order (α_1, α_2) is defined by:

$$(I^{\alpha_1,\alpha_2}_{b_1^-,b_2^-} f)(x,y) = \frac{1}{\Gamma(\alpha_1)\Gamma(\alpha_2)} \int_x^{b_1} \int_y^{b_2} (t-x)^{\alpha_1-1}(s-y)^{\alpha_2-1} f(t,s)\, ds\, dt, \tag{2}$$

where $0 < \alpha_1, \alpha_2 < 1$. This operator can be viewed as a tensor product of 1D operators: $I^{\alpha_1,\alpha_2} = I_x^{\alpha_1} \otimes I_y^{\alpha_2}$.

To approximate the 2D integral (2), we utilize the tensor product of the 1D optimal quadrature rules described in (1).

Let us define a grid on the integration domain $[x, b_1] \times [y, b_2]$:

$$t_i = x + ih_1, \quad h_1 = \frac{b_1 - x}{N}, \quad i = 0, \ldots, N,$$
$$s_j = y + jh_2, \quad h_2 = \frac{b_2 - y}{M}, \quad j = 0, \ldots, M.$$

The cubature formula gives the approximate value of the integral:

$$(I^{\alpha_1,\alpha_2}_{b_1^-,b_2^-} f)(x,y) \approx S_{N,M}(f) = \sum_{i=0}^{N} \sum_{j=0}^{M} C_{i,j}^{(\alpha_1,\alpha_2)} f(t_i, s_j). \tag{3}$$

Due to the separable nature of the kernel $(t-x)^{\alpha_1-1}(s-y)^{\alpha_2-1}$, the optimal coefficients $C_{i,j}^{(\alpha_1,\alpha_2)}$ factorize into the product of 1D optimal coefficients:

$$C_{i,j}^{(\alpha_1,\alpha_2)} = C_i^{(\alpha_1)} \cdot C_j^{(\alpha_2)}.$$

Here, $C_i^{(\alpha_1)}$ and $C_j^{(\alpha_2)}$ are the optimal weights for the 1D integrals over $[x, b_1]$ and $[y, b_2]$ respectively. This factorization significantly reduces the computational complexity from finding $(N+1)(M+1)$ unknowns to finding $(N+1)+(M+1)$ unknowns.

5. Numerical results and discussion

We test the proposed optimal cubature formula (3) on a function with an analytical solution. The numerical experiments in Python use the optimal weights C_β described in Theorem 3.1.

We calculate the right-sided mixed Riemann-Liouville fractional integral numerically:

$$(I^{\alpha_1,\alpha_2}_{b_1^-,b_2^-} f)(x,y) = \frac{1}{\Gamma(\alpha_1)\Gamma(\alpha_2)} \int_x^{b_1} \int_y^{b_2} (t-x)^{\alpha_1-1}(s-y)^{\alpha_2-1} f(t,s)\, ds\, dt.$$

The integration domain is $\Omega = [0,1] \times [0,1]$, which means $x = 0$, $y = 0$, $b_1 = 1$, and $b_2 = 1$. We choose a test function of the following form:

$$f(t,s) = (1-t)^\mu (1-s)^\nu, \quad \mu, \nu > 0.$$

This function is chosen because its exact fractional integral can be derived analytically using the properties of the Beta and Gamma functions:

$$I_{exact} = \frac{\Gamma(\mu+1)}{\Gamma(\alpha_1+\mu+1)}(1-x)^{\alpha_1+\mu} \times \frac{\Gamma(\nu+1)}{\Gamma(\alpha_2+\nu+1)}(1-y)^{\alpha_2+\nu}.$$

For our experiment, we set the parameters as follows: $\mu = 2$, $\nu = 2$, and the fractional orders $\alpha_1 = 0.5$, $\alpha_2 = 0.5$. The exact value of the integral at $(x,y) = (0,0)$ is:

$$I_{exact} = \left(\frac{\Gamma(3)}{\Gamma(3.5)}\right)^2 \approx 0.518679.$$

We compute the approximate value of the integral $S_{N,M}(f)$ using the proposed optimal cubature formula for different numbers of grid points $N = M$. The optimal coefficients $C_i^{(\alpha_1)}$ and $C_j^{(\alpha_2)}$ are calculated using the infinite series representations derived in Theorem 3.1.

The table presents the comparison between the exact value and the approximate result, along with the absolute error $\varepsilon = |I_{exact} - S_{N,M}(f)|$.

N = M	**Exact Value**	**Approximate Value**	**Absolute Error**
10	1.137777778	1.142777077	0.4999299×10^{-2}
20	1.137777778	1.139042239	0.1264461×10^{-2}
40	1.137777778	1.138096615	0.318837×10^{-3}
80	1.137777778	1.137859052	0.81274×10^{-4}

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Measuring International Efficiency: Applications of Data Envelopment Analysis in International Relations

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DOI 10.5281/zenodo.18217118

Evaluating the performance of states, international organizations and global policies remains a persistent challenge in the field of International Relations (IR). Traditional assessment methods often struggle to account for the complexity of inputs and outputs involved in international cooperation, diplomacy and development. This paper explores the application of Data Envelopment Analysis (DEA)—a non-parametric efficiency measurement technique originally developed in operations research—as a tool for assessing the relative efficiency of international actors. By examining how DEA can be used to compare resource use and policy outcomes across multiple contexts, the study aims to demonstrate its value in identifying best practices and performance gaps. Through theoretical analysis and empirical case studies, this paper argues that DEA offers a systematic and replicable framework for evaluating efficiency in IR, providing insights often overlooked by conventional methods.

Introduction

In an increasingly complex and interconnected international system, evaluating the performance of states, international organizations and transnational policies has become

a central concern for scholars and practitioners alike. Whether assessing the effectiveness of foreign aid, the success of climate diplomacy or the operational output of multilateral institutions, the challenge lies in identifying objective, comparable and replicable methods to measure **efficiency**. Traditional evaluations often rely on qualitative or aggregated metrics that fail to capture the nuanced interplay between inputs (such as resources, diplomatic capital or economic aid) and outputs (such as developmental outcomes, agreements reached or conflict mitigation). As global challenges intensify, so too does the need for robust tools that can provide insight into the relative performance of international actors across a wide range of domains.

Data Envelopment Analysis (DEA), originally developed in the field of operations research by Charnes, Cooper, and Rhodes (1978), is a non-parametric method used to assess the relative efficiency of decision-making units (DMUs) based on multiple input and output variables. Unlike parametric approaches, DEA does not require a predefined functional form, making it especially suitable for evaluating performance in diverse, multi-dimensional contexts. While DEA has been extensively applied in sectors such as healthcare, education and public administration, its potential in International Relations (IR) remains underexplored. This paper proposes that DEA can be a valuable methodological addition to the IR scholar's toolkit, offering a means to quantitatively assess how international actors utilize resources to achieve policy goals.

The primary aim of this study is to explore how Data Envelopment Analysis can be applied to the field of International Relations in order to assess the efficiency of states and institutions in pursuing foreign policy objectives, development goals and multilateral cooperation efforts. Through both theoretical discussion and empirical examples, this paper examines the benefits and limitations of DEA as a framework for efficiency analysis in IR.

This investigation is guided by the following research questions:

1. In what ways can DEA provide meaningful insights into the performance of international actors?
2. How does DEA compare to traditional evaluation methods used in IR?
3. What are the methodological challenges and theoretical implications of applying DEA to IR contexts?

This paper argues that DEA provides a powerful framework for evaluating efficiency in international relations. By facilitating the comparative analysis of inputs and outputs across diverse actors and policies, DEA reveals insights often overlooked by conventional evaluation methods. In doing so, it enables a more nuanced understanding of how international actors translate resources into strategic and developmental outcomes.

Literature Review

Data Envelopment Analysis (DEA) was first introduced by **Charnes, Cooper, and Rhodes (1978)** as a method for evaluating the relative efficiency of decision-making units (DMUs) based on multiple inputs and outputs. The original CCR model assumed constant returns to scale and laid the groundwork for non-parametric frontier analysis. This model was later extended by **Banker, Charnes, and Cooper (1984)**, who developed the

BCC model to account for variable returns to scale, allowing for greater flexibility in real-world applications where scale efficiency may vary. DEA has since evolved into a widely used analytical tool, especially in contexts where benchmarking performance across heterogeneous units is essential.

Beyond its origins in operations research and production analysis, DEA has been extensively applied in public policy domains, including healthcare, education, transportation and environmental policy. These applications demonstrate DEA's capacity to manage multidimensional data and provide efficiency scores for organizations or systems where inputs and outputs are not easily reduced to single metrics. In international contexts, DEA has been utilized to compare national education systems, public health outcomes and the administrative efficiency of public institutions. Studies have shown that DEA can effectively benchmark performance, identify best practices and inform resource allocation decisions (e.g., Thanassoulis et al., 2001; Emrouznejad & Yang, 2018).

Although DEA has gained traction in public administration and global development research, its application in core International Relations (IR) scholarship remains relatively limited. However, several studies demonstrate its potential. In the area of **foreign aid evaluation**, DEA has been employed to assess the efficiency of donor countries by comparing aid disbursements (inputs) against developmental outcomes (outputs) (e.g., Wane, 2004). Similarly, DEA has been used to evaluate **military spending efficiency**, particularly in NATO member states, by comparing defense budgets with strategic or security outcomes (e.g., Afonso & Aubyn, 2006). **Climate policy** is another emerging area, where DEA models have assessed the environmental performance of countries based on carbon emissions and energy use relative to environmental investments.

Despite these promising applications, the integration of DEA into mainstream IR scholarship remains underdeveloped. Most studies employing DEA in international contexts do so from an economic or public administration perspective, with little engagement with IR theory or strategic considerations. There is a notable **lack of theoretical integration**, where efficiency analysis is not linked to foundational IR paradigms such as realism, liberal institutionalism or constructivism. Furthermore, DEA is rarely applied to evaluate **diplomatic networks**, **multilateral engagement** or **strategic performance**—domains central to contemporary IR. This methodological gap presents an opportunity to broaden the analytical tools available to IR scholars and policymakers by incorporating efficiency analysis into the evaluation of international behavior and institutional performance.

Theoretical Framework

Data Envelopment Analysis (DEA) is a non-parametric linear programming method used to assess the relative efficiency of *decision-making units* (DMUs) that convert multiple inputs into multiple outputs. In the context of international relations (IR), DMUs may include states, international organizations or transnational programs. DEA estimates the **efficiency frontier**—the set of best-performing units—against which all other units are compared.

Consider n decision-making units (DMUs), each using m inputs to produce s outputs. Let:

- x_{ij} = amount of input i used by DMU j

- y_{rj} = amount of output r produced by DMU j
- x_{i0}, y_{r0} = inputs and outputs of the DMU under evaluation (DMU_0)

The most basic form of DEA, the **CCR model** (Charnes, Cooper and Rhodes, 1978), assumes constant returns to scale. The efficiency of a DMU is calculated as the ratio of a weighted sum of outputs to a weighted sum of inputs:

$$\text{Efficiency}_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$$

Subject to:

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad \text{for all } j = 1, ..., n \; u_r, v_i \geq 0$$

Where:

- u_r and v_i are the weights for outputs and inputs, respectively

To address situations where efficiency may vary with scale, the **BCC model** (Banker, Charnes and Cooper, 1984) incorporates **variable returns to scale** by adding a convexity constraint, allowing more realistic comparisons in settings where size matters (e.g., comparing large and small states or institutions).

BCC Model (Variable Returns to Scale)

Input-Oriented BCC Model : The objective is to **minimize inputs** while maintaining at least the observed output levels.

$$\begin{aligned}
\min_{\theta,\lambda} \quad & \theta \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{i0}, \quad i = 1, \ldots, m, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{r0}, \quad r = 1, \ldots, s, \\
& \sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \geq 0, \quad j = 1, \ldots, n.
\end{aligned}$$

Here:

- $\theta \in (0, 1]$ is the **pure technical efficiency score**.
- The constraint $\sum_{j=1}^{n} \lambda_j = 1$ imposes **variable returns to scale**.

Output-Oriented BCC Model: The objective is to **maximize outputs** given the

current input levels.

$$\begin{aligned}
\max_{\varphi,\lambda} \quad & \varphi \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{i0}, \quad i = 1, \ldots, m, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geq \varphi y_{r0}, \quad r = 1, \ldots, s, \\
& \sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \geq 0, \quad j = 1, \ldots, n.
\end{aligned}$$

where:

- $\varphi \geq 1$ measures proportional **output expansion**.

Dual (Multiplier) Form of the Input-Oriented BCC Model

$$\begin{aligned}
\max_{u,v,u_0} \quad & \sum_{r=1}^{s} u_r y_{r0} + u_0 \\
\text{s.t.} \quad & \sum_{i=1}^{m} v_i x_{i0} = 1, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0 \leq 0, \quad j = 1, \ldots, n, \\
& u_r \geq 0, ; v_i \geq 0.
\end{aligned}$$

where:

- u_r and v_i are output and input weights,
- u_0 is a **free (unrestricted) variable** capturing scale effects.

Key Interpretation

- The **BCC efficiency score** measures **pure technical efficiency**, excluding scale inefficiency.
- Comparison with the CCR model allows decomposition into:

$$\text{Scale Efficiency} = \frac{\text{CCR Efficiency}}{\text{BCC Efficiency}}.$$

DEA thus provides a flexible, data-driven method for comparing efficiency across diverse units without requiring a pre-specified functional form—an important feature when applying it to the complexity of international systems.

In the figure 1, we compare the **CCR** and **BCC** models of DEA. The left panel illustrates the **CCR model (Constant Returns to Scale)**, where the efficiency frontier is a single ray from the origin, implying that outputs change proportionally with inputs; DMUs located on the frontier (e.g., A, B, C) are technically efficient, while interior points

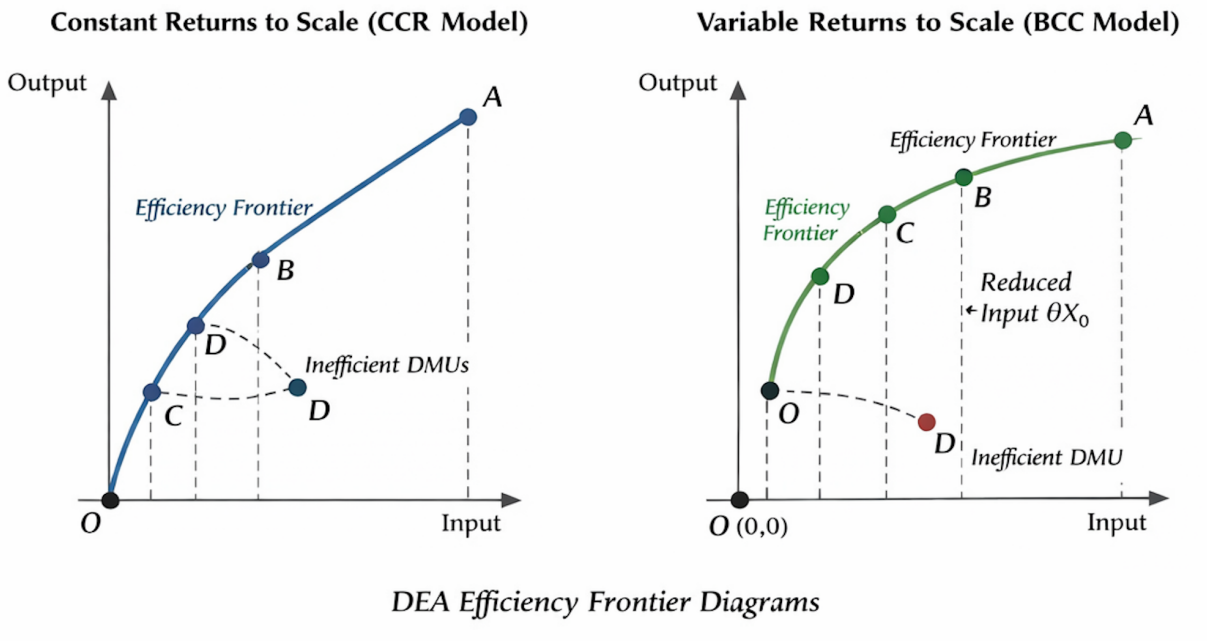

Fig 1.DEA efficiency frontier diagrams (CCR vs. BCC

(e.g., D) are inefficient and could proportionally reduce inputs (or increase outputs) to reach the frontier. In contrast, the right panel presents the **BCC model (Variable Returns to Scale)**, where the frontier is curved, allowing for increasing and decreasing returns to scale; efficiency is assessed **conditional on scale**, so an inefficient DMU (D) is projected to the frontier via an input contraction θX_θ that reflects **pure technical inefficiency** rather than scale effects. The comparison highlights that CCR efficiency conflates technical and scale efficiency, whereas BCC isolates **pure technical efficiency**, making BCC more appropriate when countries (or DMUs) operate at heterogeneous scales—common in international trade analyses.

Linking DEA to International Relations Theory

While DEA originates in operations research, its application in IR invites theoretical engagement with key paradigms in the discipline:

- **Realism** emphasizes power and material capabilities as the primary drivers of international behavior. DEA can be used to assess the *efficiency of power projection*—for instance, how states convert military spending or diplomatic effort into strategic outcomes (e.g., deterrence, alliances, influence). From a realist lens, efficiency becomes a proxy for state capacity and survival in an anarchic system.
- **Liberalism** focuses on institutions, cooperation and interdependence. DEA is well suited to measure the *effectiveness of institutional mechanisms*, such as how efficiently international organizations use financial and human capital to produce public goods (e.g., development, peacekeeping, health responses). It can also help evaluate how states contribute to and benefit from international regimes.
- **Constructivism** foregrounds the role of norms, identity and discourse in shaping state behavior. While DEA is a quantitative method, it can be adapted to include norm-based outputs, such as compliance with international agreements, soft power initiatives, or normative leadership. In this context, DEA can help assess how states

efficiently align their material capabilities with ideational goals.

By aligning DEA with these theoretical lenses, scholars can go beyond simple efficiency scores and interpret results through a broader conceptual framework-bridging the quantitative with the qualitative dimensions of IR.

DEA offers practical value in international policy analysis by providing **benchmarking and optimization** tools. Policymakers and international organizations often operate under constraints—limited resources, competing priorities and political pressures. DEA enables them to identify efficient performers, diagnose inefficiencies, and simulate scenarios for improving outcomes without necessarily increasing inputs.

For example, in **global governance**, DEA can help multilateral institutions assess which member states or programs are achieving the greatest impact per unit of investment. In **foreign aid**, it can inform more strategic allocation of resources by identifying which donors are delivering the most effective development outcomes. In **diplomacy**, it may help assess the productivity of diplomatic missions relative to their staffing, budgets and international influence.

DEA introduces a **rigorous, comparative and multidimensional** approach to performance evaluation in IR. Its integration with theoretical perspectives enhances both methodological robustness and conceptual depth, making it a valuable addition to the field's analytical toolkit.

Methodology

This study employs a **comparative efficiency analysis** using **Data Envelopment Analysis (DEA)** to evaluate the performance of selected international actors across policy domains. DEA is well-suited for this purpose, as it accommodates multiple input and output variables and allows for the identification of relatively efficient decision-making units (DMUs) without assuming a predefined production function. The research design is cross-sectional and comparative, focusing on how states or organizations utilize resources (inputs) to achieve international policy outcomes (outputs).

The objective is not to rank actors by absolute performance but to determine which actors form the *efficiency frontier* and how others perform relative to this benchmark.

DEA Case Study in International Trade

In this DEA-based case study, each country is treated as a decision-making unit that transforms **economic and structural resources**—GDP, labor, and capital—into **international trade outcomes**, measured by exports and imports.

GDP reflects the overall economic capacity and production scale supporting trade activities. **Labor** captures the human resources engaged in production, logistics, and trade-related services, while **capital** represents infrastructure, investment, and technological capability that facilitate participation in global markets.

On the output side, **exports** indicate a country's external competitiveness and ability to supply goods and services internationally, whereas **imports** reflect the depth of integration into the global economy and access to foreign inputs and technologies.

From an international relations perspective, a country is considered **trade-efficient** if it achieves relatively high export and import volumes given its economic size and factor endowments. Inefficiency suggests that existing economic resources are not fully translated into international trade performance, potentially due to **trade policy barriers, weak institutions, limited connectivity, geopolitical constraints or underdeveloped regional cooperation**.

Thus, DEA results provide not only an economic efficiency measure but also an **indirect assessment of a country's effectiveness in leveraging international economic relations and trade diplomacy**.

Data Description and DEA Input–Output Specification

This study employs **Data Envelopment Analysis (DEA)** as a non-parametric frontier technique to evaluate the **relative trade efficiency of countries**. DEA is particularly appropriate for cross-country analysis in international economics, as it allows for the simultaneous consideration of multiple inputs and outputs without imposing a predefined functional form on the production technology. Each country is treated as a **Decision-Making Unit (DMU)** that transforms economic resources into trade outcomes.

The selection of inputs and outputs is guided by **international trade theory**, **macroeconomic growth models**, and prior empirical DEA applications in international relations and economics.

Input Variables: The input variables represent the **productive capacity and resource endowments** that underpin a country's ability to participate in international trade.

1. **Gross Domestic Product (GDP)** GDP is used as a proxy for overall economic size and production potential. In the context of trade efficiency, GDP reflects the scale of domestic economic activity, industrial capacity, and the ability to generate tradable goods and services. Larger economies typically possess broader production bases and more diversified export structures; however, DEA allows assessment of whether such scale advantages are translated efficiently into trade outcomes.
2. **Labor** Labor represents the human input available for production and trade-related activities. It captures workforce size and, indirectly, the capacity for manufacturing, services, logistics, and export-oriented industries. Including labor as an input aligns with classical and neoclassical trade theories, where labor is a fundamental factor of production influencing comparative advantage.
3. **Capital** Capital reflects the stock of physical and financial resources used in production, such as machinery, infrastructure, and investment assets. Capital accumulation enhances productivity, supports export competitiveness, and facilitates integration into global value chains. In DEA, capital serves as a critical input capturing long-term investment capacity and structural economic strength.

Output Variables: The output variables measure **actual trade performance**, reflecting how effectively countries convert economic resources into international trade flows.

1. **Exports** Exports represent a country's ability to supply goods and services to international markets. As a core indicator of outward trade performance, exports capture competitiveness, production efficiency, and global demand for domestic output. Higher export levels, given comparable inputs, indicate greater trade efficiency.

2. **Imports** Imports reflect integration into the global economy and access to foreign goods, intermediate inputs, and technologies. From an efficiency perspective, imports signal a country's capacity to participate in international exchange and global value chains. Including imports as an output recognizes that efficient trade systems involve both export capability and effective sourcing from international markets.

Conceptual Interpretation in DEA Context: Within the DEA framework, a country is considered **efficient** if it achieves relatively high levels of exports and imports given its GDP, labor, and capital endowments. Inefficiency, conversely, indicates unrealized trade potential—suggesting institutional, structural, logistical, or policy-related constraints that prevent the country from fully leveraging its economic resources in international trade. This input–output specification enables a **balanced assessment of trade efficiency**, accounting for both production capacity (inputs) and realized trade engagement (outputs), and provides a robust foundation for comparative analysis across countries and regions.

All input and output variables used in the DEA model are obtained from the World Bank's World Development Indicators (WDI) database, which provides standardized, internationally comparable macroeconomic and trade statistics: `https://databank.worldbank.org/source/world-development-indicators`

Table 1. Sample Data for DEA Analysis (2024)

country	gdp	labor	capital	exports	imports
Bangladesh	4.501194e+11	77355168	1.381889e+11	4.708784e+10	7.344548e+10
China	1.874380e+13	773879678	7.473099e+12	3.753056e+12	3.219343e+12
India	3.909892e+12	607691498	1.168910e+12	8.274052e+11	9.183337e+11
Indonesia	1.396300e+12	143143940	4.069598e+11	3.097456e+11	2.846963e+11
Iran, Islamic Rep.	4.752521e+11	28574661	1.335799e+11	1.119289e+11	1.354975e+11
Japan	4.027598e+12	69382089	1.050562e+12	9.170437e+11	9.520416e+11
Kazakhstan	2.914803e+11	10285416	7.088095e+10	9.207035e+10	7.454369e+10
Korea, Rep.	1.875388e+12	29713473	5.631673e+11	8.318892e+11	7.554862e+11
Kyrgyz Republic	1.747826e+10	3196163	4.192528e+09	7.593079e+09	1.471802e+10
Malaysia	4.222270e+11	18264000	8.673690e+10	3.012213e+11	2.787605e+11
Pakistan	3.715700e+11	83643815	4.256903e+10	3.860781e+10	6.386551e+10
Philippines	4.616175e+11	50979290	1.088056e+11	1.189747e+11	1.851642e+11
Saudi Arabia	1.239805e+12	17168015	3.549347e+11	3.608973e+11	3.170117e+11
Singapore	5.473866e+11	3722265	1.198585e+11	9.785975e+11	7.860206e+11
Sri Lanka	9.896319e+10	8498980	1.859817e+10	1.968014e+10	2.227778e+10
Thailand	5.265177e+11	40623017	1.171136e+11	3.688242e+11	3.511735e+11
Turkiye	1.359124e+12	36080817	4.256195e+11	3.746863e+11	3.673791e+11
Turkmenistan	5.138721e+10	2444797	9.063429e+09	9.030078e+09	5.735835e+09
Uzbekistan	1.149653e+11	13974298	4.260144e+10	2.617276e+10	4.364272e+10
Viet Nam	4.763882e+11	57133476	1.382558e+11	4.294841e+11	3.987741e+11

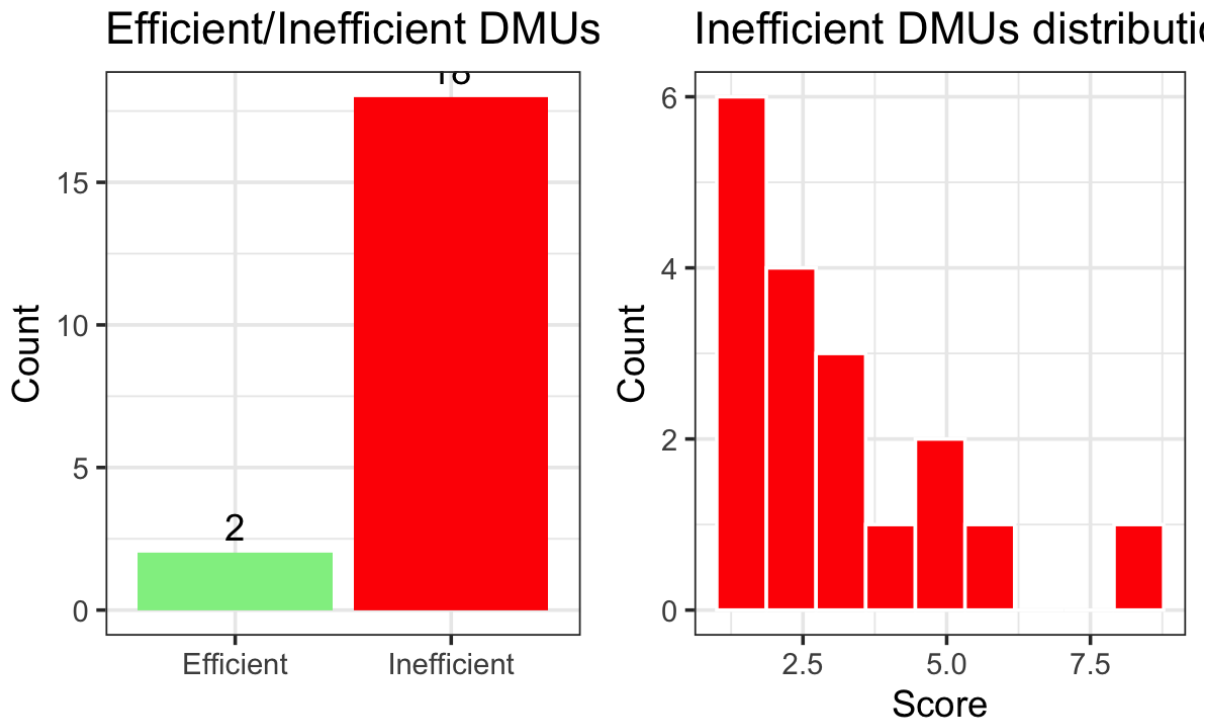

Fig 2. Distribution of Efficient and Inefficient DMUs

Analysis and Results

Figure 2 summarizes the classification of decision-making units (DMUs) into efficient and inefficient categories based on the output-oriented DEA trade efficiency model. Out of the total sample, only **two countries are identified as efficient**, lying on the DEA frontier, while **eighteen countries are classified as inefficient**. This pronounced imbalance indicates that the majority of Asian economies in the sample operate **below the best-practice trade frontier**, given their existing levels of economic inputs. The small number of efficient DMUs reflects the **stringent nature of DEA benchmarking**, where only countries that exhibit best-practice combinations of inputs and outputs are assigned full efficiency. These frontier countries serve as reference points for all inefficient DMUs and play a crucial role in defining the production possibility frontier. From an international relations perspective, this result suggests that **trade efficiency is relatively rare**, even among economically significant countries. Most states appear unable to fully convert their economic capacity into proportional trade outcomes, highlighting the importance of structural, institutional, and policy factors beyond mere resource endowments.

Figure 2 presents the distribution of inefficiency scores among the inefficient DMUs. The histogram reveals a **right-skewed distribution**, with a concentration of countries exhibiting **moderate inefficiency levels**, and a smaller number of countries displaying **high inefficiency scores**. Most inefficient DMUs cluster in the lower range of scores, indicating that while they do not operate on the efficiency frontier, their deviation from best practice is relatively limited. These countries may require **incremental improvements** in trade facilitation, export diversification or institutional quality to approach the frontier. In contrast, a small subset of countries exhibits **substantially**

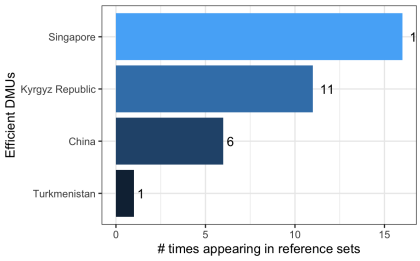

Figure 3.DEA reference sets

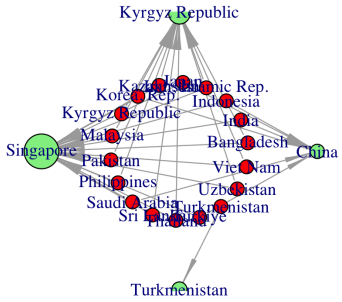

Figure 4. Peer Reference Network

higher inefficiency scores, suggesting more profound structural mismatches between inputs and trade outcomes. For these countries, inefficiency is likely driven by deeper constraints, such as limited integration into global value chains, weak logistics infrastructure, or restrictive trade policies. Addressing such inefficiencies would require **comprehensive structural reforms** rather than marginal adjustments.

Taken together, the two figures highlight that **inefficiency is the dominant empirical outcome** in the sample, but its **severity varies considerably across countries**. While only a few countries define the efficiency frontier, most inefficient DMUs are not extremely distant from it, implying significant scope for improvement without expanding resource inputs.

In policy terms, the results indicate that improving trade efficiency in Asia is less a question of increasing economic size and more a matter of **enhancing the effectiveness with which existing resources are utilized**. From an international relations standpoint, these findings underscore the role of trade governance, regional integration, and institutional coordination in shaping countries' relative performance in international trade.

The figure 3 shows how often each country appears in the **DEA reference (peer) sets**, meaning how frequently it serves as a **benchmark for inefficient countries**. **Singapore** appears most frequently, indicating that it represents a best-practice trade structure and is the dominant efficiency benchmark in the sample. The **Kyrgyz Republic** also appears often, suggesting that despite its smaller economic size, its trade performance is relatively efficient given its GDP, labor and capital endowments. **China's** moderate number of appearances reflects selective benchmarking—efficient in certain dimensions but not universally dominant across all comparisons. **Turkmenistan's** rare appearance implies limited benchmarking relevance, indicating that its trade outcomes are less efficient relative to its input endowments. Overall, higher frequencies signal stronger **role-model status in international trade efficiency**, rather than sheer economic size.

Figure 4 illustrates the **DEA peer (reference) network** derived from the output-oriented VRS trade efficiency model for Asian countries. In this network representation, **nodes correspond to countries**, while **directed edges indicate benchmarking**

relationships generated by the DEA solution. Specifically, an arrow from country i to country j indicates that country j serves as a **reference (peer)** for country i in constructing the efficiency frontier.

Countries highlighted as **frontier or reference units** occupy central positions in the network and receive multiple incoming links from inefficient countries. These benchmark economies represent best-practice trade performers within the sample and define the production possibility frontier against which other countries are evaluated. In the present network, countries such as **Singapore, China and the Kyrgyz Republic** emerge as prominent reference nodes, indicating that their combinations of inputs and trade outputs are frequently used to benchmark less efficient peers.

The majority of countries appear as **peripheral nodes**, characterized by outgoing links directed toward one or more benchmark countries. This structure reflects the fact that inefficient countries are evaluated as convex combinations of efficient peers rather than being compared to a single benchmark. The density of outgoing links suggests that inefficiency is not driven by a single dominant performer but instead by multiple best-practice trade patterns within the region.

From an economic interpretation perspective, the peer relationships reveal **heterogeneous pathways to trade efficiency**. For example, some countries benchmark primarily against large, globally integrated economies, while others reference smaller but highly open economies. This implies that trade efficiency in Asia is not solely determined by economic size, but also by structural factors such as openness, export composition and integration into regional and global value chains.

The network structure further highlights **regional clustering effects**. Several Central and South Asian economies share common reference countries, indicating similar trade inefficiency profiles and comparable adjustment paths toward the frontier. This finding suggests that policy learning and regional cooperation may play a role in improving trade efficiency through the diffusion of best practices.

Overall, the DEA peer network complements the efficiency score and slack analyses by providing **qualitative insight into benchmarking relationships**. While efficiency scores quantify the degree of inefficiency, the network identifies **which countries serve as role models** and **how inefficient countries are positioned relative to the frontier**. Together, these results enable a richer interpretation of trade performance and offer a basis for targeted, country-specific policy recommendations.

Figure **??**. presents the results of the output-oriented DEA analysis of trade efficiency for selected Asian countries. The horizontal bars report relative efficiency scores derived from the DEA model, with higher values indicating greater trade performance relative to peer countries given comparable levels of economic inputs.

The results reveal **substantial heterogeneity in trade efficiency across Asian economies**. Countries such as **Bangladesh and Kazakhstan** appear at the upper end of the distribution, indicating that they achieve comparatively high trade outcomes relative to their input endowments. These countries can be interpreted as **best-performing reference units** within the sample, serving as benchmarks for other economies.

A middle group of countries—including **Iran, Sri Lanka, Pakistan, Uzbekistan and the Philippines**—exhibits moderate efficiency levels. These economies demonstrate reasonable trade performance but remain below the frontier defined by the most efficient

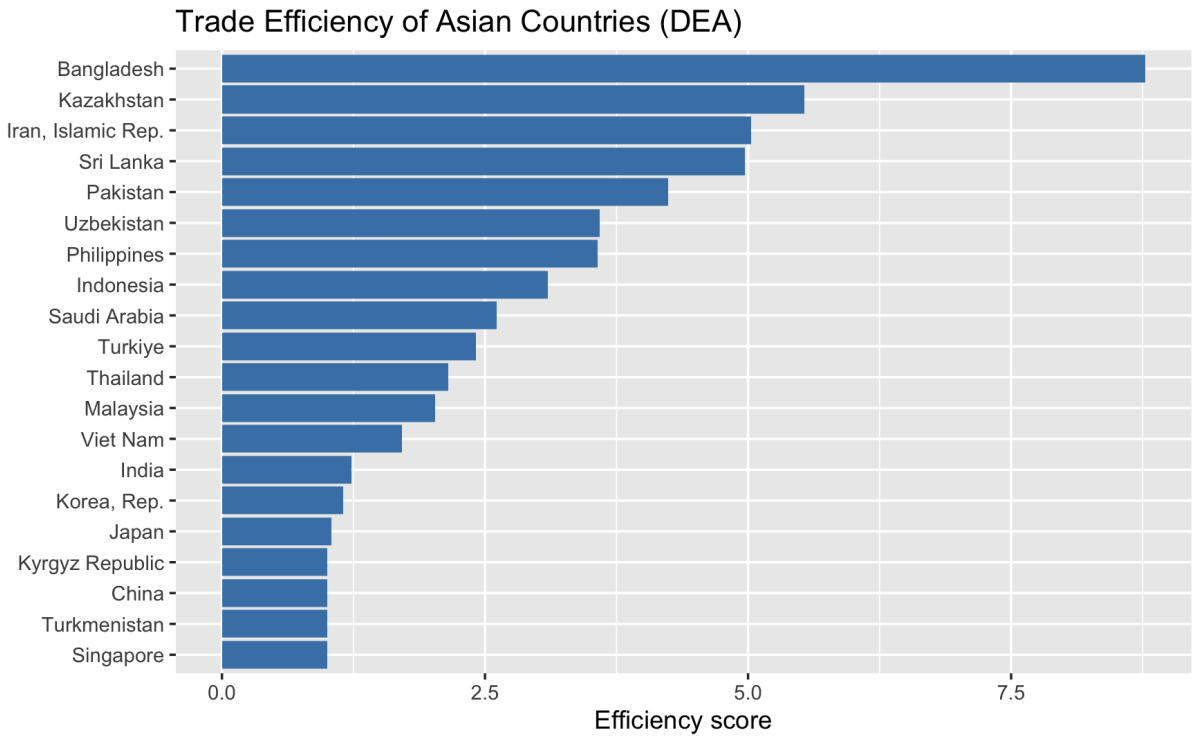

Figure 5. Trade Efficiency of Asian countries

peers. For these countries, DEA suggests the presence of **unrealized trade potential**, meaning that exports and imports could be expanded without a proportional increase in inputs such as GDP, labor force, or capital formation.

At the lower end of the efficiency distribution are countries such as **China, Singapore, Turkmenistan, and the Kyrgyz Republic**. Importantly, lower efficiency scores do **not** imply weak trade performance in absolute terms. Rather, they indicate that, relative to their substantial economic resources, trade outcomes are proportionally smaller than those observed in more efficient countries. In the DEA framework, these countries may face structural, policy or institutional constraints that limit the conversion of economic capacity into trade flows.

Trade Efficiency of Uzbekistan

The DEA-based analysis indicates that **Uzbekistan operates below the trade efficiency frontier**, suggesting the presence of **unrealized trade potential** relative to its existing economic resources. Although Uzbekistan does not emerge as an efficient decision-making unit (DMU), its efficiency score places it within the **middle range of inefficient countries**, implying that its deviation from best-practice performance is **moderate rather than extreme**.

Slack analysis reveals that Uzbekistan's inefficiency is **primarily output-driven**, rather than the result of excessive input usage. In particular, the presence of export and import slacks indicates that current levels of GDP, labor force, and capital formation could support **substantially higher trade volumes** if utilized more effectively. This finding

suggests that Uzbekistan's trade inefficiency is not rooted in insufficient resources, but rather in **structural and institutional constraints affecting trade performance**.

The DEA peer reference network further clarifies Uzbekistan's position relative to benchmark economies. Uzbekistan is repeatedly benchmarked against **Singapore, the Kyrgyz Republic and China**, indicating that its optimal adjustment path toward the efficiency frontier is defined by a combination of **high trade openness (Singapore)**, **efficient utilization of limited resources (Kyrgyz Republic)** and **scale-based integration into global markets (China)**. This combination highlights that Uzbekistan's trade efficiency gap is not attributable to a single missing factor, but to the need for coordinated improvements across trade facilitation, market access and export diversification.

Importantly, Uzbekistan does not serve as a reference country for other DMUs, which implies that its current trade structure does not yet represent a best-practice model within the regional context. However, given its moderate inefficiency and proximity to the frontier, Uzbekistan appears to be a **strong candidate for efficiency gains through targeted reforms**, rather than through large-scale expansion of economic inputs.

From an international relations perspective, the results suggest that Uzbekistan's trade performance could be significantly enhanced by strengthening **regional trade integration**, improving **logistics and customs efficiency**, and deepening participation in **global and regional value chains**. Such measures would allow Uzbekistan to convert existing economic capacity into greater international trade outcomes, thereby improving its relative position in the regional trade efficiency landscape.

Overall, the DEA findings indicate that **Uzbekistan's trade inefficiency is reversible and policy-sensitive**. By aligning trade governance and international economic strategy with the practices of its most relevant benchmark countries, Uzbekistan has the potential to move closer to the efficiency frontier without requiring proportional increases in resource endowments.

Comparative DEA Model Specifications for International Relations

This study employs a set of complementary Data Envelopment Analysis (DEA) model specifications to evaluate multiple dimensions of state performance in international relations. Rather than relying on a single efficiency metric, the analysis adopts a comparative modeling strategy that reflects the multidimensional nature of international power, encompassing economic diplomacy, soft power, diplomatic engagement, security provision, development-driven influence and composite state power. Each model treats countries as decision-making units (DMUs) and assesses how efficiently national resources are transformed into internationally relevant outcomes.

Table 2. Comparative DEA Model Specifications for International Relations Analysis with Data Sources

Model	Inputs	Outputs	IR_Focus	Interpretation	Sources

Model A: Trade Efficiency	GDP; Labor force; Capital formation	Exports; Imports	Economic diplomacy and trade performance	Efficiency of converting economic capacity into trade outcomes	WDI; OECD
Model B: Soft Power & Engagement	GDP; Education expenditure; Population	International students; Tourism arrivals; Cultural exchanges	Soft power and public diplomacy	Efficiency of generating international attractiveness	WDI; UNESCO; UNWTO
Model C: Diplomatic Network	GDP; Government expenditure	Embassies abroad; Treaties signed; IO memberships	Bilateral and multilateral diplomacy	Efficiency of diplomatic presence and engagement	WDI; UN Treaty Collection; MFA data
Model D: Security & Defense	Military expenditure; Armed forces personnel	Defense readiness; Peacekeeping missions	Security studies and defense economics	Efficiency of translating military resources into security outcomes	SIPRI; UN Peacekeeping
Model E: Development & Influence	GDP; Human capital index	FDI inflows; ODA provision; GVC participation	Development diplomacy and emerging powers	Efficiency of converting development into global influence	WDI; OECD; UNCTAD
Model F: Composite State Power	GDP; Population; Military expenditure	Trade volume; Diplomatic network size; Soft power indicators	Overall international power and influence	Balanced efficiency of economic, demographic, and military power	WDI; SIPRI; OECD; UN

Model A focuses on **trade efficiency** and captures the economic diplomacy dimension of international relations. By specifying gross domestic product, labor force and capital formation as inputs, and exports and imports as outputs, this model evaluates the capacity of states to translate economic potential into trade performance. Efficiency shortfalls in this framework indicate unrealized trade potential arising from market access barriers, structural competitiveness constraints or suboptimal trade policy design.

Model B examines **soft power and international engagement efficiency**, emphasizing non-coercive forms of influence. Inputs such as economic size, education expenditure and population are related to outputs reflecting international attractiveness, including international student mobility, tourism inflows and cultural exchanges. This

specification is particularly relevant for analyzing public diplomacy strategies and the ability of states to convert domestic social and educational investments into international visibility and appeal.

Model C addresses **diplomatic network efficiency**, evaluating how effectively states deploy economic and governmental resources to build and sustain international diplomatic presence. Outputs such as embassies abroad, international treaties and memberships in international organizations reflect both bilateral and multilateral engagement. Inefficiency in this model suggests underdeveloped diplomatic networks relative to available national resources.

Model D is designed to assess **security and defense efficiency**, a core concern in international relations and security studies. Military expenditure and armed forces personnel are treated as inputs, while outputs capture security outcomes such as defense readiness and participation in peacekeeping missions. This model enables a comparative assessment of how effectively military resources contribute to security provision and international stability.

Model E evaluates **development and global influence efficiency**, focusing on how domestic development translates into external economic and political impact. Inputs include economic size and human capital, while outputs encompass foreign direct investment inflows, official development assistance provision and participation in global value chains. This framework is particularly suitable for studying emerging powers and development diplomacy.

Model F provides a **composite state power efficiency** perspective by integrating economic, demographic and military resources on the input side with a broad set of outputs reflecting trade performance, diplomatic reach and soft power. This advanced specification captures the balanced use of national power across multiple domains and offers an overarching efficiency assessment of a state's international influence.

Across all specifications, the models rely on internationally comparable data from authoritative sources, including the World Bank's World Development Indicators, SIPRI, OECD, UN agencies, and related international databases. The use of output-oriented DEA with variable returns to scale ensures that cross-country heterogeneity in size and capacity is appropriately accounted for. Taken together, the comparative DEA framework provides a systematic and multidimensional assessment of efficiency in international relations, enabling both cross-model robustness checks and policy-relevant interpretations.

Conclusion

This policy brief applies **Data Envelopment Analysis (DEA)** to evaluate international trade efficiency across Asian countries, with particular emphasis on **Uzbekistan's regional performance**. DEA assesses how effectively countries transform key economic inputs—GDP, labor, and capital—into trade outcomes measured by exports and imports, offering a multidimensional alternative to traditional trade indicators.

The results show that **trade efficiency in Asia is not driven by economic size alone**. Several smaller economies emerge as efficiency benchmarks, demonstrating that institutional quality, trade facilitation, logistics and regional integration play a decisive role. For **Uzbekistan**, the findings suggest that despite growing economic capacity, trade outcomes remain below potential, indicating scope for efficiency gains through improved

export diversification, customs procedures, and deeper participation in regional value chains.

From a policy perspective, DEA identifies **peer benchmark countries** that provide practical models for reform. Targeted alignment with best-performing Asian economies—particularly in trade infrastructure, regulatory efficiency and trade diplomacy—can enhance Uzbekistan's international trade efficiency without requiring proportional increases in resource inputs.

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Numerical Modeling of some Problem of Financial Mathematics

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Keywords: option prices, stock prices, equity diffusion markets, options of American and European types, Stefan's problem, numerical modeling.

This article examines the calculation of the option price $V(t;x)$, the stock price $x(t)$, and the optimal stopping (execution) time τ; ($\equiv t$) over both finite and infinite time horizons. It then delves into determining a fair value for American-style options, leveraging the optimal stopping time within the framework of diffusion processes in stock markets, represented by (B,S). Additionally, the article explores the pricing of European-style options, starting with the buyer's perspective and then transitioning to the seller's viewpoint. The problems are solved either analytically, when the optimal stopping time is pre-determined, or numerically using methods like the sweep method and finite element techniques.These methods are applied by reducing the problem to Stefan's problem, where $Y^*(t,x)$ represents the rational option value, τ_T^* indicates the rational execution time, and $x^*(t)$ corresponds to the rational stock price.

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The impact of the technological revolution on the composition of labor resources due to automation: The era of Industry 4.0 and artificial intelligence

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DOI 10.5281/zenodo.18217134

Abstract This article analyzes the impact of the technological revolution, particularly Industry 4.0, artificial intelligence (AI) , and automation technologies on the composition of labor resources. The study examines structural changes in the labor market, professional transformation, and evolution of qualification requirements resulting from the

digitalization of production processes and the introduction of intelligent technologies and robotics. Additionally, using Uzbekistan as an example, the dynamics of labor resources in recent years, employment levels, and the emergence of new professions are analyzed. The results indicate that automation is leading to a reduction in low-skilled jobs while increasing demand for specialists with intellectual, analytical, and technological skills. Furthermore, workforce retraining, modernization of the education system, and strategic human resource management are identified as key factors in ensuring a stable labor market in the context of the technological revolution.

The research results provide practical recommendations for developing labor policies, optimizing investment directions in human capital, and increasing employment stability in the digital economy.

Keywords: Technological revolution, Industry 4.0, Artificial Intelligence (AI), Automation, Labor resources, Digital economy, Labor market transformation, Human capital, Professional qualifications, Innovative employment

At the beginning of the 21st century, the world economy entered a new stage - the fourth industrial revolution. This period is characterized by the unprecedented pace of development in technologies, digital systems, artificial intelligence, and automation. Various industrial revolutions have had a profound impact on the evolution of labor resources, each characterized by significant technological advancements.

The first industrial revolution began at the end of the 18th century and was characterized by the introduction of mechanical methods of production, including water and steam power in textile manufacturing. This period enabled mass production and shifted labor from agriculture to industrial work, leading to significant socio-economic changes[1][2].

In the late 19th and early 20th centuries, the second industrial revolution introduced electricity and conveyor belt production. These innovations transformed production processes, further mechanized the workforce, and ensured high efficiency and product manufacturing in various sectors[1][17].

The third industrial revolution occurred in the 1960s, driven by the development of automation and computer technologies. At this stage, the integration of digital technologies and machine programming into production led to a more data-driven approach in industrial processes[1][2].

The fourth industrial revolution, often referred to as Industry 4.0, builds upon the foundations of previous revolutions, incorporating artificial intelligence (AI), machine learning, and extensive use of data. This current stage is distinguished by the potential to automate decision-making processes, which significantly reduces the need for human intervention in various tasks [1][18].

The technological revolution characterized by Industry 4.0, along with advancements in artificial intelligence (AI) and automation, has significantly transformed the workforce. This transformation has led to both the creation and elimination of various professions, altering the employment landscape across different sectors [3][19].

Diagram title: The evolution of industrial revolutions and their impact on labor resources

Industrial stage	Key technology	Nature of workforce	Result / Impact
1. Revolution (18th century)	Water and steam power, mechanization	Shift from rural to factory work, predominance of physical labor	Centralization of production, urbanization
2. Revolution (late 19th - early 20th century)	Electricity, assembly line	Specialized workers, mass employment	Increased production efficiency
3. Revolution (1960s)	Computerization, automation	Technically skilled specialists	Increased labor productivity, growth in white-collar jobs
4. Revolution (current stage)	Artificial intelligence, Big Data, digital systems	Adaptable, digitally skilled workforce	Automation, diversification of job types, emergence of new professions

One of the main concerns associated with the rise of artificial intelligence and automation is the possibility of large-scale job losses. Studies show that low-wage and low-skilled workers are particularly vulnerable in this shift, with an estimated job displacement rate of 6-7% across various professions.[4][20]. However, while some conventional roles, such as cashiers, have experienced a decline in employment rates due to automation, they have been replaced by other non-conventional low-paying jobs, especially those requiring interpersonal skills. This phenomenon suggests that the overall impact on low-wage employment may be neutral, as losses in routine-oriented positions are offset by gains in non-routine sectors [5],[21],[6].

The increase in the minimum wage also plays an important role in this process. Studies have shown that an increase in the minimum wage leads to job losses in typical professions;

However, they do not have a significant impact on the overall employment levels in low-wage sectors. As labor-intensive firms may face high costs and disproportionately fail, capital-intensive firms that can absorb this growth often expand and create new jobs [5],[4]. The shift towards more capital-intensive production further complicates the employment situation, as it requires workers to adapt to new, often technologically advanced roles.

The technological revolution, especially Industry 4.0, artificial intelligence, and automation, plays a crucial role in shaping the structure of labor resources in enterprises and socio-economic sectors. For example, Marguerit (2025) demonstrates in her research that automation and AI may lead to job losses in low-skilled professions but could also expand opportunities for high-skilled and technical/intellectual capabilities. MГ¤kelГ¤ and Stephany (2024) emphasize that AI is further increasing the demand for human skills - complementary skills such as digital literacy, teamwork, and problem-solving. Analyses in Uzbekistan, for instance Anafiyaev et al. (2024) , show that the introduction of AI in industry increases efficiency by optimizing processes and improving product quality, while also requiring new technological competencies and knowledge from workers. wos.academiascience.org Thus, in general, according to the literature, the technological revolution is causing the following changes in the structure of labor resources:

- reduction of low-skilled jobs and automation-related threats;
- increased demand for highly qualified, digital/intellectual skills;
- redistribution of work-related tasks among professions;

- the need for worker retraining (reskilling/upskilling) in the labor market.

Scientific research and development have shown that approximately 50% of skills are affected by automation. Routine experiments, data analysis, and literature reviews are becoming increasingly automated, allowing scientists to focus on developing creative hypotheses and interpreting complex results. This transition indicates a shift towards higher-level cognitive tasks in the scientific community [43].

Literature review

In their research, Tamrin et al. (2026) systematically analyzed how artificial intelligence-based chatbots play a role in the field of mental health. They selected 24 out of 392 articles published between 2020 and 2024, and using the systematic literature review (SLR) methodology, identified four main areas:
1) Delivering cognitive-behavioral therapy (CBT) methods to users,
2) Assisting in the process of diagnosis and assessment,
3) Automating monitoring and feedback systems,
4) Detecting emotional state in real-time.

The research results indicate that the main factors encouraging people to use AI chatbots for mental health are affordability, speed, convenience, and accessibility. Additionally, key technological factors for successful implementation include Natural Language Processing (NLP) , emotional intelligence, user-friendly interface, and data security. The findings of this study provide a theoretical foundation for the implementation of AI technologies in the social sphere, particularly in healthcare and psychological support systems, within the context of Uzbekistan.

Direction	**Key sources and ideas**
Industry 4.0 and the labor market	Schwab, K. *The Fourth Industrial Revolution* (2016) - the interdependence of humans and technology; Brynjolfsson & McAfee *The Second Machine Age* (2014) - AI and automation redistribute jobs.
Automation and workforce	Frey & Osborne (2017): 47% of occupations are at risk of automation. Arntz et al. (OECD, 2020): automation changes the nature of jobs, but does not completely eliminate them.
Artificial intelligence and new competencies	Autor, D. (2020): new occupations require "enhanced human competencies" - critical thinking, creativity, digital literacy.
Digital transformation and education	UNESCO (2022): the education system in the digital economy needs to develop adaptable skills.
Uzbekistan's experience	Decree of the President of the Republic of Uzbekistan dated April 28, 2022 No. PP-221 - "On Measures for Developing the Personnel Training System in the Field of Digital Economy"; 2023 "AI Strategy" project.

Abdurakhmanov K. and co-authors (2022) empirically analyzed the limitations and challenges in developing human resources and technological talents in the era of artificial intelligence. The study was conducted based on a sample of 65 countries out of 193 countries, using AI Readiness Index and IQ level (2021) as the main independent variables. The results were assessed using OLS and robust regression models and were found to be statistically significant at the $p<0.05$ level. The authors note that cooperation between

humans and AI is emerging as a new type of socio-technical system. In this system, the complementary skills of humans and machines are optimally combined, increasing labor productivity and national economic well-being. At the same time, levels of technological IQ, cognitive adaptability, and AI readiness play an important role in the development of human capital. The study is also relevant for Uzbekistan, as it substantiates the need to strengthen the technological potential of human resources by increasing the level of digital economy, IT literacy, and AI integration.

The rapid transformation of jobs necessitates a focus on skills development and retraining. As automation takes over routine tasks, workers need to cultivate skills that are not easily replicated by machines, such as creativity, problem-solving, and emotional intelligence[8][6]. Roles that combine technical skills with human qualities are on the rise, underscoring the importance of lifelong learning and adaptability in the workforce. For instance, positions such as AI assistant engineers or digital trust advisors are increasingly in demand as businesses evolve in response to technological changes [8][6].

Industry 4.0 is characterized by the integration of advanced technologies that enhance the efficiency, adaptability, and interconnectedness of production processes. This fourth industrial revolution will fundamentally transform how industry operates and interacts with the environment, fostering a more interconnected and intelligent manufacturing ecosystem.

Big data is crucial for Industry 4.0, as it encompasses large volumes of data from various sources, including IoT sensors, sales data, and supply chain factors. The ability to analyze this data quickly and cost-effectively allows manufacturers to gain actionable insights that drive strategic decision-making[22][23]. As industry increasingly utilizes big data, the effectiveness of other Industry 4.0 technologies improves.

Interconnectedness is a core principle of Industry 4.0, enabling seamless communication between machines, devices, and systems in the manufacturing environment.

This interconnected ecosystem facilitates real-time data exchange and collaboration, enabling more informed decision-making and operational transparency across the organization[22][24].

The Industrial Internet of Things (IIoT) involves the deployment of small sensors that collect and monitor data in real time across the entire business. These sensors can be integrated with a wide variety of equipment, from lighting to factory machinery, providing valuable information that aids in better decision-making[25][22]. The advantages of IIoT are enormous, as the connected equipment improves operational visibility and efficiency.

Artificial Intelligence (AI) and Machine Learning are crucial for maintenance and demand forecasting in manufacturing. These technologies can process large amounts of data from sensors to anticipate market changes and operational issues, enabling manufacturers to respond more effectively to dynamic conditions [-22][26]. Artificial intelligence's ability to analyze complex datasets distinguishes it as an essential tool in the digital transformation of industry.

Autonomous robots play a crucial role in increasing operational efficiency in the production environment. They are utilized for various tasks, including moving heavy loads and picking orders in warehouses. By automating these processes, companies can reduce the risk of human injuries and improve production rates, as robots can operate continuously without breaks[22][24].

Digital twins create virtual copies of physical objects or systems, enabling simulations and analyses that aid in making operational decisions. This technology allows manufacturers to visualize and test changes in a risk-free environment, optimize production processes, and accelerate the time to market for new products [22][24].

Augmented Reality (AR) integrates additional sensory data, usually visual information, into the real-world environment. In manufacturing, AR can enhance training and operational capabilities by providing workers with real-time information and visual tools, thereby increasing efficiency, productivity, and safety [22][24].

Cloud and edge computing facilitate the storage and processing of large volumes of data generated by IoT devices. This layered approach ensures efficiency in data processing - from immediate processing at the edge to in-depth analysis in the cloud - thereby supporting real-time capabilities that are central to Industry 4.0 [27][9].

As Industry 4.0 technologies become increasingly integrated into production, the need for upskilling and retraining employees is growing. Companies must invest in continuous education to ensure their workforce remains qualified in the context of rapid technological progress and thereby maintain a competitive advantage in the market [26][10].

Through the harmonization of these components, "Industry 4.0"is radically transforming the production landscape, leading to high efficiency, adaptability, and innovation across various sectors. The rapid development of technologies, particularly in the fields of Industry 4.0, artificial intelligence (AI), and automation, is creating significant challenges and opportunities for the workforce. As enterprises integrate these technologies, adapting the workforce becomes crucial for minimizing job losses and ensuring consistent productivity.

Results

One of the main concerns associated with the implementation of automation and artificial intelligence is the workforce's resilience to changes. Many employees feel threatened by these technologies, fearing job loss and layoffs. To alleviate these fears, employers need to cultivate a culture of innovation, emphasizing that technologies can augment human labor rather than replace it[28]. Furthermore, as automation primarily affects medium-skilled jobs, concerns about increasing inequality are growing. Job polarization has become a significant trend, with high-skilled and low-skilled jobs remaining the only segments showing growth [17].

1. **H1:** The technological revolution (AI, automation) will shift the composition of labor resources in favor of a highly skilled workforce.

2. **H2:** Automation reduces low-skilled jobs but creates new professions (data analyst, AI engineer, digital manager).

3. **H3:** When the education system lacks flexibility, technological growth leads to structural unemployment in the labor market.

4. **H4:** Digital transformation increases the importance of soft skills in human resources.

It is important to invest in employee development through upskilling and retraining. Upskilling focuses on improving existing skills by teaching technologies such as AI, robotics, and data analysis to those who are developing them. On the other hand, reskilling prepares employees for completely new roles by teaching them a different set of skills.

Programs tailored to the specific needs of the industry can significantly increase the flexibility and resilience of the workforce[30]. A Brookings study found that in order to keep pace with technological progress, approximately 40% of workers may need to undergo retraining in the coming years.[9]

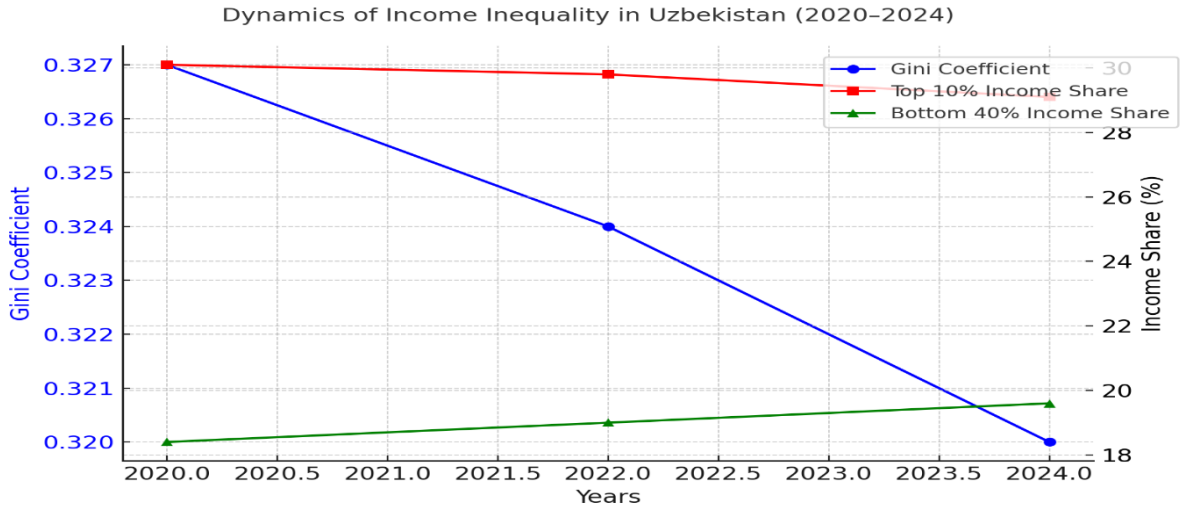
Establishing partnerships between companies and educational institutions can create a pool of qualified personnel that meets the requirements of the future labor market. These partnerships can help develop curricula that meet the needs of Industry 4.0, ensuring a sustainable flow of qualified graduates who are well-versed in relevant technologies[28][6].

It is crucial to promote a culture of continuous learning for workforce adaptation. Companies can encourage employees to obtain certifications, attend conferences, and pursue opportunities for continuing education[28]. This not only prepares employees for changing job roles but also increases their morale and engagement, fostering a loyal and motivated workforce[30].

Governments play a crucial role in supporting workforce adaptation through state policies and training initiatives. By investing in subsidized training programs and ensuring equal access to educational opportunities, governments can help workers transition to roles less susceptible to automation[6][29]. Furthermore, policies that encourage collaboration among various stakeholders - governments, educational institutions, and businesses - are essential for creating a comprehensive framework for skill development[31].

The emergence of Industry 4.0, characterized by the integration of automation and artificial intelligence (AI) into various fields, is causing significant economic consequences, particularly related to labor markets and forms of employment.

The shift towards increased automation is reshaping employment dynamics, leading to a significant decline in traditional middle-income jobs such as machine operators and clerks in developed economies like the United States and Western Europe. This trend, known as "job polarization,"has resulted in a growing segmentation of the workforce, leading to an increase in both low-paid service jobs and high-skilled, high-paying positions, such as engineers and healthcare workers[11][17].



Conversely, many developing economies are facing challenges in industrialization, as automated manufacturers gain competitive advantages in developed regions. As

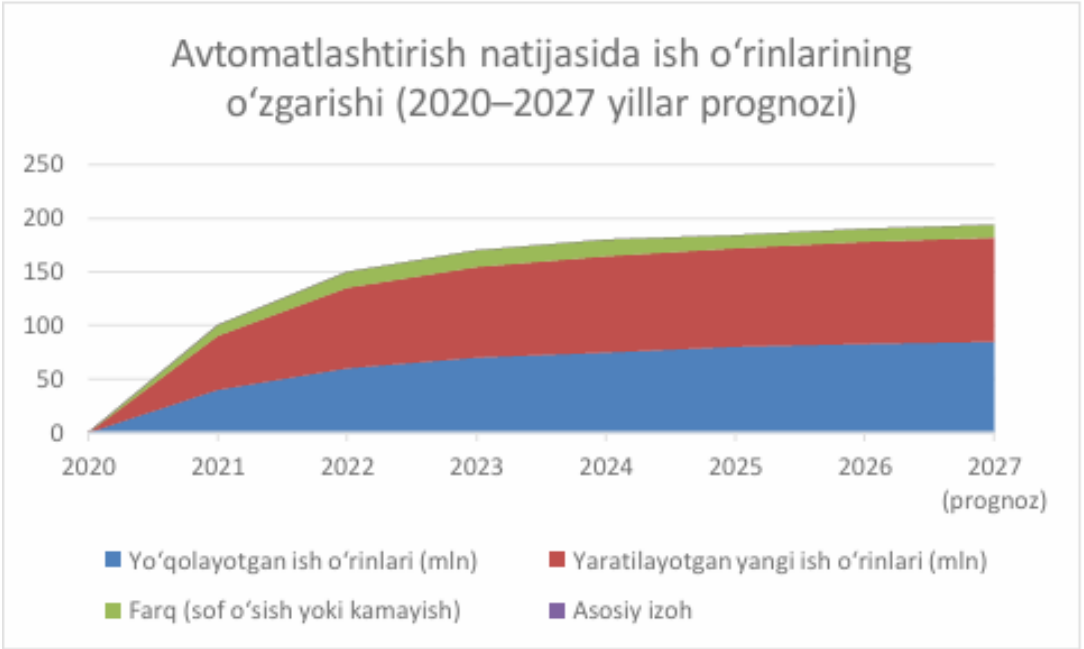
automation reduces production costs, small manufacturers in places such as Latin America and sub-Saharan Africa are struggling to compete, leading to a shift in employment from industrial jobs to lower-paid service sectors.[17].

Dynamics of Income Inequality in Uzbekistan (2020-2024), illustrating the decline in the Gini coefficient and a gradual reduction in income disparity between the top 10% and bottom 40% of the population. The impact of automation, beyond affecting employment levels, is also contributing to an increase in income inequality. As mid-skilled jobs decline, the benefits of automation are increasingly concentrated among highly skilled workers and corporations that develop and implement new technologies. This growing disparity may intensify concerns about social stability, prompting debates about government intervention in areas such as inequality and labor regulation [11][17].

Although new high-paying jobs have been created over the past two decades, the total wages and benefits these workers receive do not compensate for the losses suffered by laid-off employees. This disparity highlights a broader economic problem where the advantages of technological progress are not evenly distributed across the workforce [17][14].

Automation is expected to significantly increase productivity, with evidence suggesting that output per worker could increase substantially in sectors such as healthcare and construction [32][12]. For example, forecasts indicate that even if some jobs become redundant, labor productivity in the US may increase by up to 14% from 2017 to 2024 due to automation [32]. While such productivity growth could lead to higher efficiency, it raises concerns about potential job reductions, as fewer workers may be required to achieve the same results [33].

Furthermore, studies show that high-income workers are more likely to benefit from productivity-boosting opportunities offered by artificial intelligence technologies, while low-income workers face the risk of job loss and wage stagnation [12][34]. This indicates that the economic landscape is increasingly tilting in favor of those who possess the skills and capabilities to effectively utilize new technologies.



Despite the challenges posed by automation, it is projected that approximately 97 million new jobs will be created by 2027. However, the nature of these jobs may differ from those being displaced, requiring a workforce that is adaptable and equipped with new skills[7]. The continuous evolution of the labor market due to technological advancements emphasizes the need for strategic workforce development and policy measures to address these dynamic shifts in employment patterns[16].

The emergence of Industry 4.0 and the rapid integration of artificial intelligence (AI) and automation into various sectors are causing significant social consequences, especially related to labor resources in rural areas. Understanding these consequences requires an interdisciplinary approach based on sociology, anthropology, and behavioral economics. Sociological theories can illuminate the following:

The distribution of AI across different social groups, highlighting issues of social stratification and income inequality[13].

The socio-economic impact of artificial intelligence is not limited to simply creating or eliminating jobs; it intersects with income inequality and broader issues of access to education and skill development. While in rural settings, artificial intelligence can expand opportunities for small business owners by optimizing operations and expanding market reach, it may also widen the gap between successful and struggling enterprises. The use of AI-based tools often depends on financial resources and technical expertise, which can disadvantage those who lack such assets[13][14]. This disparity can exacerbate existing inequalities, so it is necessary to consider equitable outcomes when implementing artificial intelligence technologies.

Furthermore, the overall distribution of productivity gains derived from AI advancements may already be shifting towards knowledgeable employees who receive high salaries, potentially leaving low-skilled workers behind, especially in personal service and manual labor roles. This dynamic poses a risk of deepening wage inequality, as the wage premium for human skills decreases due to the development of artificial intelligence capabilities [12]. There are concerns that generative artificial intelligence may inadvertently exacerbate inequalities across multiple dimensions, such as income, wealth, gender, race, education level, and geography, as its implementation continues to reshape labor markets without sufficient guidance or ethical considerations [14][12].

To mitigate the negative social impact of artificial intelligence and automation, upskilling and retraining employees are of crucial importance. Training programs designed to enhance workers' qualifications can increase morale and efficiency, while simultaneously fostering a sense of expanding opportunities among employees. Such initiatives not only prepare the workforce for new technological requirements but also cultivate loyalty and reduce staff turnover -[35]. Organizations that invest in employee development can benefit from increased productivity and engagement, thereby adapting their workforce capabilities to the changing landscape of work [35][7].

Nevertheless, an important problem remains: there is a need for comprehensive strategies that ensure retraining alongside the adoption of automation technologies. If employers focus solely on replacing workers without implementing retraining programs, the resulting job cuts could lead to increased unemployment rates and heightened social unrest in affected communities[36]. Therefore, proactive measures, including skills assessment and tailored learning opportunities, are crucial for an effective transition to

an AI-based economy.

Analysis and recommendations

The implementation of Industry 4.0 technologies creates numerous challenges that organizations must face in order to take advantage of this technological revolution. These challenges can be broadly categorized as integration complexity, data management and security, initial investment issues, change management, and the need to adapt a skilled workforce.

One of the most significant challenges organizations face is the complexity of integrating new Industry 4.0 technologies with existing legacy systems. The convergence of Information Technology (IT) and Operational Technology (OT) requires seamless communication between machines, sensors, and data analysis platforms. This often involves substantial investments in robust integration strategies to ensure the effective collaboration of all components [37][38]. Many manufacturing plants are already using various legacy systems, which raises concerns about incorporating new technologies that may not integrate well with these existing tools [39].

The volume of data generated by Industry 4.0 technologies can be enormous. Organizations need to develop effective data management strategies for efficiently collecting, storing, and analyzing this data. Furthermore, as connectivity increases, so do the associated cybersecurity risks. Protecting confidential information and ensuring the security of connected devices is a crucial challenge that requires implementing robust cybersecurity protocols and training employees in best practices for preventing breaches[22][28]. Companies are advised to invest in reliable analytics tools to ensure efficient use of this data and to ensure that their workforce possesses the necessary data interpretation skills [40].

The initial costs associated with implementing Industry 4.0 solutions can be substantial, which often raises questions about the return on investment (ROI). Although there is potential for cost savings and increased efficiency, decision-makers should view these investments as long-term strategies. This requires clearly articulating the business benefits to justify the initial costs associated with introducing new technologies [37][23].

Adopting Industry 4.0 requires a skilled and technologically knowledgeable workforce capable of managing and supporting complex technologies. Upskilling and training employees are crucial for the successful implementation of new technologies[41]. Organizations may need to partner with training providers to develop comprehensive programs that equip their employees with the knowledge and experience necessary for effective operation [42].

Advancements in automation within the legal field are reshaping the skill set required for lawyers. Document review, legal research, and contract analysis can be largely automated, allowing lawyers to focus more on strategy, client counseling, and complex legal arguments. As a result, it is predicted that approximately 20 percent of skills in this field will undergo significant changes due to automation technologies [43].

Significant changes are also being observed in the sales sector, with approximately 52% of skills undergoing transformation. Automated systems now manage lead generation, initial contacts, and follow-up communications. As a result, sales professionals are focusing on building relationships and negotiating complex deals, emphasizing the importance of interpersonal skills and strategic thinking over routine tasks [43].

The hospitality and tourism industries face automation of approximately 49% of their skills, with tasks such as booking and answering basic inquiries increasingly managed by artificial intelligence systems. This allows hospitality staff to focus on providing personal service and enhancing guests' experiences. Similarly, in retail, where 51 percent of skills are changing, automation is taking over inventory management and basic customer interactions, enabling retail staff to offer personalized shopping assistance[43][40].

A case study by Larsen & Toubro (L&T) demonstrates how effective change management can overcome cultural resistance to technology adoption. By involving employees in the planning process, providing thorough training, and ensuring clear communication, L&T successfully reduced resistance to the implementation of new digital tools, which facilitated an easier transition to Industry 4.0 technologies[40]. This emphasizes the need for companies to address cultural challenges and engage their workforce in technology adoption processes.

India's "Make in India"initiative

The "Make in India"initiative serves as a practical example of how government support can facilitate the adoption of Industry 4.0 solutions among small and medium-sized enterprises (SMEs). By encouraging the introduction of digital technologies, many SMEs were able to implement Industry 4.0 technologies without bearing the full financial burden, demonstrating the important role of policy interventions in stimulating technological progress in the industrial sector[40][10].

These case studies demonstrate the multifaceted impact of Industry 4.0 on various sectors, showcasing the opportunities and challenges of automation and technological advancement in the labor market.

The impact of Industry 4.0, artificial intelligence (AI), and automation on the labor market is expected to be profound, leading to significant changes in employment forms and professional structures in the coming years. The World Economic Forum's 2023 "Future of Jobs"report outlines several key trends that could influence the future work environment.

One of the main forecasts is the emergence of new jobs, although some roles may be replaced due to technological progress. By 2027, approximately 97 million new jobs are expected to be created, especially in fields such as data science, artificial intelligence, and cybersecurity, which are anticipated to thrive under these ongoing developmental conditions[7][39]. Simultaneously, the labor market transformation characterizes structural changes in employment, with new positions emerging and existing ones gradually being phased out, demonstrating the dynamic nature of the labor market[7][3].

The implementation of automation and artificial intelligence across various fields is expected to significantly alter the demand for labor. By 2045, estimates suggest that many jobs could be automated, particularly in sectors vulnerable to technological disruptions such as law, finance, and manufacturing[44][17]. As employers increasingly adopt AI tools to enhance productivity, there is a risk of job polarization, where mid-skilled positions are eliminated while opportunities at the lower and upper ends of the wage scale expand[17]. This trend reflects a widespread transformation in the labor market, which may exacerbate inequality, as highly skilled workers and technologically adept companies are likely to disproportionately benefit from the advancements offered by automation[17][45].

As countries grapple with the consequences of these technological changes, future labor markets will also face challenges related to social stability and inequality. Governments will

need to implement effective policies to mitigate the impact of job losses and growing skills shortages. This includes investing in retraining programs and ensuring equitable access to the new opportunities created by the technological revolution[17][13]. The COVID-19 pandemic has further accelerated the need for enterprises to implement "Industry 4.0"technologies, with 91% of industrial companies acknowledging the necessity to invest in digital transformation to remain competitive in the long term[39] .

Conclusion The impact of the technological revolution, particularly through Industry 4.0, artificial intelligence (AI), and automation, has fundamentally transformed the workforce worldwide. This transformation marks a pivotal stage in the evolution of work, as it significantly alters the forms of employment, work structures, and the overall dynamics of labor markets. Industry 4.0 encompasses the integration of advanced technologies such as the Internet of Things (IoT), machine learning, and big data analytics, which collectively enhance productivity and operational efficiency across various sectors. As these technologies become increasingly prevalent, they are raising serious concerns about workplace transformation and the future viability of many professions, especially those involving routine and manual labor [1][2][3][4].

One of the most pressing issues emerging from this technological shift is the phenomenon of job polarization, where medium-skilled jobs are declining while opportunities at the lower and upper ends of the labor spectrum are expanding. Automation has led to the obsolescence of certain roles, such as cashiers and assembly line workers.

Continuous workers - new job categories are emerging, especially in fields that require advanced technical skills and interpersonal abilities.[5][6][7] However, the transition process is fraught with difficulties, as workers must adapt to rapidly changing demands for skills and qualifications. This requires significant investment in upskilling and retraining initiatives to ensure workforce readiness [8][9][10].

Moreover, the consequences of these changes extend beyond personal job losses, leading to broader economic issues related to income inequality and social stability. As automation increasingly favors highly skilled workers and capital-intensive sectors, there is a greater risk of exacerbating existing disparities in wealth and opportunities [11][12][13] Policymakers face the crucial task of developing strategies to address the multifaceted challenges brought about by this technological revolution, including supporting displaced workers and promoting fair access to new employment opportunities [14][15][13].

In conclusion, the impact of Industry 4.0, artificial intelligence, and automation on labor resources involves a complex interplay of innovations, adaptation, and socio-economic challenges. As the world of work continues to evolve, understanding and responding to these changes will be crucial to ensure a sustainable and inclusive labor market that benefits all stakeholders [7][16][13].

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On a coefficient-inverse problem with semi-nonlocal boundary conditions for the three-dimensional Tricomi equation in an unbounded parallelepiped

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DOI 10.5281/zenodo.18217197

Keywords: three-dimensional Tricomi equation, coefficient-inverse problem with semi-nonlocal conditions, Fourier and "ε— regularization" methods, a priori estimates and sequences of approximations.

1. Introduction

Linear inverse problems for second-order mixed-mode equations are used to model and identify processes occurring in transient regimes where the equation switches from elliptic to hyperbolic. Historically, such models arose in transonic aerodynamics to study the transition of a gas flow through the speed of sound, where inverse problems are used to reconstruct the coefficients characterizing the properties of the medium from experimental data. Similar formulations are used in acoustics and the theory of wave propagation in inhomogeneous media to diagnose the parameters of waveguides and extended channels. In geophysics, linear inverse problems serve as a tool for reconstructing the characteristics of subsurface structures from seismic data. In models of filtration and transport in porous media, such problems allow for the identification of permeability coefficients and sources. Linear inverse problems play a significant role in control theory and the calibration of mathematical models, as they provide robust algorithms for reconstructing parameters. From a mathematical point of view, they are important for the development of the theory of correctness, stability and methods for analyzing degenerate operators that underlie more complex nonlinear models.

In the study of nonlocal problems, a close relationship was revealed between problems with nonlocal boundary conditions and inverse problems [1]. Inverse problems for classical equations such as parabolic, elliptic, and hyperbolic equations have been studied quite well [2,3].

Linear inverse problems (with the search for solutions to the problem and right-hand sides of the equations) for second-order mixed-type model equations in a plane were studied in the works of A.G. Megrabov, K.B. Sabitov and their students [4-6], and for multidimensional second-order mixed-type equations in bounded domains with local boundary conditions were studied and developed in the work of S.Z. Dzhamalov, S.G. Pyatkov [7], and with non-local boundary conditions were studied and developed in the works of S.Z. Dzhamalov, R.R. Ashurov [8].

Linear inverse problems for second-order mixed-type equations of both the first and second kind in unbounded domains have been studied in the works of S.Z. Dzhamalov

and his students [9,10]. As far as we know, coefficient inverse problems (involving solutions, coefficients, and right-hand sides of the equations) for degenerate and mixed-type equations have been very little studied to date. Coefficient inverse problems with nonlocal boundary conditions in a parallelepiped for the Tricomi equation were first proposed and studied in the works of S.Z. Dzhamalov, A.A. Shokirov [11]. Coefficient inverse problems for mixed-type equations in unbounded domains have remained virtually unexplored. We will attempt to partially fill this gap in this paper.

To study the unique solvability of the coefficient inverse problem for the three-dimensional Tricomi equation in an unbounded parallelepiped, we propose a method based on reducing the coefficient inverse problem to direct problems for families of loaded Tricomi integro-differential equations with semi-nonlocal boundary conditions in a bounded rectangular domain.

Let Ω be a domain in R^n. Recall that a differential equation is called a loaded equation if it contains a trace of some operations from the desired solution on manifolds belonging to the closure of $\bar{\Omega}$ the domain Ω whose dimension is strictly less than n [12].

In the domain

$$G = (-1,1) \times (0,T) \times (-\infty,+\infty) = Q \times (-\infty,+\infty) =$$
$$= \{(x,t,z)|\ -1 < x < 1,\ 0 < t < T,\ -\infty < z < +\infty\}$$

we consider the three-dimensional Tricomi equation:

$$Lu = xu_{tt} - bu_{xx} - u_{zz} + \alpha(x,t)u_t = c(x,t)u + \psi(x,t,z) \tag{1}$$

here $\alpha(x,t)$ is a sufficiently large function, b is a sufficiently large positive number. Let $\psi(x,t,z) = g(x,t,z) + h(x,t) \cdot f(x,t,z)$, where $g(x,t,z)$,$f(x,t,z)$ are given functions, and $h(x,t)$ and $c(x,t)$ are to be determined.

Preliminary information. The Fourier transform of the function $u(x,t,z)$ with respect to the variable z is denoted by

$$\hat{u}(x,t,\lambda) = \frac{1}{(2\pi)^{1/2}} \int\limits_{-\infty}^{+\infty} u(x,t,z)e^{-i\lambda z}dz,$$

and the inverse Fourier transform is denoted by

$$u(x,t,z) = \frac{1}{(2\pi)^{1/2}} \int\limits_{-\infty}^{+\infty} \hat{u}(x,t,\lambda)e^{i\lambda z}d\lambda.$$

We introduce spaces of continuous functions together with derivatives up to order l via $C^l(Q)$ or simply C^l, with the norm

$$\|u\|_{C^l(Q)} = \sum_{|\alpha|=0}^{l} \max_{(x,t)\in Q} |D^\alpha u|.$$

Let $W_2^l(Q)$ denote Sobolev spaces with norm

$$\|u\|^2_{W_2^l(Q)} = \|u\|_l^2 = \sum_{|\alpha|=0}^{l} \int\limits_Q (D^\alpha u)^2 dxdt.$$

We introduce the anisotropic Sobolev space by $W_2^{l,s}(G)$, consisting of functions whose derivatives up to the l-th order in x and t, and up to the s-th order in z belong to $L_2(G)$, with the norm

$$[u]_{l,s}^2 \equiv \|u\|_{W_2^{l,s}(G)}^2 = \int_{-\infty}^{+\infty} (1+\lambda^2)^s \, \|\hat{u}\|_{W_2^l(Q)}^2, \qquad s \geq 3, \quad l = 0, 1, 2, ...$$

where $\hat{u}$ is the inverse Fourier transform of u.

2. Problem statement

Find a triple of functions $\{u(x,t,z),\ h(x,t),\ c(x,t)\}$ satisfying equation (**??**) almost everywhere in the domain G, such that $u(x,t,z)$ satisfies the following boundary conditions:

1) semi-nonlocal boundary conditions

$$\gamma D_t^p u|_{t=0} = D_t^p u|_{t=T}, \quad p = 0, 1, \tag{2}$$

$$u_x|_{x=-1} = u_x|_{x=1} = 0, \tag{3}$$

$$\lim_{|z|\to\infty} u = 0, \quad \lim_{|z|\to\infty} u_z = 0, \tag{4}$$

where γ is a positive number such that $|\gamma| > 1$;

2) additional conditions

$$u(x,t,\ell_1) = \varphi_1(x,t), \tag{5}$$

$$u(x,t,\ell_2) = \varphi_2(x,t), \tag{6}$$

$$-\infty < \ell_1 < \ell_2 < +\infty.$$

and, together with the functions $h(x,t)$, (x,t) belongs to the class

$$U = \left\{(u,\ h,\ c)|\ u \in W_2^{2,3}(G),\ h \in W_2^2(Q),\ c \in W_2^2(Q)\right\}.$$

Definition 1. A generalized solution of problem (1)-(6) is a function $\{u, h, c\}$ from class U that satisfies equation (1) almost everywhere in domain G subject to conditions (2)-(6).

Let the following conditions be satisfied regarding the coefficients and the right-hand side of equation (1):

Condition 1:

1) non-local conditions:

$$\alpha(x,0) = \alpha(x,T), \quad \alpha(-1,t) = \alpha(1,t),$$

$$\gamma g(x,0,z) = g(x,T,z), \quad \gamma f(x,0,z) = f(x,T,z);$$

2) smoothness:

$$f(x,t,\ell_i) = f_i \in C_{x,t}^{0,1}(\bar{Q}), \quad g(x,t,\ell_i) = g_i \in C_{x,t}^{0,1}(\bar{Q}),$$

$$f \in W_2^{3,3}(G), \quad g \in W_2^{1,3}(G);$$

3) $2\alpha(x,t)+(\mu+2)x>1$, $\mu\alpha(x,t)-\alpha_t(x,t)-\mu^2>2, b>2$, for all $(x,t)\in Q$, where $\mu=\frac{2}{T}\ln|\gamma|>0$.

Condition 2:

$$\varphi_i\in W_2^3(Q),\quad \gamma D_t^p\varphi_i|_{t=0}=D_t^p\varphi_i|_{t=T},\quad p=0,1,\quad \varphi_{i\,x}|_{x=-1}=\varphi_{i\,x}|_{x=1}=0,\quad \varphi_i\neq 0.$$

3. Main part

We introduce the following notation, through

$$q\equiv\gamma_1+2\gamma_0\gamma_2=m_1\left(\varepsilon_{\varphi,g}+m_3\left[g\right]_{1,3}^2+(m_2+\varepsilon_{\varphi,g}m_3)\left[f\right]_{2,3}^2\right),$$

where

$$\varepsilon_{\varphi,g}=\left(\left(1+b+\|\alpha\|_{C^1}^2\right)\sum_{i=1,2}\sum_{|\alpha|=1}^{3}\|D^\alpha\varphi_i\|_C^2+\sum_{i=1,2}\|g_i\|_{C^1}^2\right).$$

The main result is the following theorem.

Theorem 1. *Let the above Conditions 1 and 2 be satisfied for the coefficients of equation (1), and let the following inequalities be satisfied for the coefficients of the equation and their traces:*

$$2q<1.$$

Then there exists a unique solution to problem (1)-(6) from the class U.

Remark 1. We can achieve the inequality $2q<1$ by choosing the norms of f and g small, and taking g_i with its derivatives small, and also taking the derivatives of φ_i small.

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On a linear inverse problem with nonlocal boundary conditions for a three-dimensional equation of mixed type of the second kind of the fourth order in a parallelepiped.

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DOI 10.5281/zenodo.18217182

Keywords: three-dimensional equation of mixed type of the second kind of the fourth order, linear inverse problem with nonlocal boundary conditions, correctness of the problem, Fourier method, methods of "ε-regularization a priori estimates, sequence of approximations.

1. Introduction

In the process of studying nonlocal problems, a close relationship was revealed between problems with nonlocal boundary conditions and inverse problems. By now, inverse problems for classical equations such as parabolic, elliptic and hyperbolic types of the second order have been studied quite well [1],[2]. Linear inverse problems for equations of mixed type of the second order in the plane have been studied in the works of A.G. Megrabov, K.B. Sabitov and their students [3],[4].

For three-dimensional equations of mixed type of both first and second kind of the second order, they were studied in the works of S.Z. Dzhamalov, and for multidimensional

equations of mixed type of the second order of both first and second kind with local and non-local conditions in bounded domains, they were studied and developed in the works of S.Z. Dzhamalov, R.R. Ashurov and S.Z. Dzhamalov, S.G. Pyatkov [5]-[8].

For high-order mixed-type equations, such problems have not been studied in practice. We will try to partially fill this gap in this paper.

In this paper, for the study of unique solvability of inverse problems for a three-dimensional equation of mixed type of the second kind, fourth order in a rectangle, a new method is proposed. Which is based on reducing the inverse problem to direct non-local boundary value problems of periodic type for a family of loaded differential equations of mixed type of the second kind, fourth order[5]-[9].

Let us recall that a loaded equation is usually called an equation with partial derivatives that contains in the coefficients or on the right-hand side the values of certain functionals from the solution of the equation [10].

In the domain

$$G = (0,1)\times(0,T)\times(0,\ell) = Q\times(0,\ell) = \{(x,t,y) : 0 < x < 1;\ 0 < t < T < +\infty;\ 0 < y < \ell\}.$$

Let's consider equations of mixed type of the second kind of the fourth order.

$$Lu = Pu + Mu + Nu = \psi(x,t,y), \tag{1}$$

where

$$Pu = \sum_{i=0}^{4} K_i(x,t)D_t^i u, \quad Mu = a(x)u_{xxxx} - b(x)u_{xxtt} - c(x)u_{xx}, \quad Nu = u_{yyyy}.$$

Here,$(K_4(x,t) = K_4(t),\ K_4(0) = K_4(T) = 0;\ a(x) > 0, b(x) > 0, c(x) > 0$ and $D_t^i u = \dfrac{\partial^i u}{\partial t^i}$, where $i = 0,1,2,3,4$.

Equation (1) refers to mixed-type equations of the second kind, since no restrictions are imposed on the sign of the function $K_4(t)$ with respect to the variable t inside the segment $[0,T]$ [11].

In the future, we will assume that $\psi(x,t,y) = g(x,t,y) + h(x,t)\cdot f(x,t,y)$, where $g(x,t,y)$ and $f(x,t,y)$ are given functions, and the function $h(x,t)$ is subject to definition.

Linear inverse problem. Find functions $\{u(x,t,y), h(x,t)\}$, satisfying equation (1) in the domain G, such that the function $u(x,t,y)$ satisfies the following nonlocal boundary conditions:

$$\gamma D_t^q\, u|_{t=0} = D_t^q\, u|_{t=T}\,, \quad q = 0,1,2 \tag{2}$$

$$\rho D_x^p\, u|_{x=0} = D_x^p u|_{x=1}\,, \quad p = 0,1,2,3 \tag{3}$$

$$u|_{y=0} = u|_{y=1} = 0;\ u_{yy}|_{y=0} = u_{yy}|_{y=\ell} = 0 \tag{4}$$

and an additional condition

$$u(x,t,\ell_0) = \varphi_0(x,t), \quad \text{where} \quad 0 < \ell_0 < \ell < +\infty \tag{5}$$

and together with the functions $h(x,t)$ belongs to the class

$$U = \left\{(u,h) \mid u \in W_2^{4,3}(G),\ h \in W_2^4(Q)\right\}.$$

Here, by $W_2^{4,3}(G)$ we denote the anisotropic Sobolev space with the norm

$$\|u\|^2_{W_2^{4,3}(G)} = \sqrt{\frac{2}{\ell}} \sum_{k=1}^{\infty} (1+\mu_k^4)^3 \|u_k(x,t)\|^2_{W_2^4(Q)},$$

where the functions $u_k(x,t)$, $k = 1,2,3,\ldots$, are the Fourier coefficients of the function $u(x,t,y)$ with respect to the system $Y_k(y) = \{\sqrt{2\ell^{-1}} \sin \mu_k y\}$, $\mu_k = \pi k \ell^{-1}$, $k = 1,2,3,\ldots$,

and by $W_2^l(Q)$ we denote the Sobolev space with the norm

$$\|\vartheta\|^2_{W_2^4(Q)} = \sum_{|\alpha|\le 4} \int_Q |D^\alpha \vartheta|^2 \, dx\, dt,$$

where α is a multi-index and D^α is the generalized derivative with respect to the variables x and t.

We will assume the following conditions are satisfied. Let all coefficients of equation (1) be sufficiently smooth functions in the domain $\overline{Q}$, and let the following conditions hold with respect to the coefficients, the right-hand side, and the given function $\varphi_0(x,t)$:

Condition 1: ***Non-local condition***

$$K_{4t}(0) = K_{4t}(T);\ K_i(x,0) = K_i(x,T); \quad i = 0,2,3, \quad \forall x \in [0,1],$$
$$\gamma g(x,0,z) = g(x,T,z), \quad \gamma f(x,0,z) = f(x,T,z).$$
$$K_1(x,t) > 0, \quad K_3(x,t) > 0$$

are sufficiently large functions. In addition, let the following conditions be met for the coefficients of equation (1):

$$-(2K_3 + (2j-3)K_{4t} + 3\lambda K_4) \ge \delta_3 > 0, \quad j = 0,1,2;$$
$$2K_1 - K_{2t} + \lambda K_2 \ge \delta_2 > 0, \quad \lambda K_0 - K_{0t} \ge \delta_1 > 0,$$

for any $(x,t) \in \overline{Q}$, where $\lambda = \frac{2}{T} \ln|\gamma| > 0$, $\quad |\gamma| > 1, \quad |\rho| \ge 1$.

$a(x) \ge a_0 > 0$, $b(x) \ge b_0 > 0$, $c(x) \ge c_0 > 0$.

***Smoothness*:**

$$g(x,t,l_0) = g_0(x,t) \in C^{0,1}_{x,t}(\bar{Q}), \quad f(x,t,l_0) = f_0(x,t) \in C^{0,1}_{x,t}(\bar{Q}),$$
$$|f_0(x,t)| \ge \eta > 0, \quad f \in W_2^{3,3}(G), \quad g \in W_2^{1,3}(G).$$

Condition 2:

$$\varphi_0(x,t) \in W_2^5(Q); \quad \gamma D_t^q \varphi_0|_{t=0} = D_t^q \varphi_0|_{t=T}, \quad q = 0,1,2,3,4;$$
$$\rho D_x^p \varphi_0|_{x=0} = D_x^p \varphi_0|_{x=1}, \quad p = 0,1,2,3., \rho \ge 1$$

Theorem 1. The main result. *Let the above listed conditions 1 and 2 be satisfied for the coefficients of equation (1), and let there exist a positive number σ such that for some (arbitrarily small) positive constants the following relations hold: $\delta_0 - 31\, e^{\lambda T} \sigma^{-1} = \delta > 0$, $q = M \cdot \sigma \cdot \|f\|^2_{W_2^{3,3}(G)} < 1$, (where $\delta_0 = \min\{\delta_3, \lambda a_0, \lambda b_0, \lambda c_0, \delta_2, \delta_1\}$, M is a positive constant). Then there exists a unique solution to the linear inverse problem (1)-(5) from the specified class U.*

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Homotopy perturbation method for second order nonlinear integro-differential equations with boundary value problems

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Keywords: Approximate solution; Boundary condition; Homotopy analysis method; Adomian decomposition method; integro-differential equations.

Abstract

Homotopy Perturbation method (HPM) is a famous techniques for solving a wide range of linear and non-linear problems in general functional equations. In this study, new modification HPM is introduced as semi-analytical solutions for nonlinear Volterra-Fredholm integro-differential equations (VF-IDEs) of order 2 with boundary conditions. As a comparison, the Adomian decomposition method (ADM) is also introduced. In addition, the method for determining unknown parameter in boundary conditions is explained in detail. Proposed HPM is tested and compared with ADM, series solution method (SSM) and polynomial approximation method (PAM) for few examples of IDEs with boundary conditions. Numerical results indicate that HPM outperforms over ADM, SSM and PAM for linear and non-linear VF-IDEs with boundary conditions.

One of the popular approach for solving linear Volterra or Fredholm type integro-differential equations (IDEs) is the method of quadrature and polynomial approximation. However, these methods are less suitable for non-linear IDEs. Consequently, alternative methods are needed to develop. One effective semi-analytical method is the homotopy perturbation method (HPM), proposed by He [1] in 1999. The HPM has been further developed by various researchers [2]-[3]. Standard HPM and modified HPM were applied in many research works [4]-[5] on many areas of integral and integro-differential equations (IDEs).

In this study, we examine second order non-linear IDEs of the Volterra-Fredholm type

$$\begin{aligned} a_2(t)u''(t) + a_1(t)u'(t) + a_0(t)u(t) &= g(t)P_1[u(t), u'(t)] \\ &+\mu \int_a^t K_1(t,\tau)P_2[u(\tau), u'(\tau)]\, d\tau \\ &+\lambda \int_a^b K_2(t,\tau)P_3[u(\tau), u'(\tau)]\, d\tau, \end{aligned} \tag{1}$$

subject to boundary conditions

$$u(a) = \alpha_0, \qquad u(b) = \alpha_1, \tag{2}$$

where $u(t)$ is a continuous unknown function along with its derivatives, $K_1(t,\tau)$ and $K_2(t,\tau)$ are kernels, $a_k(t)$ and $g(t)$ are given analytic functions, P_j $(j = 1, 2, 3)$ represents nonlinear functions, and $\mu, \lambda, \alpha_0, \alpha_1, \ldots$ are real constants.

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Homotopy perturbation method for singular initial value problems of ordinary differential equations of second order

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Keywords: Approximate solution; Boundary condition; Homotopy analysis method; Adomian decomposition method; Integro-differential equations.

Abstract: Ordinary differential equations (ODEs) are mathematical tools that model dynamic systems and their changes over time. They are essential in fields such as physics, engineering, biology, and economics. Solving nonlinear ODEs is a central problem in computational mathematics. The homotopy perturbation method (HPM) is a well-known technique for solving a wide range of linear and nonlinear integral and differential equations. In this study, a new modification of HPM is implemented to obtain semi-analytical solutions for singular and nonlinear second-order ODEs with initial conditions. For comparison, the Adomian decomposition method (ADM) is also considered. The proposed HPM is tested against ADM, the series solution method (SSM), and the polynomial approximation method (PAM). Numerical results show that the modified HPM outperforms these methods for both linear and nonlinear problems.

1. Introduction

Ordinary differential equations play a key role in mathematical modelling and are widely used to describe real-life processes in science, engineering, and economics. However, obtaining analytical solutions for nonlinear ODEs is often difficult. Therefore, various approximate and semi-analytical methods have been developed to address this issue.

The homotopy perturbation method (HPM), introduced by Ji-Huan He in 1999, is a powerful semi-analytical approach for solving nonlinear problems. It has been successfully applied to nonlinear oscillators, boundary-value problems, reactionвЂ“diffusion models, and other applications. The construction of homotopies and series expansions forms the basis of HPM.

Another widely used technique is the Adomian decomposition method (ADM), first proposed by G. Adomian in 1994. ADM employs Adomian polynomials to handle nonlinear terms and provides rapidly convergent solutions for many classes of problems.

In this study, we consider a second-order ordinary differential equation of the generalized LaneвЂ“Emden type

$$u''(t) + \frac{\alpha}{t-a}u'(t) - b(t)u(t) = g(t,u),$$

subject to the initial conditions

$$u(a) = \alpha_0, \qquad u'(a) = 0,$$

where $b(t)$ is a given smooth function, $g(t,u)$ is a real-valued nonlinear function, and $u(t)$ is an unknown continuous function together with its derivatives.

1. Methodology

Equation (1) can be rewritten in the form

$$\big((t-a)^{\alpha}u'(t)\big)' = (t-a)^{\alpha}\big(b(t)u(t) + g(t,u)\big),$$

which is suitable for applying the homotopy perturbation framework.

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Discrete model of interaction between two populations

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DOI 10.5281/zenodo.18217163

Keywords: Interaction between two populations, Friedman's model, Discrete model.

We consider a model features interactions of several strategically distinct populations, so as to represent economic relationships between buyers and sellers, officials and supplicants, or between residents in different jurisdictions, etc.

Discrete Friedman's model

In [1] the author provided the following model of interaction between two populations. It is considered a stylized interaction between two populations, called "buyers"and "sellers". Each seller has available two possible actions or strategies, "honest"($i = 1$) and "cheat"($i = 2$); and each buyer also has two alternative strategies, "inspect"($i = 1$) and "don't inspect"($i = 2$). Since the strategy/state simplices here are 1-dimensional the state $s = ((s_1^1, s_2^1), (s_1^2, s_2^2))$ can be described by a point (p, q) in the square $[0, 1]$x$[0, 1]$, with $s_1^1 = p$ as the fraction of buyers who inspect and $s_1^2 = q$ as the fraction of honest sellers, so $s_2^1 = 1 - p$ and $s_2^2 = 1 - q$.
As for dynamics, it is assumed with Malthus that the growth rate of a strategy is proportional to or (with an appropriate choice of time scale) equal to its relative fitness and produced the following dynamic system

$$\begin{aligned} \dot{p} &= p(1-p)(1-2q) \\ \dot{q} &= q(1-q)(2p-1) \end{aligned} \tag{1}$$

It is shown that the system (1) of coupled differential equations has five fixed points (i.e., steady-states), at the center and at the four corners of the p - q square and all other points are on periodic trajectories circling counterclockwise [1].
We produce a discrete dynamical systems for model considered by Friedman.
The discrete time dynamical system corresponding to (1) is defined as follows

$$\begin{aligned} p' &= p(2-p-2q+2pq) \\ q' &= q(2p+q-2pq) \end{aligned} \tag{2}$$

It is proven the following statement.
Theorem 1 The non-linear dynamical system (2) is the regular transformation.

The new discrete model of interaction between two populations

In this section we present new discrete model of interaction between two populations. Let $\Omega_s = \{(\sigma_1, \sigma_2)\}$ and $\Omega_b = \{(\sigma_3, \sigma_4)\}$, where σ_1 is the honest seller, σ_2 is cheat seller, σ_3 is the inspecting buyer and σ_4 is the non-inspecting buyer. For brevity assume 1 denote honest seller, 2-cheat seller, 3-inspecting buyer and 4- non-ispecting buyer. On the set $\Omega = \{1, 2, 3, 4\}$ we consider the set of all probability measures

$$S(\Omega) = \{(x_1, x_2, x_3, x_4) :\in R^4 : \text{ for any } i\ x_i \geq 0, \text{ and } \sum_{i=1}^{4} x_i = 1\},$$

where x_1 and x_2 are the fraction of honest and cheat sellers respectively; and x_3 and x_4 are the fraction of buyers who inspect and don't inspect respectively. Since Ω consists of 4 elements, the set $S(\Omega)$ coincides with 3-dimensional simplex S^3.
Assume that $p_{13,1}$ is the probability that after interaction honest seller and inspecting buyer the next moment of time selected seller will be honest. Similarly one can define $p_{13,k}, p_{14,k}, p_{23,k}$ and $p_{24,k}$ for $k = 1, 2, 3, 4$, where $p_{ij,1} + p_{ij,2} = 1$ and $p_{ij,3} + p_{ij,4} = 1$. Let us define a hyper-simplex $S_H(\Omega)$ as follows:

$$S_H(\Omega) = \{(x_1, x_2, x_3, x_4) \in S^3 : x_1 + x_2 = x_3 + x_4 = 0.5\}$$

that is the fraction of sellers is equal to fraction of buyers in total population.
As shown in [2] one can define the dynamic on hyper-simplex $V : S_H(\Omega) \to S_H(\Omega)$ as follows

$$\begin{aligned} x_1' &= 2p_{13,1}x_1x_3 + 2p_{14,1}x_1x_4 + 2p_{23,1}x_2x_3 + 2p_{24,1}x_2x_4 \\ x_2' &= 2p_{13,2}x_1x_3 + 2p_{14,2}x_1x_4 + 2p_{23,2}x_2x_3 + 2p_{24,2}x_2x_4 \\ x_3' &= 2p_{13,3}x_1x_3 + 2p_{14,3}x_1x_4 + 2p_{23,3}x_2x_3 + 2p_{24,3}x_2x_4 \\ x_4' &= 2p_{13,4}x_1x_3 + 2p_{14,4}x_1x_4 + 2p_{23,4}x_2x_3 + 2p_{24,4}x_2x_4. \end{aligned} \tag{3}$$

Examples

Example 1. Assume

$$\begin{array}{llll} p_{13,1} = 1 & p_{14,1} = 1 & p_{23,1} = 1 & p_{24,1} = 0 \\ p_{13,2} = 0 & p_{14,2} = 0 & p_{23,2} = 0 & p_{24,2} = 1 \\ p_{13,3} = 1 & p_{14,3} = 0 & p_{23,3} = 1 & p_{24,3} = 1 \\ p_{13,4} = 0 & p_{14,4} = 1 & p_{23,4} = 0 & p_{24,4} = 0 \end{array}$$

For this model in long run all sellers are honest and all buyers don't inspect, i.e. in long run we will have ideal society.
Example 2. Assume

$$\begin{array}{llll} p_{13,1} = 0 & p_{14,1} = 0 & p_{23,1} = 0 & p_{24,1} = 0 \\ p_{13,2} = 1 & p_{14,2} = 1 & p_{23,2} = 1 & p_{24,2} = 1 \\ p_{13,3} = 0 & p_{14,3} = 1 & p_{23,3} = 0 & p_{24,3} = 1 \\ p_{13,4} = 1 & p_{14,4} = 0 & p_{23,4} = 1 & p_{24,4} = 0 \end{array}$$

Then for this model all sellers are cheat and all buyers inspect, i.e. in long run we will have society without trust.

Example 3. Perturbations of first Example Assume

$$\begin{array}{cccc} p_{13,1}=1-\varepsilon & p_{14,1}=1-\varepsilon & p_{23,1}=1-\varepsilon & p_{24,1}=\varepsilon \\ p_{13,2}=\varepsilon & p_{14,2}=\varepsilon & p_{23,2}=\varepsilon & p_{24,2}=1-\varepsilon \\ p_{13,3}=1-\varepsilon & p_{14,3}=\varepsilon & p_{23,3}=1-\varepsilon & p_{24,3}=1-\varepsilon \\ p_{13,4}=\varepsilon & p_{14,4}=1-\varepsilon & p_{23,4}=\varepsilon & p_{24,4}=\varepsilon \end{array}$$

Then for this model in long run 77.5% of sellers be honest and 85.2% of buyers will not inspect.

Conclusion.

We can consider also a stylized interaction between two populations, called "officials"and "supplicants". Each official has available two possible actions or strategies, "not briber"($i = 1$) and "bribe taker"($i = 2$); and each supplicant also has two alternative strategies, "corrupter"($i = 1$) and "not bribing"($i = 2$). As shown in Example 3 rather very small perturbation of ideal model leads to very fast growing of corruption, since for $\varepsilon = 0.05$ one can see that 22.5% of officials will be bribe taker in long run.

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Exponential-algebraic optimal quadrature formulas in a Hilber space

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DOI 10.5281/zenodo.18217201

Keywords: Hilber space, quadrature formula, error functional, extremal function, optimal coefficients, inner product.

Introduction and statement of the problem

In the present work we consider a quadrature formula of the following form

$$\int_0^1 f(x)dx \cong \sum_{\beta=0}^{N} C_{\beta,0} f(x_\beta) + \sum_{\beta=0}^{N} C_{\beta,1}(f'(x_\beta) + f(x_\beta)), \tag{1}$$

where $C_{\beta,0}$, $C_{\beta,1}$ are coefficients, $x_\beta = h\beta$ are nodes and $h = 1/N$, N is a natural number, an integrand f belongs to the Hilbert space $W_2^{(2,1)}$ of functions that are square integrable with second derivative [1,2]. The space $W_2^{(2,1)}$ is equipped by the norm

$$\|f\|_{W_2^{(2,1)}} = \sqrt{\int_0^1 (f'(x) + f(x))dx}.$$

We note that in (1) the coefficients $C_{\beta,0}$ are known and have the form

$$C_{0,0} = \frac{e^h - 1}{e^h + 1}, \quad C_{\beta,0} = \frac{2(e^h - 1)}{e^h + 1}, \quad \beta = 1, 2, ..., N-1, \text{ and } C_{N,0} = \frac{e^h - 1}{e^h + 1}. \tag{2}$$

Furthermore, they are coefficients of the optimal quadrature formula of the form

$$\int_0^1 f(x)dx \cong \sum_{\beta=0}^{N} C_{\beta,0} f(x_\beta)$$

in the space $W_2^{(1,0)}$ [2].

Then we have possiblity to minimize the error of the formula (1) by the coefficients $C_{\beta,1}$. It should be noted that the following difference is called *the error* of the quadrature formula (1)

$$(\ell, f) = \int_0^1 f(x)dx - \sum_{\beta=0}^{N} C_{\beta,0} f(x_\beta) - \sum_{\beta=0}^{N} C_{\beta,1}(f'(x_\beta) + f(x_\beta)). \tag{3}$$

This difference defines a linear functional ℓ acting to the function f as

$$(\ell, f) = \int_{-\infty}^{\infty} \ell(x) f(x) dx$$

and ℓ is called *the error functional* of the quadrature formula (1) and is expressed by the following expression

$$\ell(x) = \varepsilon_{[0,1]}(x) - \sum_{\beta=0}^{N} C_{\beta,0}\delta(x - x_\beta) + \sum_{\beta=0}^{N} C_{\beta,1}(\delta'(x - x_\beta) - \delta(x - x_\beta)), \tag{4}$$

where $\varepsilon_{[0,1]}(x)$ is the characteristic function of the interval $[0, 1]$, $\delta(x)$ is the Dirac delta-function.

It is known that the norm of the error functional ℓ is expressed as

$$\|\ell\|_{W_2^{(2,1)*}} = \sup_{f, \|f\|_{W_2^{(2,1)}} \neq 0} \frac{|(\ell, f)|}{\|f\|_{W_2^{(2,1)}}}.$$

From here, we get the following upper estimate for the absolute value of the error of the quadrature formulas (1)

$$|(\ell, f)| \leq \|\ell\|_{W_2^{(2,1)*}} \cdot \|f\|_{W_2^{(2,1)}}. \tag{5}$$

Hence, taking into account the definition of the norm of the space $W_2^{(2,1)}$, it is easy to see that the error of the quadrature formula (1) is zero for the functions $f(x) = e^{-x}$ and $f(x) = 1$. That is the quadrature formula (1) is exact for any linear combination of the functions $f(x) = e^{-x}$ and $f(x) = 1$ and we have the following equations

$$(\ell, e^{-x}) = 0 \text{ and } (\ell, 1) = 0. \tag{6}$$

One can easily check that the first equation from (6) is satisfied due to coefficiets $C_{\beta,0},\ \beta = 0, 1, ..., N$, given by (2). Indeed, taking into account (2), we have

$$(\ell, e^{-x}) = \int_0^1 e^{-x}dx - \sum_{\beta=0}^{N} C_{\beta,0}e^{-x_\beta} - \sum_{\beta=0}^{N} C_{\beta,1}(-e^{-x_\beta} + e^{-x_\beta})$$

$$= \int_0^1 e^{-x}dx - \sum_{\beta=0}^{N} C_{\beta,0}e^{-x_\beta} = 0.$$

Then from the second equation of (6) we get the existence condition of the quadrature formula of the form (1) in the space $W_2^{(2,1)}$. Thus for the coefficients $C_{\beta,1}$ of the quadrature formula (1) we have the following

$$(\ell, 1) = \int_0^1 1dx - \sum_{\beta=0}^{N} C_{\beta,0} \cdot 1 - \sum_{\beta=0}^{N} C_{\beta,1}(0 + 1) =$$

$$= 1 - \sum_{\beta=0}^{N} C_{\beta,0} - \sum_{\beta=0}^{N} C_{\beta,1} = 0.$$

Hence, using (2), for the coefficients $C_{\beta,1}$ we get

$$\sum_{\beta=0}^{N} C_{\beta,1} = 1 - \frac{2(e^h - 1)}{h(e^h + 1)}. \tag{7}$$

Therefore, for existence of a quadrature formula of the form (1) it is needed satisfying the condition $N \geq 1$. Furthermore, a quadrature formula of the form (1) that satisfies the conditions (6) we call *an exponential-algebraic quadrature formula with derivative* in the space $W_2^{(2,1)}$.

Among quadrature formulas of the form (1) with given coefficients $C_{\beta,0}$ in the Hilbert space $W_2^{(2,1)}$ *an optimal one* is a quadrature formula of the form (1) with coefficients $C_{\beta,1}$ that give the absolute minimum value to the norm $\|\ell\|$ of the error functional (4). The coefficients $C_{\beta,1}$ that give this minimum are called *the optimal coefficients* and correspending quadrature formula of the form (1) we call *the exponential-algebraic optimal quadrature formula* of the form (1) in the space $W_2^{(2,1)}$.

Thus, to get the optimal quadrature formula of the form (1) we should solve the following problems.

Problem 1. Find an upper bound for the error of the quadrature formula (1) in the space $W_2^{(2,1)}$, that is, calculate the norm $\|\ell\|_{W_2^{(2,1)}}$ of the error functional (4).

We note that the norm of the error functional (4) becomes a multivariable function with respect to coefficients $C_{\beta,1}$, $\beta = 0, 1, ..., N$. In order to get an optimal quadrature formula it is natural to solve the following problem.

Problem 2. Find coefficients $C_{\beta,1}$ that give the mimimum to the norm $\|\ell\|_{W_2^{(2,1)}}$ of the error functional ℓ.

Thus, solving Problems 1 and 2 successively we get an optimal quadrature formula of the form (1). Here we present solution of Problems 1 and 2. Therefore, the aim of the paper is to solve Problems 1 and 2.

For this, in the next sections, we calculate the norm $\|\ell\|_{W_2^{(2,1)}}$ of the error functional ℓ and get the coefficients $C_{\beta,1}$ that give the minimum to this norm.

An extremal function of the quadrature formula(1)

In the present section we find an exptremal function for the quadrature formula (1). Then, in the next section, using the obtained extremal function we get the norm of the error functional (4) that allows us to obtain the sharp upper bound for the error of the quadrature formula (1).

It should be noted that the definition of an extremal function of a cubature formula with corresponding error functional on a Banach space was given by S.L.Sobolev (see, for exapmle, [3]). The extremal function allows us to get a sharp upper bound for the error of the cubature formula.

Here, to estimate the error of the quadrature formula (1) on functions of the space $W_2^{(2,1)}$ we use the Riesz representation theorem about general form of a linear continuous functional on Hilbert spaces and apply the definition of the extremal function of the quadrature formula (1).

By the definition (see, for instance, [1]) for any linear continuous functional (in particular, for the error functional (4)), defined on the Hilbert space $W_2^{(2,1)}$, an extremal function ψ_ℓ satisfies the following equality

$$(\ell, \psi_\ell) = \|\ell\|_{W_2^{(2,1)*}} \|\psi_\ell\|_{W_2^{(2,1)}}. \tag{8}$$

Based on the Riesz theorem the following equality holds

$$\|\ell\|_{W_2^{(2,1)*}} = \|\psi_\ell\|_{W_2^{(2,1)}} \tag{9}$$

and from the last two equations we have

$$\|\ell\|^2_{W_2^{(2,1)*}} = (\ell, \psi_\ell). \tag{10}$$

In the Hilbert space $W_2^{(2,1)}(0,1)$ the extremal function that satisfies equalities (8) and (9) has the following representation (see, for example, [1])

$$\psi_\ell(x) = \ell(x) * G_2(x) + p_0 + de^{-x}, \tag{11}$$

where

$$G_2(x) = \frac{\text{sign}x}{2}\left(\frac{e^x - e^{-x}}{2} - x\right), \tag{12}$$

p_0 and d are constants.

For the expremal function we have the following result.

Theorem 1. *The extremal function (11) of the quadrature formula (1) in the space $W_2^{(2,1)}$ has the following analytic representation*

$$\psi_\ell(x) = \sum_{\beta=0}^{N} C_{\beta,1}\left(\frac{\text{sign}(x-x_\beta)}{2}(e^{-(x-x_\beta)} + x - x_\beta - 1)\right) -$$

$$-\sum_{\beta=0}^{N} C_{\beta,0}G_2(x-x_\beta) + \int_0^1 G_2(x-y)dy + p_0 + de^{-x}, \tag{13}$$

where $G_2(x)$ is defined by (12).

Proof. Indeed, calculating the convolution in (11) we get (13). Let us do it. By the definition of the convolution for continuous variable functions for the convolution $(\ell * G_2)(x)$ we have

$$(\ell * G_2)(x) = \int_{-\infty}^{\infty} \ell(y)G_2(x-y)dy.$$

Hence, using (4), we get

$$(\ell * G_2)(x) = \int_{-\infty}^{\infty}\left(\varepsilon_{[0,1]}(y) - \sum_{\beta=0}^{N} C_{\beta,0}\delta(y-x_\beta) + \right.$$

$$\left. + \sum_{\beta=0}^{N} C_{\beta,1}(\delta'(y-x_\beta) - \delta(x-x_\beta))\right)G_2(x-y)dy.$$

Then, using properties

$$\int_{-\infty}^{\infty}\delta(x-a)f(x)dx = f(a) \text{ and } \int_{-\infty}^{\infty}\delta'(x-a)f(x)dx = -f'(a)$$

of the Dirac delta-function, for the last expression we have

$$(\ell * G_2)(x) = \int_0^1 G_2(x-y)dy - \sum_{\beta=0}^{N} C_{\beta,0}G_2(x-x_\beta) -$$

$$-\sum_{\beta=0}^{N} C_{\beta,1}\left((G_2(x-y))'_{y=x_\beta} + G_2(x-x_\beta)\right). \tag{14}$$

Now, using the well known formula $(\text{sign}(x))' = 2\delta(x)$, we have

$$(G_2(x-y))'_{y=x_\beta} = -\frac{\text{sign}(x-x_\beta)}{2}\left(\frac{e^{x-x_\beta} + e^{-(x-x_\beta)}}{2} - 1\right).$$

Putting the last expression into (14) and taking into account (12) for the convolution we come

$$(\ell * G_2)(x) = \int_0^1 G_2(x-y)dy - \sum_{\beta=0}^{N} C_{\beta,0} G_2(x-x_\beta) -$$

$$-\sum_{\beta=0}^{N} C_{\beta,1}\left(\frac{\operatorname{sign}(x-x_\beta)}{2}\left(-\frac{e^{x-x_\beta}+e^{-(x-x_\beta)}}{2}+1+\frac{e^{x-x_\beta}-e^{-(x-x_\beta)}}{2}-(x-x_\beta)\right)\right).$$

Simplifying the last expression and putting it into (11) we get (13).

Theorem 1 is proved.

An upper bound for the error of the quadrature formula (1)

Now we are ready to calculate the norm of the error functional (4) that gives us an upper bound for the error of the quadrature formula (1). Using Theorem 1 and (4), from (10) by direct calculation we have the following main result which is the solution of Problem 1.

Theorem 2. *The norm of the error functional (4) for quadrature formulas of the form (1) is expressed as follows*

$$\|\ell\|^2_{W_2^{(2,1)}} = \sum_{\beta=0}^{N}\sum_{\gamma=0}^{N} C_{\beta,1}C_{\gamma,1}\frac{|x_\beta - x_\gamma|}{2} + 2\sum_{\beta=0}^{N} C_{\beta,1}\left(\sum_{\gamma=0}^{N} C_{\gamma,0}\frac{\operatorname{sign}(x_\beta - x_\gamma)}{2}(e^{-(x_\gamma - x_\beta)}+\right.$$

$$\left.+x_\gamma - x_\beta - 1) + \int_0^1 \frac{\operatorname{sign}(x-x_\beta)}{2}(e^{-(x-x_\beta)} + x - x_\beta - 1)dx\right)+$$

$$+\sum_{\beta=0}^{N} C_{\beta,0}\left(\sum_{\gamma=0}^{N} C_{\gamma,0}G_2(x_\beta - x_\gamma) - 2\int_0^1 G_2(x-x_\beta)dx\right) + \int_0^1\int_0^1 G_2(x-y)dxdy. \quad (15)$$

Next we consider the problem of minimization of the norm (15) with respect to coefficients $C_{\beta,1}$ under the condition (7).

The coefficients of the optimal quadrature formula (1)

In this setion we give the main result of the paper. We present the coefficients of the optimal quadrature formula (1). In order to get the optimal coefficients we use the Lagrange method of undetermined multipliers.

We consider the following Lagrange function for minimization of the expression (15) under the condition (7)

$$\Lambda(C_{\beta,1}, \lambda) = \|\ell\|^2 - 2\lambda(\ell, 1).$$

For the critical point of the last function we get the system

$$\sum_{\gamma=0}^{N} C_{\gamma,1}\frac{|h\beta-h\gamma|}{2}+\lambda=F\cdot\left((h\beta)^2+(h\beta)-\frac{1}{2}\right),\ \beta=0,1,...,N,$$
$$\sum_{\gamma=0}^{N} C_{\gamma,1}=-2F,$$

where $F=\frac{e^h-1}{h(e^h+1)}-\frac{1}{2}$.

It should be noted that the solution of the last system gives the minimum to the norm (15) under the condition (7).

The following is true.

Theorem 3. *In the space $W_2^{(2,1)}$ the coefficients of the optimal quadrature formula (1) have the following representations*

$$C_{0,0}=\frac{e^h-1}{e^h+1},$$

$$C_{\beta,0}=\frac{2(e^h-1)}{e^h+1},\quad \beta=1,2,...,N-1,$$

$$C_{N,0}=\frac{e^h-1}{e^h+1},$$

$$C_{0,1}=hF,$$

$$C_{\beta,1}=2hF,\quad \beta=1,2,...,N-1,$$

$$C_{N,1}=(h-4)F,$$

where

$$F=\frac{e^h-1}{h(e^h+1)}-\frac{1}{2}.$$

Thus, we have solved Problem 2.

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The Role of Artificial Intelligence on Economic Modelling: An Analytical Reinterpretation

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DOI 10.5281/zenodo.18217207

1. Introduction: Artificial Intelligence and Economic Modelling

In recent decades, artificial intelligence (AI) has increasingly penetrated both natural and social sciences, offering new methodological possibilities through machine learning, algorithmic reasoning, and advanced computational systems. Economic modelling, as a field that seeks to explain and forecast complex socio-economic phenomena, has been particularly influenced by these developments. The core objective of AI is to replicate selected aspects of human cognition and decision-making within electronic or mechanical systems, thereby enhancing problem-solving capacity and analytical efficiency.

Although often perceived as a modern innovation, AI has historical roots extending back to the mid-twentieth century. Its formal institutionalization occurred during World War II, with the emergence of early computing hardware and manually programmed algorithms designed for cryptographic analysis. A landmark contribution in this period was made by Alan Turing, whose work on algorithmic logic and cryptographic machines laid the intellectual foundations of both modern computing and artificial intelligence (Hodges, 1983).

Within analysis of contemporary economics AI presents both methodological challenges and transformative opportunities. Although early applications were largely confined to programming and numerical computation, recent research demonstrates a rapid expansion into areas such as financial markets, policy modelling, innovation economics, and labor analysis (Acemoglu & Restrepo, 2018; Cockburn et al., 2019; Bickley et al., 2022). Moreover, Kumar (2024) sheds light on AI‘s transformative potential on economic analysis along with data privacy and security in model transparency and interpretability. Collectively, this literature highlights AIвЂ™s potential to enhance analytical precision, scalability, and scenario-based evaluation in economic modelling.

2. Economic Modelling: Definition, Classification, and Historical Foundations

Economic modelling may be defined as a systematic analytical process that employs theoretical frameworks and quantitative or qualitative techniques to examine the causes and consequences of economic phenomena across time and space. As Ruiz Estrada (2011) argues, economic models function as conceptual and technical instruments designed to address societal challenges through structured empirical and theoretical reasoning.

Building on this definition, economic modelling can be classified into twenty thematic categories, ranging from trade and finance to health, digital economies, and environmental

management. This classification framework underpins the database architecture of the Autonomous Economic Decision Maker Simulator (AEDM-Simulator), which draws on approximately 2,000 articles published in the Journal of Economic Modelling between 1984 and 2023.

Historically, the emergence of economic modelling is closely associated with the late 1970s, when economistsвЂ”particularly within the United Nations system recognized the limitations of overly abstract and mathematically detached economic theories. As Kumar & Suresh(2024) and later scholars observed, economics had become increasingly disconnected from practical policy relevance, prompting a shift toward empirically grounded and application-oriented modelling approaches.

The transformation was accompanied by broader intellectual shifts. During the mid-twentieth century, development economics emphasized capital accumulation and industrialization, while later decades introduced concerns related to market liberalization, structural adjustment, social development, and governance. By the early twenty-first century, issues such as globalization, financial instability, inequality, and environmental sustainability had become hot topics for economic modelling debates (Ruiz Estrada & Park, 2018).

3. Evolution of Analytical Tools in Economic Modelling

Economic modelling has undergone several major methodological transformations. Initially dominated by qualitative approachesвЂ”such as historical analysis and political economyвЂ”the field gradually incorporated formal economic theory and quantitative techniques. This transition marked a shift from narrative explanations to mathematically structured models supported by statistics and econometrics. The first major analytical transformation involved the formalization of political economy, which introduced systematic concepts, assumptions, and theoretical structures. This was followed by the mathematical revolution initiated by Cournot (1838), whose work applied algebraic methods to market behavior, laying the groundwork for static equilibrium analysis.

A third transformation occurred with the rise of econometrics in the early twentieth century, driven by scholars such as Frisch, Tinbergen, and Klein. Econometrics enabled economists to move beyond static representations toward dynamic, data-driven modelling, significantly enhancing the explanatory and predictive power of economic analysis. Throughout the twentieth century, economic modelling increasingly relied on advanced mathematical and statistical tools, including calculus, optimization theory, time-series analysis, and general equilibrium frameworks. These tools underpinned influential contributions in welfare economics, growth theory, monetary policy, and rational expectations (Samuelson, 1947; Solow, 1956; Friedman, 1957; Barro, 1976).Looking forward, economic modelling faces three interconnected challenges: real-time simulation, artificial intelligence, and organic computational intelligence. AI-based modelsвЂ”particularly those employing neural networksвЂ”offer promising capabilities for pattern recognition, scenario generation, and policy evaluation. However, these systems remain fundamentally dependent on historical data and predefined algorithms, limiting their capacity for creativity, ethical judgment, and contextual reasoning.

4. The Autonomous Economic Decision Maker Simulator (AEDM-Simulator)

The AEDM-Simulator represents an innovative attempt to integrate artificial

intelligence, big data analytics, neural networks, and econographicology into a unified decision-support system. Its primary objective is to assist researchers and policymakers in identifying efficient and evidence-based solutions to economic problems. Particularly, the AEDM-Simulator database is found on enormous quantity of published papers with various volumes along with issues from the Journal of Economic modeling (JEM) by Elsevier between 1984 and 2023.
The system operates through five interconnected modules: input processing, database storage, analytical evaluation, solution generation, and final reporting. By systematically analyzing high-citation research from the Journal of Economic Modelling, the AEDM-Simulator generates multidimensional visualizations and policy recommendations with a reported accuracy rate of approximately 85%.

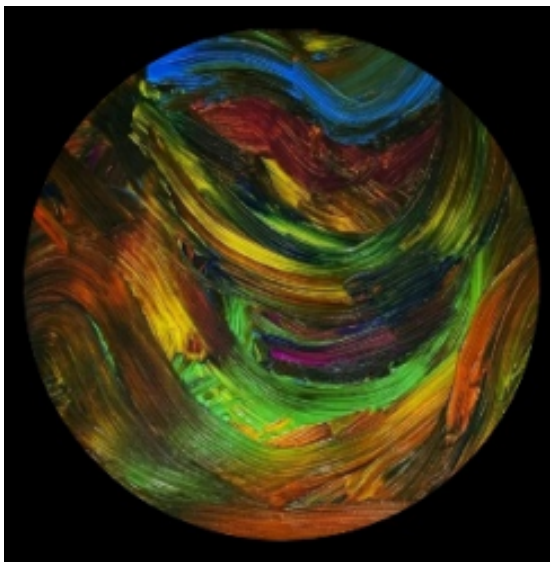
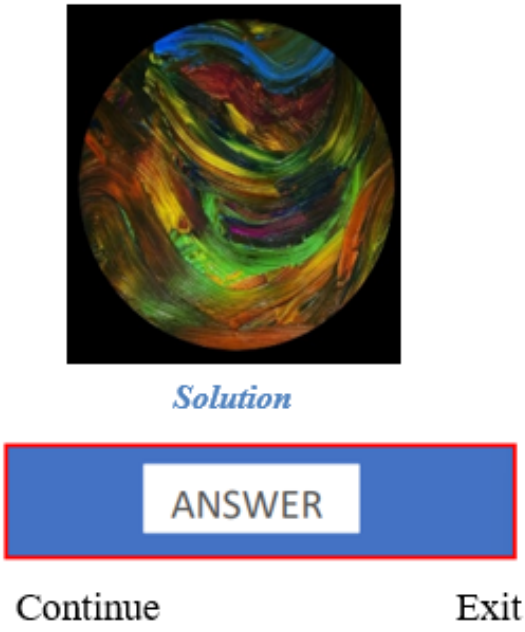


Fig. 1: AEDM-Simulator system framework
Front Page



Final Results Page
"Please type here your is sue"

A key feature of the AEDM-Simulator is its capacity to classify economic issues across twenty thematic databases, enabling rapid cross-referencing and scenario-based evaluation. Through iterative filtering and partial differentiation, the system identifies robust policy recommendations while minimizing risk and analytical vulnerability.

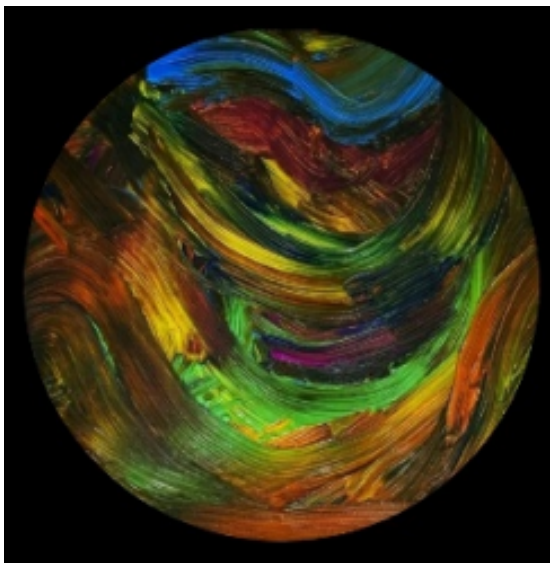


Fig. 2: Example of the Input Access Window to AEDM-Simulator

How to deal with Government Expenditure and Public Debt Policy in the UK?

To answer the question AEDM simulation distinguishes data into 3 keywords such as primary, secondary, complementary. This structure reflects the AEDM assumption that public debt sustainability cannot be analyzed in isolation, but must be jointly evaluated with growth, monetary conditions, and labor-market outcomes. **AEDM-Simulator Database Mapping for the UK Case** According to **Table 1**, the AEDM-Simulator distributes fiscal-policy-related knowledge as follows:

- **Main database**
  - **DB4**: Fiscal, Government Expenditure, and Public Debt Policy Modeling (180 papers, 7%)
- **Strong complementary databases**
  - **DB11**: Economic Growth and Development Economic Modeling (260 papers, 10%)
  - **DB7**: Monetary, Financial Markets, and Investment Policy Modeling (800 papers, 32%)
  - **DB16**: Unemployment, Inflation, and Poverty Economic Modeling (40 papers, 2%)

Based on AEDM database ranking and algorithmic logic, the optimal UK strategy for monetary market along with Public Debt Policy is as follows: A growth-linked, coordinated fiscal framework, where government expenditure is selectively targeted, public debt is stabilized through growth rather than austerity, and fiscal decisions are dynamically aligned with monetary and labor-market conditions.

Table 1. The Twenty Database Storage Spaces (DBxi) in the AEDM-Simulator

Number	**The Twenty Database Storage Spaces (DBxi) in the AEDM-Simulator**	Papers	Percentage
DB1	The Domestic and International Trade Pol. Mod.	300	12%
DB2	The Energy, Infra., Commu., and Trans. Pol. Mod.	120	5%
DB3	The Environmental, Agriculture, and Natural Resources Pol. Mod.	120	5%
DB4	The Fiscal, Government Expenditure, and Public Debt Pol. Mod.	180	7%
DB5	The Institutional, regulation, and Negotiation Pol. Mod.	50	2%
DB6	The Labor, employment, and Population Economic Modeling	100	4%
DB7	The Mon., Stock Market, Ex-Rate, Fin., bank, ins., and Inv. Pol. Mod.	800	32%
DB8	The Production, Savings, Income and Consumption Economic Modeling	240	9%
DB9	The Technological and R&D Economic Modeling	50	2%
DB10	The Welfare and Social Economic Modeling	30	1%
DB11	The Economic Growth and Development Economic Modeling	260	10%
DB12	The Education and Human Capital Development Economic Modeling	75	3%
DB13	The Military and National Security Economic Modeling	30	1%
DB14	The Gender, Culture, Discrimination, Racism Economic Modeling	10	0%
DB15	The Natural Disasters and Food Security Economic Modeling	10	0%
DB16	The Unemployment, Inflation, Poverty Economic Modeling	40	2%
DB17	Tourism and Entertainment P.M.	2	0%
DB18	Health Economic Modeling	35	1%
DB19	Digital Economic, ICT, AI, E-pay., Economic Modeling	3	0%
DB20	The Miscellaneous Economic Modeling	73	3%
Total		2000	100%

5. Conclusion

Artificial intelligence has emerged as a powerful analytical instrument in economic modelling, particularly in relation to data processing, simulation, and quantitative evaluation. Nevertheless, AI remains constrained by its reliance on predefined logic, historical data, and algorithmic structure. Critical elements of economic reasoningвЂ”such as creativity, ethical judgment, and contextual sensitivityвЂ”continue to require human cognition.

Based on an extensive review of nearly four decades of economic modelling research, this study concludes that AI should be understood as a complementary, rather than substitutive, tool in economic analysis. Its greatest contribution lies in enhancing computational efficiency and analytical scope, while final decision-making and policy interpretation must remain human-centered.

As economic systems grow increasingly complex and interconnected, the integration of AI into economic modelling will continue to evolve. When combined with interdisciplinary approaches and human judgment, AI holds significant promise for advancing the study of

dynamic economic phenomena across diverse social and historical contexts.

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First-Passage-Based Last-Passage Algorithm for Charge Density on a Conducting Surface

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According to probabilistic potential theory, first- and last-passage algorithms have been developed. Usually the first-passage algorithms with an enclosing sphere are used for overall charge distribution on a closed conducting object, and last-passage algorithms are used for computing the charge density at a specific point on the conducting object. The first- and last-passage algorithms are inherently connected.

In this paper, we combine the first- and last-passage algorithms. We develop an algorithm for computing the charge density at a specific point on a conducting object via the overall charge density distribution on the conducting object obtained as the simulation result of the first-passage algorithm with an enclosing sphere. We demonstrate the proposed algorithm for the charge density on a sphere and on the unit cube held at

unit potential. The numerical results show good agreement with theoretical solutions and with results obtained from other simulation methods.

An Uncertainty-Oriented Production Model for Task Allocation in Electronic Document Management Systems

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DOI 10.5281/zenodo.18217209

Abstract

This work is devoted to solving the problem of task allocation in automated electronic document management systems. To solve the stated problem, the necessity of applying methods based on fuzzy logic and fuzzy inference systems is substantiated. A specially developed uncertainty-oriented production model is proposed. In addition, the practical application of this model is described using the example of task allocation for maintaining the register of personal data operators in the electronic document management system of the territorial department. The results obtained using the developed software complex are compared with the effectiveness of other uncertainty-based task allocation methods. A conclusion is made about the adequacy of the uncertainty-oriented production model and the necessity of its practical application in supporting decision-making for task allocation in electronic document management systems.

Keywords: electronic document management, task allocation, fuzzy logic, uncertainty-oriented production model, software complex, max–min aggregation, additive aggregation.

Introduction

At present, there is a noticeable trend toward transitioning from paper-based document processing to the active use of electronic document management systems in various areas of human activity. According to [?], electronic document management is the movement of electronic documents within an organization and the activities that ensure this movement.

For organizational management, the use of electronic document management systems enables faster access to information required for managerial decision-making, as well as operational control over execution discipline. For system users, these systems provide opportunities for rapid document search, simplification of preparation for various events

and activities, and acceleration of document preparation, coordination, and approval processes.

Let us consider the electronic document management system used in the activities of the territorial bodies of . From the standpoint of protecting the rights of personal data subjects, this system solves the following main tasks [?]:

- maintaining the register of personal data operators;
- exercising control and supervision over the activities of personal data operators;
- reviewing citizens' complaints and appeals.

Among the tasks solved through this system, the most labor-intensive one is maintaining the register of personal data operators. This task includes registering operators, entering information (or changes) about operators into a unified information system, deleting operator information from the register, and providing extracts from the register upon request.
The specificity of this task lies in the requirement to register incoming applications within one day and to enter relevant information into the register no later than 15 days from the date of receipt of the application.

At the same time, the head of the department responsible for maintaining the register faces the problem of optimally allocating tasks among department employees, i.e., users of the electronic document management system. This problem is caused by the following main factors:

- uncertainty in the daily number of applications received from personal data operators;
- varying complexity of processing applications from different categories of operators;
- differences in qualification levels and current workload of executors;
- the necessity to complete tasks within deadlines established by administrative regulations.

Obviously, there is no single precise solution to this problem. Therefore, the effectiveness of the territorial bodyвЂ™s activities in maintaining the register of personal data operators depends on the qualifications of the department head and the correctness of decisions made when allocating tasks among executors [?].

Problem Statement of Task Allocation in an Electronic Document Management System

Let us consider the task allocation problem for maintaining the register of personal data operators [?, ?]. Assume that there is a set of N tasks with different levels of complexity: low (S_1), medium (S_2), and high (S_3). There is also a set of executors (alternatives):

$$A = \{a_1, a_2, a_3, a_4\},$$

where a_1 is a chief expert specialist, a_2 is a leading expert specialist, a_3 is an expert specialist, and a_4 is a first-category specialist.

The criteria for task allocation among executors (executor characteristics) are defined as follows:

- C_1 — current workload of the executor;

- C_2 — working capacity of the executor;
- C_3 — qualification level of the executor.

It is required to solve the problem of optimal task allocation, which serves as decision support for the manager in rationally selecting an executor for each of the N tasks [?].

The decision-maker has limited time to assign an executor to a task and is not inclined to accept a high risk of missing the established deadlines.

Although many approaches exist for solving optimization problems [?, ?, ?], under these conditions the decision-maker must consider uncertainty factors as well as the vagueness of the applied criteria. Therefore, uncertainty-based task allocation methodsвЂ”namely, approaches based on fuzzy logic and fuzzy inference systemsвЂ”are required [?, ?].

Such an approach corresponds to the expertвЂ™s reasoning logic in decision-making. In this work, a method based on an uncertainty-oriented production model is used to implement rational selection of alternatives [?, ?, ?].

Construction of a Task Allocation Model Based on an Uncertainty-Oriented Production Model

To solve the stated problem, we consider an approach based on constructing and applying an uncertainty-oriented production model [?, ?]. The following form of fuzzy production rules is chosen as a model for representing expert knowledge [?]:

$$\text{IF } x_1 = \bar{A}_1 \text{ AND } x_2 = \bar{A}_2 \text{ AND } \ldots \text{ AND } x_n = \bar{A}_n, \text{ THEN } y = B\ [CF], \qquad (1)$$

where x_i are input variables, $\bar{A}_i$ are fuzzy gradations of the input variables, y is the output variable, B is a deterministic output value, and $CF \in [0, 1]$ is the certainty factor of the rule.

The model input parameters are defined as:

- x_1 — workload level of executor a_1;
- x_2 — workload level of executor a_2;
- x_3 — workload level of executor a_3;
- x_4 — workload level of executor a_4;
- x_5 — task complexity level.

The workload level of executors is determined by the number of tasks they perform simultaneously. If the number of tasks equals 20, this is considered the lower bound of maximum workload, corresponding to 100% occupancy. The model also allows workload levels to be both below and above 100%.

The workload levels are divided into three categories: low (H), medium (S), and high (B). Membership functions corresponding to these categories are shown in results.

Task complexity levels correspond to categories of personal data operators: low complexity (individuals and sole proprietors), medium complexity (legal entities), and high complexity (state and municipal authorities). Membership functions for task complexity levels are shown in results.

The certainty factor CF of each rule reflects the expertвЂ™s confidence in assigning

a task to a particular executor and is based on working capacity, qualifications, and workload level.

The certainty factor for assigning an executor a_i to a task of complexity level k is calculated as:

$$CF_k^i = \mu_{\bar{C}_1}(a_i) \cdot \mu_{\bar{C}_2}(a_i) \cdot \mu_{\bar{C}_{3k}}(a_i), \tag{2}$$

where $\mu_{\bar{C}_1}(a_i)$ is the utility of selecting executor a_i based on workload, $\mu_{\bar{C}_2}(a_i)$ is based on working capacity, and $\mu_{\bar{C}_{3k}}(a_i)$ is based on qualification for tasks of complexity level k.

The workload-based utility is calculated using the heuristic formula:

$$\mu_{\bar{C}_1}(a_i) = \begin{cases} 1 - \dfrac{n_i}{N}, & N \neq 0, \\ 1, & N = 0, \end{cases} \tag{3}$$

where n_i is the number of tasks performed by executor a_i, and $N = \sum_{i=1}^{4} n_i$ is the total number of tasks.

Conclusion

The results of the conducted studies demonstrate the stability of the proposed methods and the consistency of the obtained decisions with expert opinions. An intelligent system for decision support in task allocation for maintaining the register of personal data operators has been developed. The implementation and practical application of this software complex in the electronic document management system of the functioning Administration for the Republic of Tatarstan is planned. The use of this system makes it possible to improve the quality of decisions through rational selection of effective alternatives.

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The time-optimal problem of steering two rigidly connected objects between prescribed states

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DOI 10.5281/zenodo.18217221

Abstract In this paper, the problem of optimal control of two rigidly connected objects moving in the plane is investigated. The problem is interpreted the synthesis of a minimum-time optimal control is carried out using PontryaginвЂ™s maximum principle. The structure of the controls, switching functions, and the geometric interpretation of the optimal trajectory are analyzed in detail. The developed control concepts are of significant importance for applications to motion planning problems of unmanned aerial vehicles.

Keywords: optimal control, Pontryagin's maximum principle, minimum time, switching functions.

1. Introduction

Optimal control theory studies the problems of controlling dynamical systems in the most efficient manner according to a given criterion and serves as an important theoretical foundation for the analysis of complex technical and engineering systems [6], [17]. The fundamental achievements of this theory are associated with the maximum principle developed by L. S. Pontryagin [16] and his scientific school, which is widely used in solving minimum-time and minimum-energy problems under bounded (admissible) controls. A. A. Azamov [15] focused on the application of PontryaginвЂ™s maximum principle to linear and differential control systems as well as pursuitвЂ“evasion games [14]. In the field of optimal control theory, Sh. A. Alimov and G. I. Ibragimov [20] determined control functions that minimize (or maximize) a given performance criterion under the dynamic constraints of the system [11], [19]. This approach makes it possible to identify the structural properties of optimal controls, in particular, bangвЂ“bang type controls and the role of switching functions.

In recent years, interest in optimal control theory has increased significantly, especially due to practical problems related to unmanned aerial vehicles and autonomous drones. Issues of coordinated motion of drones, as well as the optimal adjustment of their spatial orientation and positioning, have become highly important in logistics, aerial photography, search-and-rescue operations, and monitoring systems (Beard and McLain, 2012) [3]. In such applications, the boundedness of control signals, the nonlinearity of system dynamics, and the requirement to minimize time make the use of optimal control methods essential [2], [4].

The theory of pursuitвЂ“evasion type differential games with a prescribed duration and integral constraints on the controls was studied in depth by G. I. Ibragimov and A. Sh. Kuchkarov, who conducted research on optimal strategies and their applications [10]. In contemporary studies, optimal trajectory planning problems for UAV systems are investigated based on various criteria. In particular, trajectories based on minimum energy or smoothness criteria (Mellinger and Kumar, 2011) [13], as well as optimal control models relying on the differential flatness properties of drone dynamics (Faessler et al., 2018) [7], have been extensively analyzed. However, in many real systems, control signals are strictly bounded [12], which gives rise to the need for a thorough theoretical investigation of minimum-time bangвЂ“bang controls..

From this perspective, the optimal motion of mechanical systems with a rigid geometric structure attracts particular scientific interest. Such models are effective for describing a system of rigidly connected drones or the planar translational and rotational motion of a single rigid platform [9], [18]. Determining the minimum-time optimal trajectories of these systems under bounded controls is theoretically challenging and requires an in-depth analysis based on PontryaginвЂ™s maximum principle (Agrachev and Sachkov, 2004) [1].

This paper investigates the minimum-time optimal control problem for a segment of constant length moving in the plane. The problem is analyzed using PontryaginвЂ™s maximum principle, demonstrating that the optimal controls have a bangвЂ“bang structure and providing a precise geometric interpretation of the optimal trajectory. The results contribute to the development of optimal control theory and have practical significance for motion planning problems of coordinated drones and robotic systems.

2. Problem statement and its reduction to an optimal control formulation

A segment AB of length $l > 0$ is given in the plane, with its initial position at the points

$$A = (a, b), \quad B = (a + l, b),$$

and its final position at the points

$$C = (0, 0), \quad D = (l, 0).$$

During the motion of the segment, its length remains constant, and the velocity vectors of its endpoints $A(t)$ and $B(t)$ are always perpendicular to the segment $\overline{AB}$ and their magnitudes do not exceed 1.

2.1. Mathematical model

Let us denote $A(t) = (x_1(t), x_2(t))$, where $x_1(0) = a$, $x_2(0) = b$.

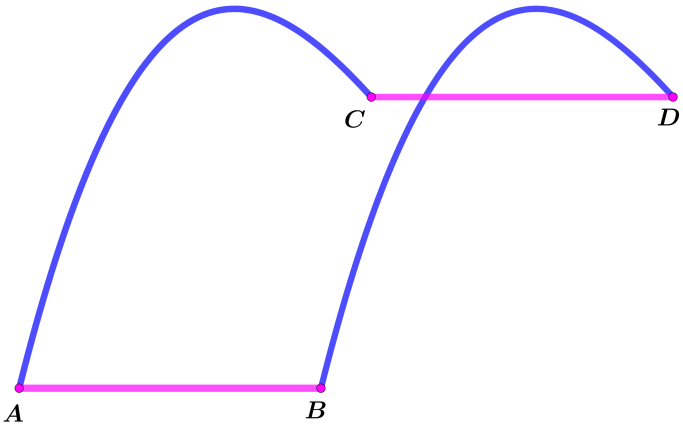
We introduce the state vector $x(t) = (x_1(t), x_2(t), \theta(t))$, where $\theta(t)$ is the angle formed by the segment $A(t)B(t)$ with the Ox axis.

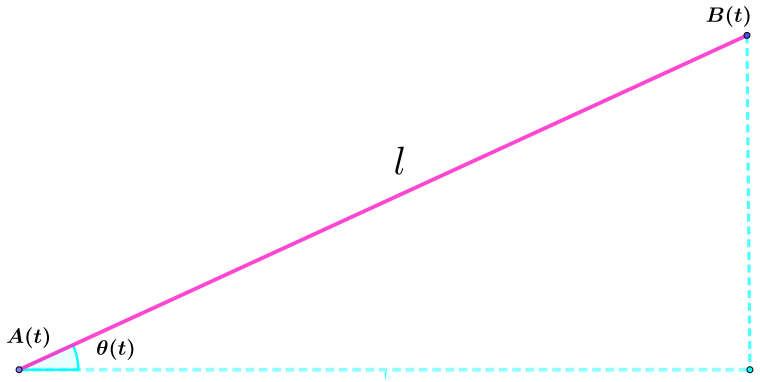
Since the segment is rigid, the position of its second endpoint $B(t)$ is determined by

$$B(t) = (x_1(t) + l\cos\theta(t), x_2(t) + l\sin\theta(t)) = A(t) + l\big(\cos\theta(t),\ \sin\theta(t)\big) \qquad (1)$$

It follows that

$$\overrightarrow{A(t)B(t)} = l\big(\cos\theta(t), \sin\theta(t)\big),$$





and the unit vector perpendicular to it has the form

$$\big(-\sin\theta(t), \cos\theta(t)\big).$$

Consequently, the velocity vectors of the segment's endpoints can be written as

$$\dot{A}(t) = u_1(t)\big(-\sin\theta(t), \cos\theta(t)\big), \qquad \dot{B}(t) = u_2(t)\big(-\sin\theta(t), \cos\theta(t)\big), \tag{2}$$

where the control functions satisfy the constraints

$$|u_1(t)| \le 1, \qquad |u_2(t)| \le 1.$$

Differentiating equation (1) with respect to time, we obtain

$$\dot{B}(t) = \dot{A}(t) + l\big(-\sin\theta(t) \cdot \dot{\theta}(t), \cos\theta(t) \cdot \dot{\theta}(t)\big) \tag{3}$$

From equations (2) and (3), it follows that

$$\dot{\theta}(t) = \frac{u_2(t) - u_1(t)}{l}.$$

As a result, the system is described by the following controlled dynamical system:

$$\begin{aligned} \dot{x}_1 &= -u_1 \sin\theta, \\ \dot{x}_2 &= \ \ u_1 \cos\theta, \\ \dot{\theta} &= \frac{u_2 - u_1}{l}, \end{aligned} \tag{4}$$

with the initial conditions

$$x_1(0) = a, \qquad x_2(0) = b, \qquad \theta(0) = 0.$$

The control set is defined as

$$U = (u_1, u_2) \in [-1, 1] \times [-1, 1].$$

The terminal state of the system is specified by the following set:

$$S = \Big\{(x_1, x_2, \theta) \in \mathbb{R}^2 \times [0, 2\pi) \;\Big|\; x_1 = 0,\ x_2 = 0,\ \theta = 0\Big\}.$$

That is, for the optimal trajectory, the condition $x(T) \in S$ must be satisfied.

2.2. Optimal control problem

Based on the dynamic model presented above, we consider the minimum-time optimal control problem.

Problem (P). The control functions

$$u(\cdot) = (u_1(\cdot), u_2(\cdot)) \in U = [-1, 1] \times [-1, 1]$$

must be chosen so as to minimize the motion time T, while the state trajectory

$$\dot{x} = f(x, u) = \begin{bmatrix} -u_1 \sin\theta \\ u_1 \cos\theta \\ \frac{1}{l}(u_2 - u_1) \end{bmatrix}, \tag{5}$$

satisfies the following initial and terminal conditions:

$$x(0) = (a, b, 0), \qquad x(T) = (0, 0, 0).$$

Remark: Since the point $B(t)$ is expressed via $A(t)$ as

$$B(t) = A(t) + l(\cos\theta(t), \sin\theta(t)),$$

no separate state equation for $B(t)$ is included in the system dynamics.

The initial and terminal orientation conditions $\theta(0) = \theta(T) = 0$ and equation (4) imply the following integral constraint:

$$\int_0^T \big(u_2(t) - u_1(t)\big)\, dt = 0.$$

This condition ensures that the final orientation of the segment coincides with the initial one, i.e., the total rotation of the segment is zero.

2.3. Pontryagin's maximum principle

We consider the following control system:

$$\dot{x} = f(x,u), \qquad u(t) \in U, \qquad x(0) = x_0, \quad x(T) = x_1.$$

For this system, the minimum-time optimal control problem is analyzed using Pontryagin's maximum principle (PMP).

If $u^*(\cdot)$ is an optimal control and $x^*(\cdot)$ is the corresponding optimal trajectory, then there exists a nonzero absolutely continuous adjoint vector function

$$p(\cdot) = (p_1(\cdot), p_2(\cdot), p_3(\cdot))$$

and a number $p_0 \leq 0$, which for the time-optimal problem is $p_0 = -1$. Furthermore, the condition

$$(p(\cdot), p_0) \neq 0$$

must be satisfied.

The Hamiltonian function is defined as

$$H(x,p,u) = p \cdot f(x,u) + p_0,$$

and the following necessary conditions must hold:

1) Hamiltonian system:

$$\begin{aligned} \dot{p}_1 &= -\frac{\partial H(x^*(t), p, u^*(t))}{\partial x_1}, \\ \dot{p}_2 &= -\frac{\partial H(x^*(t), p, u^*(t))}{\partial x_2}, \\ \dot{p}_3 &= -\frac{\partial H(x^*(t), p, u^*(t))}{\partial \theta}. \end{aligned} \tag{6}$$

2) Maximum condition:

$$H(x^*(t), p(t), u^*(t)) = \max_{u \in U} H(x^*(t), p(t), u), \qquad t \in [0,T]. \tag{7}$$

3) For the time-optimal control problem, along the optimal process,

$$H(x^*(t), p(t), u^*(t)) \equiv 0 \tag{8}$$

holds.

(4) For the system, the Hamiltonian function can be written as

$$\begin{aligned} H &= p \cdot f(x,u) - 1 \\ &= p_1(-u_1 \sin\theta) + p_2(u_1 \cos\theta) + p_3 \frac{u_2 - u_1}{l} - 1 \\ &= u_1 \Big(- p_1 \sin\theta + p_2 \cos\theta - \frac{p_3}{l} \Big) + u_2 \frac{p_3}{l} - 1 \\ &= u_1\, \varphi_1(t) + u_2\, \varphi_2(t) - 1, \end{aligned} \tag{9}$$

where

$$\varphi_1(t) = -p_1 \sin\theta + p_2 \cos\theta - \frac{p_3}{l}, \qquad \varphi_2(t) = \frac{p_3}{l}$$

are the switching functions.

From the maximum condition, the Hamiltonian reaches its maximum under the following controls:

$$u_1(t) = \text{sign } \varphi_1(t), \qquad u_2(t) = \text{sign } \varphi_2(t),$$

and if $\varphi_i(t) = 0$, then $u_i(t) \in [-1, 1]$ can be chosen arbitrarily. Therefore,

$$u_1 \in \{-1, 1\}, \qquad u_2 \in \{-1, 1\},$$

i.e., the optimal control is of the bangвЂ“bang type [8].

From the Hamiltonian system, the adjoint equations take the following form:

$$\begin{aligned} \dot{p}_1 &= -\frac{\partial H}{\partial x_1} = 0, \\ \dot{p}_2 &= -\frac{\partial H}{\partial x_2} = 0, \\ \dot{p}_3 &= -\frac{\partial H}{\partial \theta} = u_1(p_1 \cos\theta + p_2 \sin\theta). \end{aligned} \tag{10}$$

We introduce the following notations:

$$q = p_1 \cos\theta + p_2 \sin\theta,$$

$$r = -p_1 \sin\theta + p_2 \cos\theta.$$

Using these notations, the switching functions take the form:

$$\begin{aligned} \varphi_1 &= r - \frac{p_3}{l}, \\ \varphi_2 &= \frac{p_3}{l}, \\ \dot{p}_3 &= u_1 q. \end{aligned}$$

As a result, the optimal control is determined as follows:

$$u_1(t) = \begin{cases} +1, & \varphi_1(t) > 0, \\ -1, & \varphi_1(t) < 0, \end{cases} \qquad u_2(t) = \begin{cases} +1, & \varphi_2(t) > 0, \\ -1, & \varphi_2(t) < 0. \end{cases} \tag{11}$$

From the third necessary condition for the Hamiltonian, i.e., $H \equiv 0$, it follows that

$$u_1(t)\varphi_1(t) + u_2(t)\varphi_2(t) = 1.$$

3. Solving the optimal control problem

The point $M(t)$, defined as the geometric midpoint of the segment $A(t)B(t)$, plays an important role in describing the overall motion of the system. Its coordinates are given by

$$M = \left(x_1 + \frac{l}{2}\cos\theta,\ x_2 + \frac{l}{2}\sin\theta \right).$$

This expression clearly shows that the position of the point M depends on the coordinates x_1, x_2 and the orientation angle θ of the body.

The velocity vector of the point M is obtained by differentiating with respect to time:

$$\dot{M} = \left(\dot{x}_1 - \frac{l}{2} \sin\theta \cdot \dot{\theta},\ \dot{x}_2 + \frac{l}{2} \cos\theta \cdot \dot{\theta} \right).$$

Here, the first term represents the translational motion, while the second term accounts for the contribution of rotational motion to the midpoint.

Taking into account the state equations of the control system, the velocity of the point M can be simplified to

$$\dot{M} = \frac{u_1 + u_2}{2} (-\sin\theta,\ \cos\theta) = s(-\sin\theta,\ \cos\theta),$$

where

$$s = \frac{u_1 + u_2}{2}. \tag{12}$$

Thus, the point M moves at each instant in a direction perpendicular to the angle θ. The magnitude s represents the translational speed of M and equals the average of the two controls.

The modulus of the velocity vector is

$$|\dot{M}| = |s|,$$

i.e., the speed of the point M is determined solely by the parameter s.

To separate the relative influence of the controls, we introduce the following notation:

$$a = \frac{u_2 - u_1}{2}, \qquad \dot{\theta} = \frac{2a}{l}. \tag{13}$$

Here, the magnitude a represents the angular velocity of the system and is determined by the difference of the controls. Thus, if u_1 and u_2 are equal, the system moves straight without rotation; otherwise, a rotational motion occurs.

Consequently, instead of the original controls (u_1, u_2), the motion of the system can be described by the pair (s, a), which simplifies the model both geometrically and analytically.

Using the constraints $|u_1| \le 1, |u_2| \le 1$ and $u_1 = \pm k \cdot u_2,\ 0 \le k \le 1$, the following important estimate is obtained:

$$|s| + |a| \le 1. \tag{14}$$

This inequality describes the admissible region in the control space as a rhombus and indicates that the optimal control has bangвЂ“bang character.

3.1. Reduction of the Problem to its Equivalent Form

Using the notations introduced in the previous section, the problem can be represented in an equivalent geometric form. That is, the initial and final positions of the point M are given by

$$M(0) = M_0 = \left(a + \frac{l}{2},\ b \right), \qquad M(T) = M_1 = \left(\frac{l}{2},\ 0 \right).$$

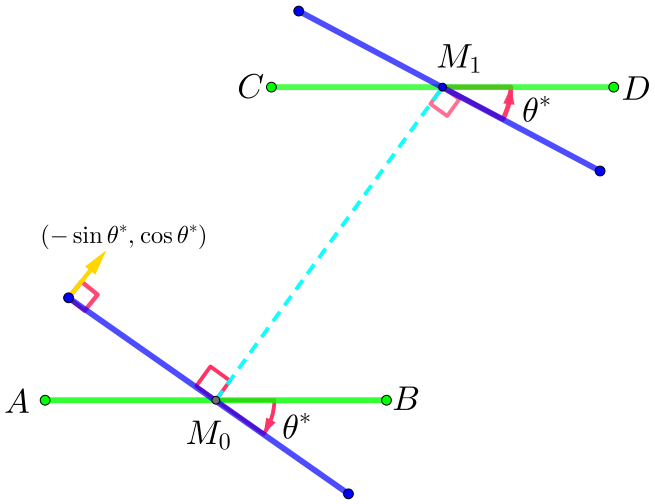
Moreover, the orientation of the system at the initial and final times is required to be the same, i.e.,

$$\theta(0) = \theta(T) = 0.$$

Among the trajectories satisfying these conditions, the goal is to find the one minimizing the final time T [5]. For simplicity, we assume

$$a < 0, \qquad b < 0,$$

i.e., the point M_0 is located in the third quadrant of the coordinate plane.



According to the maximum condition obtained in the previous section, the optimal controls in equation (11) take one of the following four combinations:

$$\begin{aligned} &1)\ u_1 = 1, \quad u_2 = 1, \\ &2)\ u_1 = 1, \quad u_2 = -1, \\ &3)\ u_1 = -1, \quad u_2 = 1, \\ &4)\ u_1 = -1, \quad u_2 = -1. \end{aligned} \tag{15}$$

We now analyze each of these cases separately.

Case 1. If

$$u_1 = u_2 = 1 \quad \text{or} \quad u_1 = u_2 = -1,$$

then, from the definitions

$$s = \frac{u_1 + u_2}{2}, \qquad a = \frac{u_2 - u_1}{2}$$

introduced in the previous section, we have

$$a = 0, \qquad s = \pm 1.$$

Hence,

$$\dot{\theta} = \frac{2a}{l} = 0,$$

i.e., the orientation of the system does not change. In this case, the motion consists purely of a parallel translation, and the trajectory is a straight line.

Case 2. If

$$(u_1, u_2) = (1, -1) \quad \text{or} \quad (u_1, u_2) = (-1, 1),$$

i.e., u_1 and u_2 have opposite signs, then

$$s = 0, \qquad a = \pm 1.$$

This means that there is no parallel translation, and the motion consists solely of rotation.

The length of the segment connecting the initial and final points is

$$|M_0M_1| = |AC| = \sqrt{a^2 + b^2}.$$

For the direction of rotation to align with the segment M_0M_1, the motion direction vector $(-\sin\theta,\ \cos\theta)$ must be parallel to this segment, i.e.,

$$(-\sin\theta,\ \cos\theta) \parallel M_0M_1.$$

From this condition, the rotation angle $\theta = \theta^*$ is determined as

$$\sin\theta^* = \frac{a}{\sqrt{a^2 + b^2}}, \qquad \cos\theta^* = -\frac{b}{\sqrt{a^2 + b^2}}.$$

Taking into account that $a < 0$ and $b < 0$, we see that

$$\theta^* \in \left(-\frac{\pi}{2},\ 0\right).$$

3.2. Transferring segment AB to segment CD in minimum Time

Based on the results obtained in the previous sections, we constructively solve the problem of transferring the segment AB from the given initial position to the position of segment CD in minimum time.

From a geometric point of view, the optimal trajectory can consist of three sequential elementary motions:

1. Rotate the segment AB to the angle θ^*;
2. Translate the resulting segment $A(\theta^*)B(\theta^*)$ parallel to the vector $\overrightarrow{M_0M_1}$;
3. Rotate the segment back from θ^* to reach the final orientation.

We now estimate the time required for these motions.

Time required for parallel translation. Since the speed of the point M is $|s|$, in order to cover the distance $R = |M_0M_1|$ from M_0 to M_1, it is necessary that

$$\int_0^T |s(t)|\, dt \geq R.$$

Time required for rotation. Since the rotational speed is $\dot{\theta} = \frac{2a}{l}$, the total rotation can be estimated as

$$\int_0^T |\dot{\theta}(t)|\, dt.$$

As the segment must first rotate from 0 to θ^* and then back from θ^* to 0, the total rotation is $2|\theta^*|$. Therefore,

$$\int_0^T |a(t)|\, dt \geq \frac{l}{2} \int_0^T |\dot{\theta}(t)|\, dt \geq \frac{l}{2} \cdot 2|\theta^*| = l|\theta^*|.$$

As indicated in the previous section, the controls satisfy

$$|s(t)| + |a(t)| \leq 1.$$

Hence,

$$T \geq \int_0^T (|s(t)| + |a(t)|)\, dt \geq R + l|\theta^*|,$$

i.e.,

$$T \geq R + l|\theta^*|$$

is a valid inequality.

We now show that it is indeed possible to transfer the segment AB to CD within the time $R + l|\theta^*|$, i.e., this bound is sharp (achievable).

Stage I: Rotation. Choose the controls as

$$u_1 = 1, \qquad u_2 = -1.$$

Then

$$s = 0, \qquad a = -1, \qquad \dot{\theta} = -\frac{2}{l}.$$

Consequently, the segment rotates only, and the time required to rotate from 0 to $\theta^* < 0$ is

$$T_1 = \frac{l}{2}|\theta^*|.$$

Stage II: Parallel translation. Now choose the controls as

$$u_1 = 1, \qquad u_2 = 1.$$

In this case,

$$s = 1, \qquad a = 0,$$

i.e., there is no rotation, only parallel translation. The velocity vector of the point M is

$$\dot{M} = (-\sin\theta^*,\ \cos\theta^*) = \left(-\frac{a}{\sqrt{a^2+b^2}}, -\frac{b}{\sqrt{a^2+b^2}}\right),$$

which is aligned with the direction of M_0M_1. Thus, the point M traverses the segment M_0M_1 with unit speed, requiring

$$T_2 = \sqrt{a^2+b^2} = R$$

time.

Stage III: Reverse rotation. In the final stage, the controls are chosen as

$$u_1 = -1, \qquad u_2 = 1.$$

Then

$$s = 0, \qquad a = 1, \qquad \dot{\theta} = \frac{2}{l}.$$

The segment rotates back from θ^* to 0. The time required for this process is

$$T_3 = \frac{l}{2}|\theta^*|.$$

Thus, the total time is

$$T = T_1 + T_2 + T_3 = R + l|\theta^*| \tag{16}$$

which coincides with the lower bound obtained above. Hence, expression (16) provides the optimal solution of the problem, i.e., the minimal achievable time.

4. Conclusion

In this paper, the time-optimal control problem for a mechanical segment with fixed length moving in a plane was analyzed using PontryaginвЂ™s maximum principle. Based on the system dynamics and imposed constraints, it was shown that the optimal controls have a bangвЂ“bang structure, and the geometric interpretation of the optimal trajectory was determined through the properties of the switching functions. The theoretical results obtained serve as a methodological basis for designing effective control strategies for rigidly connected objects and coordinated motion of drones. Furthermore, the findings expand the applicability of optimal control theory in practical areas and suggest the study of tightly constrained control problems described by higher-order differential equations in future research.

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Moment Properties of Functionals of Convex Hulls Generated by a Poisson Point Process in Unit Disk

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DOI 10.5281/zenodo.18217225

Keywords: In this paper, we investigate functionals related to the number of vertices, areas, and perimeters of the convex hull generated by an inhomogeneous Poisson point process of the interior unit disk. We utilize the properties of the independence of Poisson law increments and the stationarity of vertex processes associated with the convex hull. We demonstrate the existence of exponential moments for the functionals under consideration. As a result, we conclude that the number of vertices, area, and perimeter of the convex hull possess finite moments of any order.

Introduction

Numerous publications focus on the study of random convex hulls within polygons and on various functionals associated with them. In the 1960s and 70s, a group of researchers, including B. Efron [1], A. Renyi and R. Sulanke [2], and H. Carnal [3], sought to derive asymptotic expressions for the functionals of convex hulls in both bounded and unbounded sets on the plane. The investigation of convex hulls falls under the field of stochastic geometry, and before the research conducted by P. Groeneboom [4], the primary achievement was limited to obtaining asymptotic expressions for the averages of these functionals. It was not until later that asymptotic expressions for the variance were also derived (see [5]). P.Groeneboom [4] made a significant contribution by approximating the binomial point process with a Poisson process, proving the central limit theorem for the number of vertices of a random convex hull. This was specifically when the initial distribution was uniformly concentrated within a convex polygon or ellipse. Here, the existence of exponential moments for the number of vertices, area, and perimeter of the convex hull generated by an inhomogeneous Poisson point process inside the unit disk is proved.

Basic results

Let A be a unit disk centered at point $(0, 1)$.

We consider convex hull C_n generated by the realization $(r_k, \alpha_k), k = 1, 2, ...$ (in polar coordinate systems, the poles are $(0, b_n)$) of the contraction of an inhomogeneous Poisson point process (i.p.p.p.) $\Pi_{n,\beta}(\cdot)$ in the disk $b_n A$ with intensity

$$\Lambda_{n,\beta}(D) = \begin{cases} \dfrac{\sqrt{b_n}}{2\pi L(b_n)} \iint\limits_D \dfrac{\partial}{\partial r}\left((b_n - r)^{\beta} L\left(\dfrac{b_n}{b_n - r}\right)\right) d\alpha dr, & \text{if } D \subset b_n A, \\ 0, & \text{if } D \not\subset b_n A, \end{cases} \tag{1}$$

Here, $\alpha_k \in [-\pi, \pi]$ is uniformly distributed, $L(x)$ is the slowly varying function (s.v.f.) in the sense of Karamata (see,for example, [6]), represented as:

$$L(u) = \exp\left\{\int_1^u \frac{\varepsilon(t)}{t} dt\right\}, \quad \varepsilon(t) \to 0, \quad t \to \infty. \tag{2}$$

and b_n is the smallest root of the equation:

$$n x^{-\left(\beta + \frac{1}{2}\right)} L(x) = 1. \tag{3}$$

Let $X_i = r_i \sin \alpha_i,\ Y_i = b_n - r_i \cos \alpha_i$.

Next, we assume that $W_n(a) = (X_n(a), Y_n(a))$ for any $a \in [-\pi, \pi]$ – is such a point (X_k, Y_k) of implementation $(X_1, Y_1), (X_2, Y_2), ...$, for which $r_k \cos(\alpha_k - a)$ takes the maximum value (see Figure 1). Then it follows from the definition that $W_n(a)$ forms a jump-like stationary Markov process, generated by the contraction in $b_n A$ of implementation of i.p.p.p. $\Pi_{n,\beta}(\cdot)$ with intensive measure $\Lambda_{n,\beta}(\cdot)$.

It is worth noting that in reference [4], for the case with uniform intensive measure $\beta = 1,\ L(x) = 2 - \frac{1}{x}$, it was possible to obtain the limit distribution for the number of vertices of the convex hull. The present study is devoted to the asymptotic behavior of the exponential moments of the main functionals of the vertex process for the case $\beta \geq 1$.

Let us denote the number of jumps of the jump-like process $W_n(c)$ within $0 \leq c \leq a$ by $\nu_n(0, a)$, and the length of the broken line connecting the sequence of the states of process $W_n(c), 0 \leq c \leq a$ by $l_n(0, a)$.

Let

$$l_n^*(0, a) = b_n \left(\arctan \frac{X_n(a)}{b_n - Y_n(a)} - \arctan \frac{X_n(0)}{b_n - Y_n(0)}\right) - l_n(0, a).$$

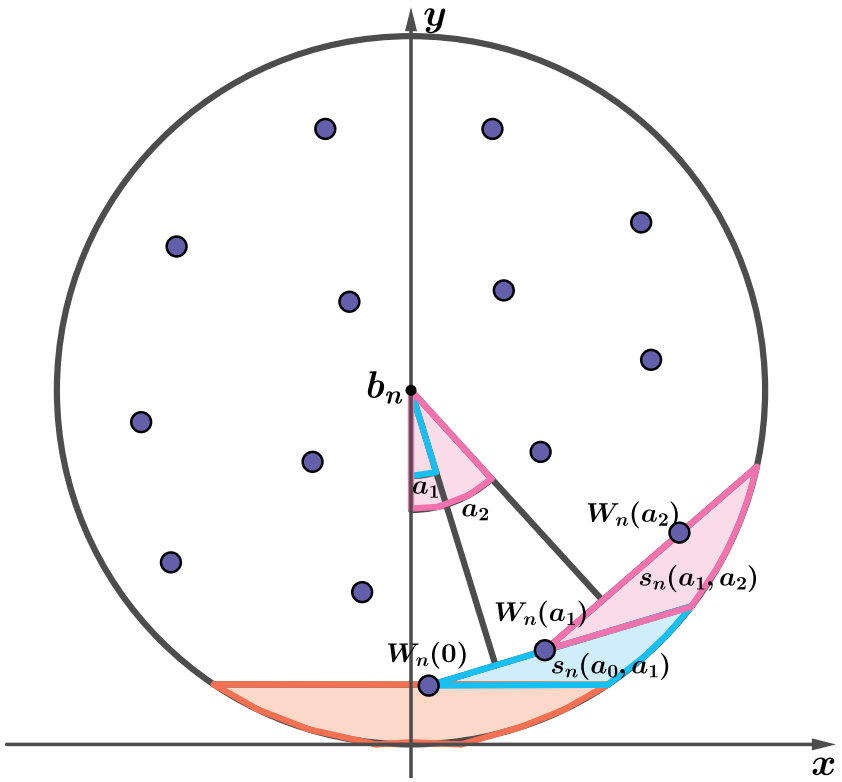

Fig.1. An illustration of the vertices $W_n(0), W_n(a_1), ...$ and the areas $s_n(a_0, a_1), s_n(a_1, a_2), ...$

Next, let $0 = a_0 < a_1 < a_2 < ... < a_k \leq a$ – be the jump times of process $W_n(c), 0 \leq c \leq a$, and $s_n(a_{i-1}, a_i)$ be the area of the figure bounded by lines $(e_{i-1}, w - W_n(a_{i-1})) = 0$, $(e_i, w - W_n(a_{i-1})) = 0$ and circles $x^2 + (b_n - y)^2 = b_n^2$. And let us denote the area of the figure bounded by lines $(e_k, w - W_n(a_k)) = 0$, $(e_a, w - W_n(a_k)) = 0$ and circles $x^2 + (b_n - y)^2 = b_n^2$, by $s_n(a_k, a)$, where $w = (x, y)$, $e_0 = (0, 1)$, $e_i = (-\tan a_i, 1)$, $i = 1, 2, ..., k$, $e_a = (-\tan a, 1)$ (see Figure 1).

If we assume that

$$s_n(0, a) = s_n(a_0, a_1) + s_n(a_1, a_2) + ... + s_n(a_k, a),$$

then $s_n(0, a)$ (similar to what was noted in [8,9] behaves as the sum of a random number of independent random variables. The following theorems to be proved ensure the existence of absolute moments of any power of the functionals under consideration

$$\nu_n(0, a), s_n(0, a) \text{ and } l_n^*(0, a)$$

for $|a| \leq a_0 = \frac{c_0}{\sqrt{b_n}}$.

Here and below, without loss of generality, we assume that $c_0, c_1, c_2, ...$ are any positive constants whose value may vary from string to string.

Given the similarity of the reasoning, we restrict ourselves to the case $L(x) = 1$, since the demonstrated technique also works for arbitrary slowly varying functions $L(x)$ in the form (2).

Theorem 1. If $|a| \leq a_1 = \frac{c_1}{\sqrt{b_n}}$ for some $c_1 > 0$, then there exists $\tau_1 > 0$ such that

$$E \exp(\tau \nu_n(0, a)) < M_1 \text{ for any } 0 < \tau < \tau_1.$$

Theorem 2. If $|a| \le a_2 = \frac{c_2}{\sqrt{b_n}}$ for some $c_2 > 0$, then there exists $\tau_2 > 0$ such that

$$E \exp\left(\tau \frac{s_n(0,a)}{\sqrt{b_n}}\right) < M_2 \text{ for any } 0 < \tau < \tau_2.$$

Theorem 3. If $|a| \le a_1 = \frac{c_3}{\sqrt{b_n}}$ for some $c_3 > 0$, then there exists $\tau_3 > 0$ such that

$$E \exp\left(\tau \sqrt{b_n} l_n^*(0,a)\right) < M_3 \text{ for any } 0 < \tau < \tau_3.$$

Proof of the basic results

Proof of Theorem 1. Let $a > a_0 = 0$. Then from the definition of mathematical expectation, we have:

$$E\left\{\exp\left(\tau\nu_n(0,a) - 1\right) / W_n(0) = (x_0, y_0)\right\} \le \sum_{k=0}^{\infty} P\left(\text{In set } A(0,a,x_0,y_0), \text{ there are } k \text{ points of}\right.$$

$$\left.\text{implementation of i.p.p.p. } \Pi_{n,\beta}(\cdot) / W_n(0) = (x_0, y_0)\right) \le \sum_{k=0}^{\infty} e^{\tau k} \frac{\left(\Lambda_{n,\beta}(A(0,a,x_0,y_0))\right)^k}{k!}.$$

$$\exp\left\{\Lambda_{n,\beta}(A(0,a,x_0,y_0))\right\} = \exp\left\{\Lambda_{n,\beta}(A(0,a,x_0,y_0))\right\} \sum_{k=0}^{\infty} \frac{\left(e^{\tau}\Lambda_{n,\beta}(A(0,a,x_0,y_0))\right)^k}{k!} =$$

$$\exp\left\{e^{\tau}\Lambda_{n,\beta}(A(0,a,x_0,y_0)) - \Lambda_{n,\beta}(A(0,a,x_0,y_0))\right\} = \exp\left\{\Lambda_{n,\beta}(A(0,a,x_0,y_0))(e^{\tau} - 1)\right\}.$$

Here $A(0,x_0,y_0)$ – is the domain bounded by lines $y = y_0$ and $x^2 + (y - b_n)^2 = b_n^2$; $A(0,a,x_0,y_0)$ – is the domain bounded by lines $y = \tan a(x - x_0) + y_0$ and $x^2 + (y - b_n)^2 = b_n^2$ (see Figure 2).

Let $y_0 \le c_0$ be for some constant $c_0 > 0$. Then it follows from the definition of measure $\Pi_{n,\beta}(\cdot)$ that there exists constant $c_1 > 0$ such that

$$\Lambda_{n,\beta}(A(0,a,x_0,y_0)) < c_1$$

Hence

$$E\left\{\exp\left(\tau\nu_n(0,a) / W_n(0) = (x_0, y_0)\right)\right\} \le c_1 \exp(\tau c_2). \tag{4}$$

Let now $y_0 > c_0$. Then there exists some other constant $c_3 > 0$ such that the following inequality holds:

$$\Lambda_{n,\beta}(A(0,a,x_0,y_0)) < c_3 \Lambda_{n,\beta}(A(0,x_0,y_0)).$$

From this and from (4), taking into account the independence of the increment of Poisson distribution, it follows that:

$$E \exp\left(\tau\nu_n(0,a)\right) = EE\left\{\exp\left(\tau\nu_n(0,a) / W_n(0) = (x_0, y_0)\right)\right\}$$

$$\le c_1 \iint_{\{b_n A\} \cap \{y_0 \le c_0\}} \exp\left\{\tau c_2\right\} P\left(W_n(x_0, y_0)\right.$$

$$\left. \in (dx_0, dy_0)\right) + c_2 \iint_{\{b_n A\} \cap \{y_0 > c_0\}} E\left\{\exp\left(\tau\nu_n(0,a) / W_n(0) = (x_0, y_0)\right)\right\} P\left(W_n(x_0, y_0) \in (dx_0, dy_0)\right)$$

$$\leq c_3 + c_4 \iint_{b_n A} \exp\left\{c_5 \Lambda_{n,\beta}(A(0,a,x_0,y_0))(e^{\tau}-1) - \Lambda_{n,\beta}(A(0,a,x_0,y_0))\right\} \|J| dx_0 dy_0 < \infty$$

for any $0 < \tau < \tau_0$, where $|J|$ – is the Jacobian of the transformation, and τ_0 is chosen such that

$$c_5 \Lambda_{n,\beta}(A(0,a,x_0,y_0))(e^{\tau}-1) - \Lambda_{n,\beta}(A(0,a,x_0,y_0)) > -c_6 \Lambda_{n,\beta}(A(0,a,x_0,y_0))$$

for some $c_6 > 0$.

The proof of Theorem 1 is complete.

Proof of Theorem 2. Let again $W_n(0) = (x_0, y_0)$.

Then it is not difficult to obtain that

$$E \exp\left(\tau \frac{s_n(0,a)}{\sqrt{b_n}}; y_0 \geq \sqrt{b_n}\right) = o(1).$$

Therefore, we will consider the case that $y_0 < \sqrt{b_n}$.

By the definition of area $s_n(0,a)$ it is easy to see that

$$s_n(0,a) \leq mes\left\{A(o,a,x_0,y_0)\right\},$$

where $mes\{D\}$ – is the area of domain D. It follows from the definition of domain $A(0,x_0,y_0)$ and $A(0,a,x_0,y_0)$ that

$$mes\left\{A(0,a,x_0,y_0)\right\} = b_n^2 \arcsin \frac{\sqrt{2b_n y_0 - y_0^2}}{b_n} - (b_n - y_0)\sqrt{2b_n y_0 - y_0^2}. \tag{5}$$

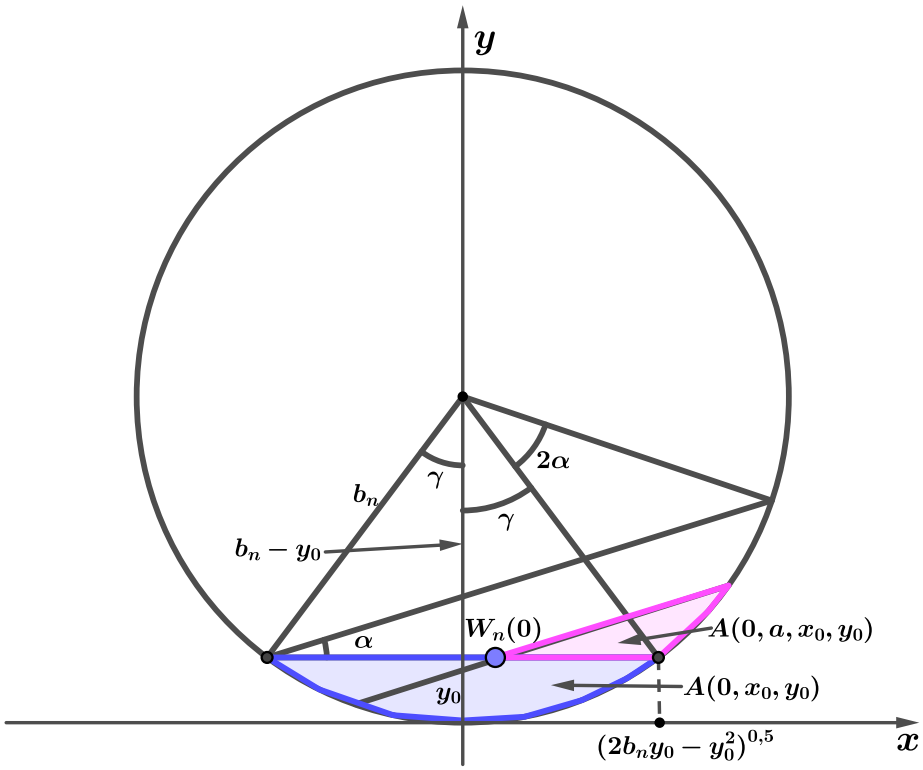

Fig.2. An illustration of the vertices $W_n(0)$ and the areas $A(0,x_0,y_0)$, $A(0,a,x_0,y_0)$.

Using the expansions of trigonometric and inverse trigonometric functions in the neighborhood of zero

$$\sin x = x - \frac{1}{6}x^3 + O(x^5),\ \tan x = x + \frac{1}{3}x^3 + O(x^5),$$

$$\arcsin x = x + \frac{1}{6}x^3 + O(x^5),\ \arctan x = x - \frac{1}{3}x^3 + O(x^5)$$

and taking into account (**??**), we obtain:

$$mes\left\{A(0,x_0,y_0)\right\} = b_n^2\left\{\frac{\sqrt{2b_ny_0-y_0^2}}{b_n} + \frac{1}{6}\left(\frac{\sqrt{2b_ny_0-y_0^2}}{b_n}\right)^3 + O\left(\frac{\sqrt{2b_ny_0-y_0^2}}{b_n}\right)^5\right\} -$$

$$-b_n\sqrt{2b_ny_0-y_0^2} + y_0\sqrt{2b_ny_0-y_0^2} = \frac{4\sqrt{2b_n}}{3}y_0^{\frac{3}{2}} + O\left(b_n^{-\frac{5}{4}}\right).$$

Similarly, for the area

$$mes\left\{A(0,a.x_0,y_0)\right\} \le b_n^2(a+\gamma) - b_n^2\sin(a+\gamma),$$

where

$$\gamma = \arcsin\left(\frac{\sqrt{2b_ny_0-y_0^2}}{b_n}\right) = \frac{\sqrt{2b_ny_0-y_0^2}}{b_n} + \frac{1}{6}\left(\frac{\sqrt{2b_ny_0-y_0^2}}{b_n}\right)^3 + O\left(\frac{\sqrt{2b_ny_0-y_0^2}}{b_n}\right)^5,$$

we obtain:

$$mes\left\{A(0,a,x_0,y_0)\right\} \le \frac{b_n^2}{6}(a+\gamma)^3 + O\left(b_n^2(a+\gamma)^5\right) = \frac{b_n^2a^3}{6} + \frac{y_0\sqrt{2b_ny_0}}{3} + O\left(b_n^2a^3 + \frac{y_0^{\frac{5}{2}}}{\sqrt{b_n}}\right)$$

see Figure 2.

Consequently, if $y_0 > c_0$, then

$$mes\left\{A(0,a,x_0,y_0)\right\} \le c_1 mes\left\{A(0,x_0,y_0)\right\} \le c_2\sqrt{b_n}y_0^{\frac{3}{2}} \le c_3 mes\left\{A(0,x_0,y_0)\right\},$$

and if $y_0 \le c_0$, then

$$mes\left\{A(0,a.x_0,y_0)\right\} \le c_3\sqrt{b_n}.$$

Hence, considering that $\beta \ge 1$, using the expression Poisson distribution we obtain:

$$E\exp\left(\tau\frac{s_n(0,a)}{\sqrt{b_n}}\right) = EE\exp\left(\tau\frac{s_n(0,a)}{\sqrt{b_n}}\Big/ W_n(0)\right) =$$

$$= \iint_{b_n\cap\{y_0\le c_0\}} E\exp\left(\tau\frac{s_n(0,a)}{\sqrt{b_n}}\Big/ W_n(0)=x_0,y_0\right) P\left(W_n(0)\in(dx_0,dy_0)\right) +$$

$$= \iint_{b_n\cap\left\{\sqrt{b_n}>y_0>c_0\right\}} E\exp\left(\tau\frac{s_n(0,a)}{\sqrt{b_n}}\Big/ W_n(0)=x_0,y_0\right) P\left(W_n(0)\in(dx_0,dy_0)\right) + o(1) \le$$

$$\le c_1 + \iint_{b_n\cap\left\{\sqrt{b_n}>y_0>c_0\right\}} E\exp\left(\tau c_2 y_0^{\frac{3}{2}}\right) P\left(W_n(0)\in(dx_0,dy_0)\right) \le$$

$$\leq c_1 + \iint_{b_n \cap \left\{\sqrt{b_n} > y_0 > c_0\right\}} E \exp\left(\tau c_3 \Lambda_{n,\beta}(A(0, x_0, y_0)\right) P\left(W_n(0) \in (dx_0, dy_0)\right)$$

$$\leq c_1 + \iint_{b_n \cap \left\{\sqrt{b_n} > y_0 > c_0\right\}} E \exp\left(\tau c_3 \Lambda_{n,\beta}(A(0, x_0, y_0)) - \Lambda_{n,\beta}(A(0, x_0, y_0))\right) |J| dx_0 dy_0 < \infty$$

for any $0 < \tau < \tau_0$. Here, as in the previous case, $|J|$ is the Jacobian of the transformation, and $\tau_0 > 0$ is chosen such that

$$\tau_0 c_3 \Lambda_{n,\beta}(A(0, x_0, y_0)) - \Lambda_{n,\beta}(A(0, x_0, y_0)) > -c_4 \Lambda_{n,\beta}(A(0, x_0, y_0))$$

for some constant $c_4 > 0$.

Theorem 2 is proven.

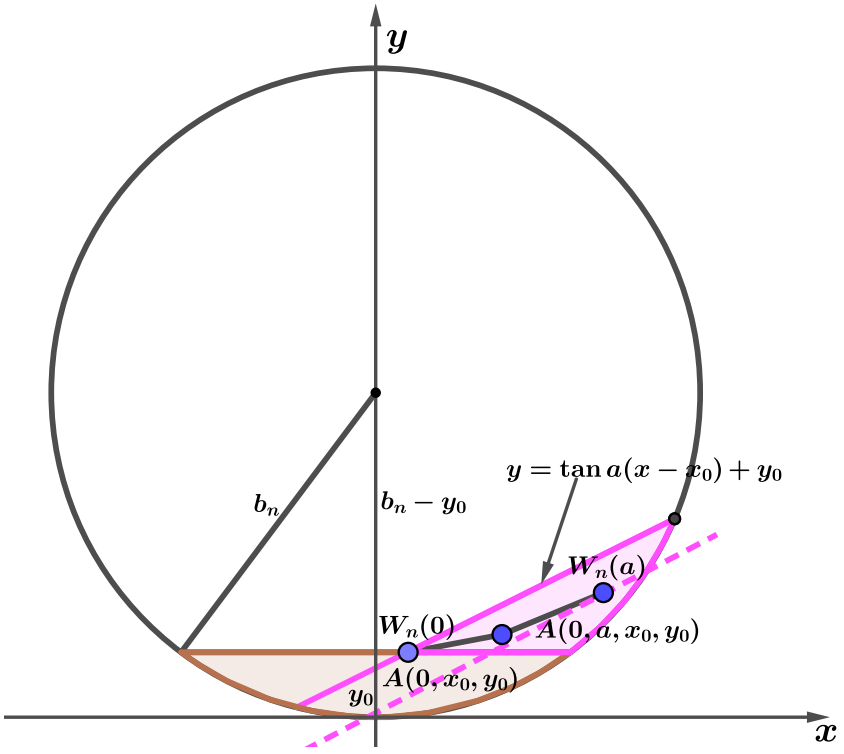

Fig.3. An illustration of the vertices $W_n(0), ..., W_n(a)$ and the areas $A(0, x_0, y_0)$, $A(0, a, x_0, y_0)$.

Proof of Theorem 3. Again, we use the expansion into a power series of the inverse trigonometric function

$$l_n^*(0, a) = b_n \left(\arctan \frac{X_n(a)}{b_n - Y_n(a)} - \arctan \frac{X_n(0)}{b_n - Y_n(0)}\right) - l_n(0, a) =$$

$$= b_n \left(\frac{X_n(a)}{b_n - Y_n(a)} - \frac{1}{3}\left(\frac{X_n(a)}{b_n - Y_n(a)}\right)^3 + O\left(\frac{X_n(a)}{b_n - Y_n(a)}\right)^5\right) -$$

$$-b_n \left(\frac{X_n(0)}{b_n - Y_n(0)} - \frac{1}{3}\left(\frac{X_n(0)}{b_n - Y_n(0)}\right)^3 + O\left(\frac{X_n(0)}{b_n - Y_n(0)}\right)^5\right) - l_n(0, a) \leq$$

$$\leq b_n\left(\frac{X_n(a)}{b_n-Y_n(a)}-\frac{X_n(0)}{b_n-Y_n(0)}\right)-\frac{1}{3}\left(\left(\frac{X_n(a)}{b_n-Y_n(a)}\right)^3-\left(\frac{X_n(0)}{b_n-Y_n(0)}\right)^3\right)+$$

$$+b_n\left(O\left(\frac{X_n(a)}{b_n-Y_n(a)}\right)^5+O\left(\frac{X_n(0)}{b_n-Y_n(0)}\right)^5\right)-(X_n(a)-X_n(0))$$

(see Figure 3).

Let $\sqrt{b_n}\geq Y_n(a)\geq c_0$, (the case of $Y_n(a)>\sqrt{b_n}$ is obvious). Then from the last relation we obtain:

$$l_n^*(0,a)\leq\frac{c_1}{\sqrt{b_n}}+\frac{c_2}{\sqrt{b_n}}Y_n^{\frac{3}{2}}(0). \tag{6}$$

Now let $Y_n(a)<c_0$. Then

$$l_n^*(0,a)\leq\frac{c_1}{\sqrt{b_n}}. \tag{7}$$

From (??) and (??), taking into account that $\beta\geq 1$, similarly to the proof of the previous theorem, we have:

$$E\exp\left(\tau\sqrt{b_n}l_n^*(0,a)\right)=E\left\{E\left(\exp\left(\tau\sqrt{b_n}l_n^*(0,a)\right)\Big/W_n(0)=(x_0,y_0)\right)\right\}=$$

$$=\iint_{(b_nA)\cap(Y\leq c_0)}E\left(\exp\left(\tau\sqrt{b_n}l_n^*(0,a)\right)\Big/W_n(0)=(x_0,y_0)\right)P\left(W_n(0)\in(dx_0,dy_0)\right)+$$

$$+\iint_{(b_nA)\cap(Y>c_0)}E\left(\exp\left(\tau\sqrt{b_n}l_n^*(0,a)\right)\Big/W_n(0)=(x_0,y_0)\right)P\left(W_n(0)\in(dx_0,dy_0)\right)+$$

$$\leq c_2+c_3\iint_{b_n\cap\left\{\sqrt{b_n}>y_0>c_0\right\}}E\exp\left(\tau c_4\Lambda_{n,\beta}(A(0,x_0,y_0))\,P\left(W_n(0)\in(dx_0,dy_0)\right)\right.$$

$$\leq c_2+c_3\iint_{b_n\cap\left\{\sqrt{b_n}>y_0>c_0\right\}}E\exp\left(\tau c_4\Lambda_{n,\beta}(A(0,x_0,y_0))-\Lambda_{n,\beta}(A(0,x_0,y_0))\right)|J|dx_0dy_0<\infty$$

for any $0<\tau<\tau_0$.

Similarly to the previous case, we choose $\tau_0>0$ such that

$$\tau_0c_4\Lambda_{n,\beta}(A(0,x_0,y_0))-\Lambda_{n,\beta}(A(0,x_0,y_0))>-c_5\Lambda_{n,\beta}(A(0,x_0,y_0))$$

for some constant $c_5>0$.

Theorem 3 is proven.

Conclusion

Let

$$\Im_n^0=\sigma\left\{W_n(c):c\leq 0\right\},\ \Im_n^{a+}=\sigma\left\{W_n(c):c>0\right\}.$$

Then, similarly to Lemma 3.3 in [9], the following lemma can be easily proved (see for example [10]).

Lemma 1. If $a = \frac{c_0 k}{\sqrt{b_n}},\ k = 1, 2, ..., [\log n]$ (c_0 – is an arbitrary constant), then there exists constant $c_1 > 0$, such that

$$|P(A \cap B) - P(A)P(B)| \leq 4 \exp\left(c_1 k^{2\beta+1-\varepsilon}\right)$$

for any $A \in \Im_n^0$, $B \in \Im_n^{a+}$ and $\varepsilon > 0$.

From Theorems 1 - 3, Lemma 1, and the Ibragimov-Linnik theorem (see, for example, Theorem 18.5.3 in [11]), it follows that stationary processes $\nu_n(0, a)$, $s_n(0, a)$ and $l_n^*(0, a))$ are asymptotic normal.

Acknowledgments

The authors sincerely thanks the editor, the associate editor, and the reviewers for their valuable comments.

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Impact of remote work and platform (gig) economy on labor force structure: demographic and institutional analysis

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DOI 10.5281/zenodo.18217235

Abstract

This article assesses the impact of remote work and the platform (gig) economy on the workforce structure. It focuses on the rapid growth of platform employment in 2015-2024, the stabilization of remote and hybrid work, as well as changes involving youth and women. Methodologically, descriptive statistics, comparison, and conceptual-institutional analysis were used. The results show structural shifts in employment contracts, income stability, social protection, and skill composition. Policy recommendations cover social guarantees (portable benefits), adaptation of tax and labor laws, as well as rapid reskilling and upskilling.

Keywords: digital platforms; remote work; gig economy; employment; labor market; social policy; digital transformation; portable benefits.

Introduction

The article analyzes the impact of digital platforms and remote work on labor relations. As a result of digital transformation and automation, short-term, project-based, and gig forms of work have emerged alongside the traditional employment model. This process is significantly affecting the demographic composition, income stability, and social security system in the labor market.

The study is based on statistical and institutional data from 2015 to 2024. The results show that while the platform economy has increased flexibility, it is causing income instability and legal uncertainties. More than 60% of gig workers engage in these activities for additional income, but most of them earn less than $2,500 per month.

"Portable benefits,"flexible taxation, and the development of digital skills have been identified as important strategic directions for a sustainable labor system. Digital transformation processes are fundamentally changing the structure of the global labor market and the nature of labor relations [1]. As a result of the rapid development of artificial intelligence, automation, cloud technologies, and online platforms, short-term, project-based, and remote forms of work have emerged alongside the traditional model of permanent staff employment [2]. This trend, coupled with the shift of economic activity from the "physical office"to the digital space, has formed a new labor paradigm in the workforce structure [3].

The modern labor market is now based on flexibility, multi-source income, and digital integration rather than permanent staff employment [4]. In particular, the rapid growth

of platform employment between 2015 and 2024, the stabilization of hybrid work forms, and the increased participation of youth and women clearly demonstrate the scale of these changes [5][6]. At the same time, new forms of employment have created unprecedented challenges for social protection systems, labor equality, and legal clarity [7][8]. While the digital platform ecosystem, known as the gig economy, offers workers freedom and flexibility, significant uncertainties are emerging in the areas of income stability, continuity of social guarantees, and protection of labor rights [9][10]. This necessitates a redesign of state policies, social institutions, and labor relations in a new format [11]. From this perspective, the present article aims to analyze the demographic, institutional, and economic dimensions of digital platforms and remote work. The main objectives of the research are to identify the systemic impact of digital labor formats on labor relations, evaluate the socioeconomic consequences of the new employment model, and develop evidence-based guidelines for sustainable labor policies.

The results of this study provide a scientific foundation for a deeper understanding of the transformational processes in human resource management within the digital economy, the modernization of employment policies, and the flexible and inclusive redesign of labor institutions [12][13]. Digital platforms and remote work have fundamentally altered labor relations: short-term, projectbased, and remote work have been added alongside the traditional permanent staffing model. This shift necessitates a reassessment of the workforce's demographic composition, income stability, social guarantees, and equality issues in the labor market. This article systematically presents the key dimensions of these trends and provides guidelines for policy design. Digital platforms and remote work have fundamentally transformed labor relations, introducing short-term, project-based, and remote forms of work alongside the traditional permanent employment model. These changes necessitate a reassessment of the demographic composition, income stability, and social security system in the labor market.

Between 2015 and 2024, there was a rapid growth in platform-based employment, stabilization of hybrid and remote work forms, and increased participation, especially among youth and women. While the gig economy enhances labor flexibility, it is characterized by income instability and weakened social protection. Empirical analyses indicate that economic growth has the most significant impact on the volume of labor resources, while education and healthcare factors are long-term determinants that shape their quality. In the gig model, the "flexibility-protection"contradiction is intensifying, demanding new approaches to labor policy. This change is characterized by its impact on labor force participation and demographic trends. While remote work has increased participation rates among adults, especially women, it simultaneously revealed disparities between different demographic groups, with men who could work remotely experiencing more favorable employment outcomes after the pandemic.[4][5]. Furthermore, while the gig economy provides independence and flexibility, it raises pressing issues regarding income stability, worker protection, and access to benefits, as many gig workers lack the traditional safety nets associated with full-time employment.[6][7][8].

The evolving labor landscape presents numerous challenges and opportunities. On the one hand, the increase in remote and gig work promotes flexibility and encourages continuous skill development, which is becoming increasingly important in a rapidly changing labor market. On the other hand, these changes contribute to the precarious nature of employment, characterized by income volatility and a lack of comprehensive benefits, leading to increased economic inequality and job insecurity for

many workers.[6][9][10]. As politicians grapple with the need for regulations that balance worker protection and the unique flexibility of gig roles, ongoing debates surrounding classification, compensation, and transparency are shaping the future of work.[11][12] Ultimately, the simultaneous rise of remote work and the gig economy signifies profound changes in job design and structure, necessitating careful consideration of long-term implications for both individuals and the broader economy.[1][3][7].

LITERATURE REVIEW

Evidence of an increase in the share of remote work and rapid expansion of gig employment has been documented in numerous international sources; these changes affect working hours, workplace, income models, and social protection structures. Also relevant are the debates surrounding the "flexibility-vulnerability"trade-off in platform work, algorithmic management, and worker classification (employee vs. independent contractor). The impact of remote work and the gig economy on workforce composition has become a key topic of discussion regarding significant technological advancements and societal changes, especially in the wake of the COVID-19 pandemic. (Lin, Peng, Au, & Baum, 2023) Direct quote (?25 words): "The COVID19 pandemic has caused the food delivery sector to boom..."(Lin et al., 2023, p. 1490). Remote work, which was previously an optional arrangement, is now perceived as the primary employment model for approximately 42% of the US workforce. Simultaneously, the gig economy, characterized by short-term, flexible employment through digital platforms, is expected to encompass 35% of the global workforce by 2025 and radically transform traditional employment paradigms.[1][2][3]. In their scientific work, researchers Ni Huang, Gordon Burtch, Yili Hong, and Paul A. Pavlou state: "a 1% increase in county unemployment is associated with a 21.8% increase in the volume of county residents actively working online on the platform."

Scientific evidence of the increase in remote work and the rapid expansion of gig employment has been extensively documented in numerous international studies in recent years. Their findings indicate that the digital transformation process is fundamentally altering working hours, workplace arrangements, income models, and social protection structures. Simultaneously, debates surrounding the "flexibility-vulnerability"trade-off in platform labor, algorithmic management, and the ambiguity in worker classification (employee vs. independent contractor) have become central issues in global labor policy. The COVID-19 pandemic has accelerated this process. As noted by Lin, Peng, Au, and Baum (2023), the food delivery sector experienced a "sharp growth phase"during the pandemic, which catalyzed the emergence of remote and gig work as a new labor model. Studies show that by 2024, nearly 42% of the US workforce is operating in remote or hybrid work formats, indicating that remote work, previously considered a form of "supplementary employment,"has become the predominant model.

Furthermore, the rapid development of the gig economy, which enables short-term and flexible employment through digital platforms, is radically transforming the global labor market. According to international estimates, platform-based employment is expected to constitute 35% of the global workforce by 2025 (ILO, 2023; OECD, 2024). Empirical evidence also links this trend to economic cycles. For example, according to research conducted by Ni Huang, Gordon Burtch, Yili Hong, and Paul A. Pavlou, a 1% increase in the unemployment rate leads to a 21.8% increase in the number of users actively working on platforms. This indicates that digital employment is emerging as an alternative source

of income even during economic crises.
In general, existing literature links the growth of remote and gig work not only to technological innovations but also to socio-economic changes in society, particularly the new work culture that emerged in the post-pandemic period. From this perspective, a deep analysis of their demographic and institutional consequences has important scientific and practical significance for today's labor policy.

3. Data and Methodology

The platform economy characterized by gig work has introduced flexibility and a different structure to employment. As businesses increasingly turn to freelancers for projectbased roles, the number of traditional full-time positions may decrease[7].
While this change allows workers to choose their work schedules and potentially improve work-life balance, it may impact income stability and Benefits[7][6]. Gig workers often lack traditional work-related benefits such as health insurance, retirement plans, and paid leave, which makes them particularly vulnerable during economic crises[6].
Moreover, the variable income associated with gig work creates challenges for financial stability, making it difficult for individuals to plan for long-term goals such as retirement savings[8]. As a result, the workforce structure is becoming more flexible but also more precarious, with the traditional employment model increasingly being supplemented by gig and remote work opportunities[7][6].
The shift to remote work and the rise of the gig economy have resulted in significant economic consequences that affect both individuals and organizations. This change has not only impacted income distribution but has also altered the nature of labor relations across various industries.
The transition to remote work has led to significant changes in labor market flows, especially regarding the geographical redistribution of jobs and human mobility. As remote work allows people to work from various locations, there is a noticeable shift in economic activity from urban centers to more rural areas, thereby reshaping local economies[9][1]. Additionally, the gig economy has introduced various short-term and flexible work arrangements through digital platforms that have transformed traditional employment structures[14][15].
One of the most significant economic consequences of gig work is income instability. Unlike traditional jobs, which typically offer a steady salary, gig work often relies on a piece-rate payment model, leading to income fluctuations. This volatility can create difficulties for financial planning and increase stress levels among gig workers [16][6]. Data show that more than 60% of gig workers participate in this economy mainly to supplement their income, with a large portion earning less than $2,500 per month. This trend has led to a widening income gap and a decline in worker protection, raising concerns about the long-term viability of gig jobs as a sustainable source of income.[17].

METHODOLOGY AND ANALYSIS

The empirical basis of the research consisted of a set of statistical and institutional observations for the period 2015-2024 provided by the user, as well as sector-specific examples. The dataset encompasses dynamic changes that demonstrate the impact of the gig economy and remote work on labor structure. The platform economy, characterized by gig work, has introduced flexibility and a new organizational structure to the employment

system. As enterprises increasingly turn to freelancers for project-based roles, the number of traditional full-time positions is decreasing [7]. While this model allows workers to independently choose their work schedules and improve work-life balance, it negatively impacts income stability and the continuity of social benefits [6], [7].

Gig workers often lack traditional work-related health insurance, pension plans, and paid leave, which makes them particularly vulnerable during economic crises [6]. Furthermore, the variable income associated with gig work creates challenges for financial stability, making it difficult for individuals to plan long-term financial goals such as retirement savings [8]. As a result, the workforce structure is becoming more flexible, but also a riskier landscape, where the traditional employment model is increasingly being supplemented by gig work and remote work opportunities [6], [7]. The transition to remote work and the rise of the gig economy have created significant economic consequences that affect both individuals and organizations.

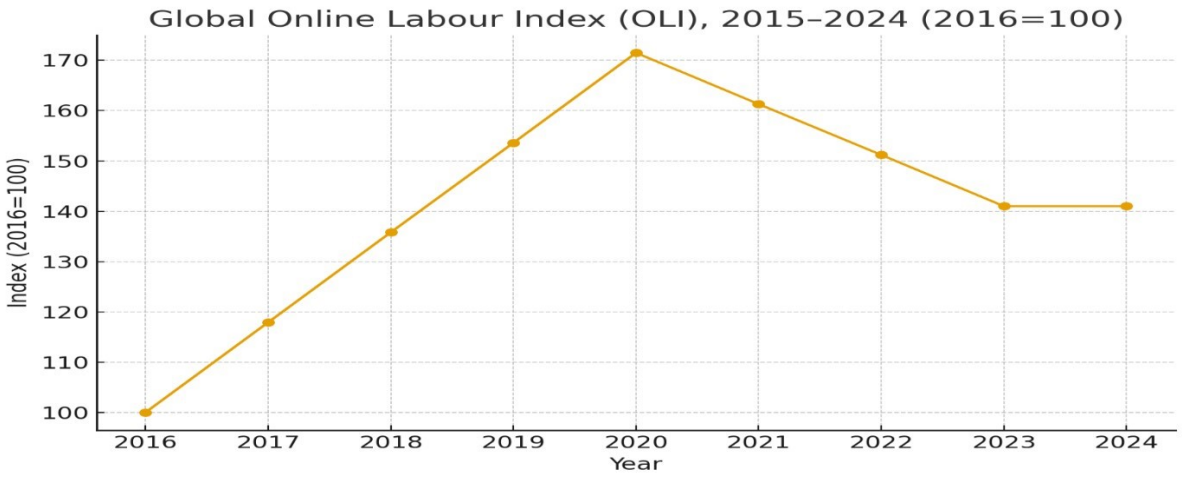
This change has impacted not only income distribution but also the nature of labor relations across various sectors. The shift to remote work has led to significant changes in labor market dynamics, especially in the geographical redistribution of jobs and population movement. Remote work has enabled people to work from different locations, encouraging the migration of economic activity from urban centers to more rural areas, thereby reshaping local economies [9], [1]. Additionally, the gig economy has introduced various short-term and flexible work arrangements through digital platforms that have transformed traditional employment structures [14], [15]. One of the most significant economic consequences of gig work is income instability. Unlike traditional employment, which typically offers a steady salary, gig work is often based on a piece-rate payment model, which leads to sharp fluctuations in income [16].

This volatility creates difficulties for financial planning and causes an increase in stress levels among gig workers [6]. Data shows that more than 60 percent of gig workers participate in this economy primarily to supplement their income, with a large portion earning less than $2,500 per month. This trend has led to a widening income gap and a decline in worker protection, intensifying concerns about the long-term viability of gig work as a stable source of income [17]. Thus, the expansion of remote work and the platform economy has made the modern labor market landscape more flexible, but less stable. This situation necessitates the development of new mechanisms in state policy aimed at enhancing social guarantees, labor rights, and digital skills.

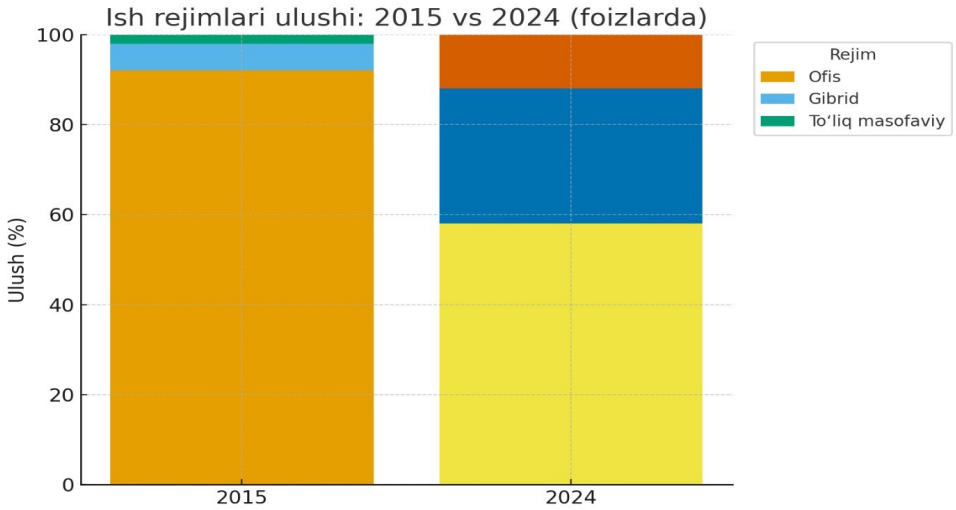
4. Results

4.1. Growth of Platform Employment

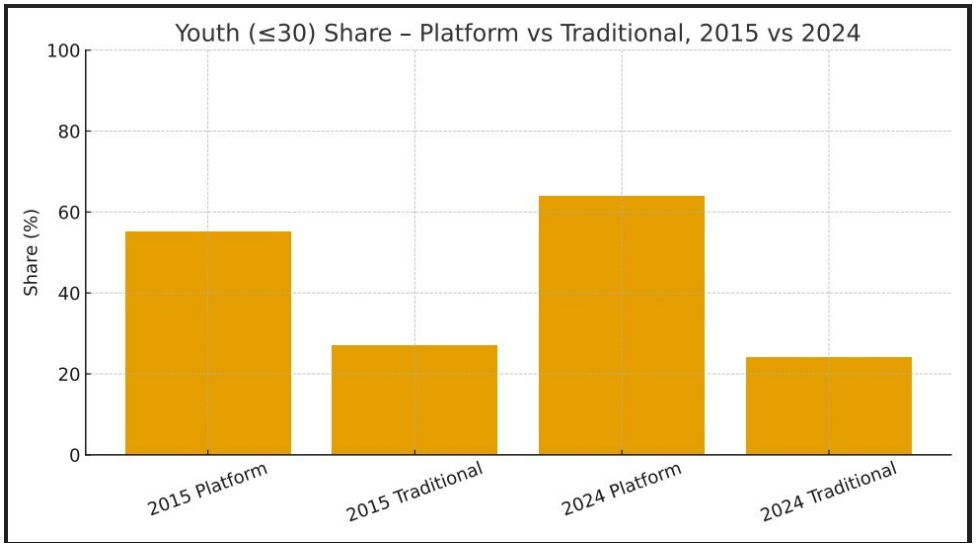
Table 1. Dynamics of gig employment from 2015 to 2024 (global estimates) (Table placeholder - indicators: number of jobs, share in total employment, growth rate). Note: Between 2015 and 2024, platform employment increased several times; IT, logistics/delivery, online education, and consulting segments are leading.



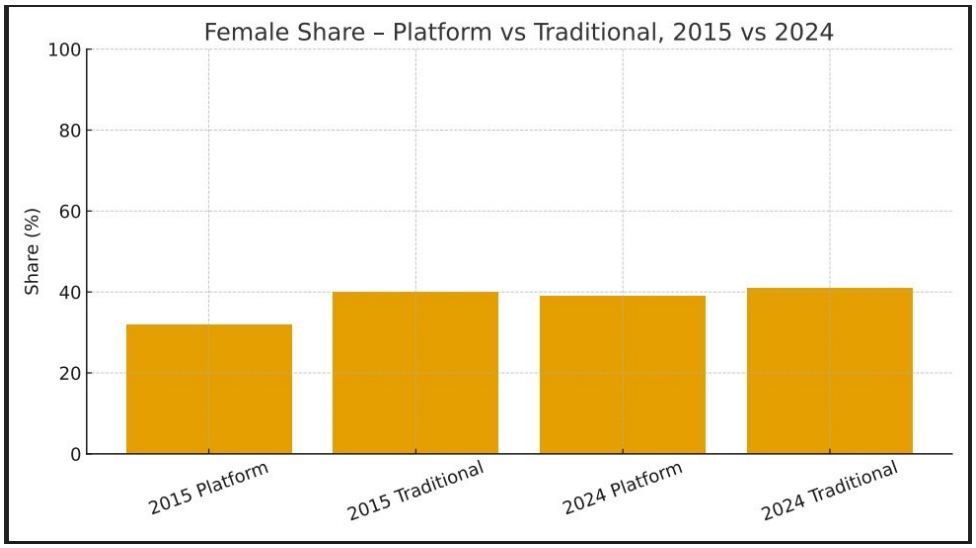
4.2. Stabilization of remote/hybrid work Table 2. Share of work modes (office, hybrid, fully remote), 2015 vs. 2024 (Table placeholder - with percentage point changes.) Result: The share of hybrid and fully remote models has increased sharply, which eases geographical constraints and shifts competition to a globalized labor market.



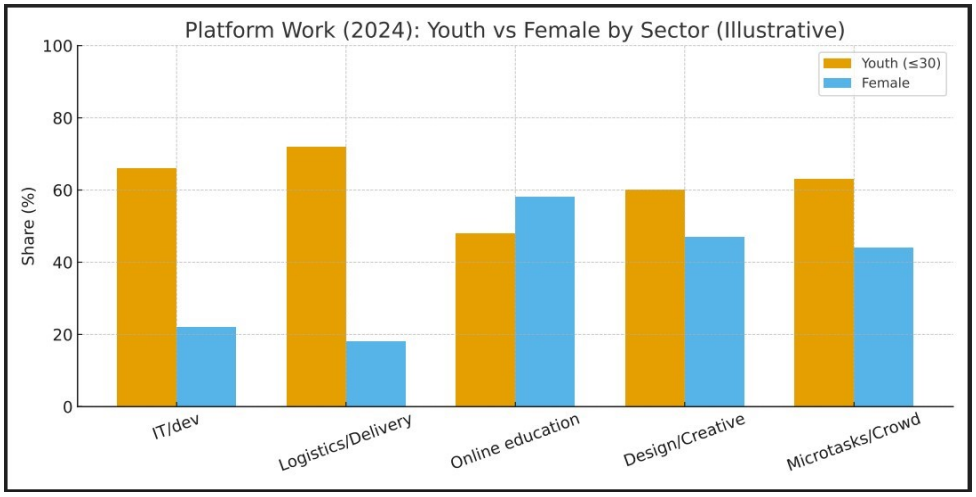
4.3. Demographic breakdown The proportion of youth (?30) and women in platform work is growing rapidly; however, the positive impact of remote work on employment flows is unevenly distributed across groups.



"Youth the proportion of workers aged 30 and younger. "Platform work performed through online labor platforms (freelance/content, micro-tasks, delivery, etc.); "traditional non-platform employment and self-employment. Indicators in percentages; p.p. - percentage point difference between 2024 and 2015. The values in the graph are illustrative (placeholders); in the final version, they will be replaced with official sources (e.g., LFS/MICS, platform admin data, ILO/WB/OII). Calculation: proportion within the group = (number of youth in the group / total number of workers in the group) ? 100.



"Share of women the percentage of female workers in the respective group. Definitions: same as above. Depending on the scope of platform segments (online freelance, micro-tasks, locationbased delivery), the definition may vary; maintain the same parameters for comparison. Graph values are illustrative; will be populated with official sources in the final version. If data is available at the task level, use task-weighted proportions.



Sector definitions: IT/Dev (programming, technical support), Logistics/Delivery (courier, delivery), Online education (online teaching/tutoring), Design/Creative (design, content), Microtasks/Crowd (annotation, micro-tasks). Indicators - the share of active workers; if only the number of orders is available, weighting by the number of orders is recommended. Graph values are illustrative; for final data, verify the compatibility of platform admin records/LFS and definitions.

5. Data and analysis

Labor resources are one of the important factors in a country's economic and social development. The working-age portion of the population not only participates in the production process but also serves as the main source for forming national income. The number of labor resources depends on various economic, social, and demographic factors, and without a comprehensive analysis, it will be difficult to effectively implement the state's employment policy. In particular, economic factors (GDP volume, total population income), health factors (healthcare institutions, quality of medical services), and educational factors (number of educational institutions, expected years of education) can have a significant impact on the formation of labor resources. In this study, the following independent variables were selected as the main determinants influencing the number of labor resources (Y):

- X1 - total population income (billion soums);
- X2 - gross domestic product (billion soums);
- X3 - number of HEIs (units);
- X4 - expected years of education;
- X5 - number of medical institutions (units);
- X6 - number of medical personnel (thousand people).

The construction of this econometric model allows for empirically determining the role and significance of changes in economic (population income, GDP), health (number of medical institutions and medical personnel), and education (number of HEIs, expected years of education) spheres in the process of labor resource formation. Based on the results of econometric analysis, statistical methods are used to verify which factors have a significant impact on labor resources and which are less significant. At the initial stage of econometric modeling, the main focus was on ensuring the following conditions:
- factor variables must have a cause-and-effect relationship with the resultant variable;

- factor variables should not duplicate each other, i.e., not be multicollinear, and should not be a structural element of the resultant variable or its function;
- it is advisable not to include factors that cause multicollinearity in the model;
- The mathematical form of the regression equation must fully correspond to the nature of the relationship between factors and the result in real conditions. If a result can be formed even when a factor or group of factors is not active, then such a condition corresponds to an additive relationship by nature. If in the absence of any of the factors it is impossible to carry out a process that ends with a result, then in such conditions it is more reasonable to use the form of a multiplicative relationship. When constructing a multifactorial econometric model, we first conduct descriptive statistics by factors. The obtained results are reflected in the following table (Table 1).

Table 1 Descriptive statistics by selected factors

Table 1. Descriptive statistics by selected factors

	Y	X_1	X_2	X_3	X_4	X_5	X_6
Mean	18565.61	340944.7	522599.6	103.3571	13.36	6327.357	348.2379
Standard Error	246.0047	67950.08	110163.7	13.84078	0.226667	173.1208	6.575744
Median	18666.3	249346.3	356453.8	71	13.35	6287	344.75
Mode	-	-	-	65	12,6	-	-
Standard Deviation	952.7719	263169.5	426662.1	51.78745	0.716783	647.7588	24.60418
Variance	907774.3	6.93E+10	1.82E+11	2681.94	0.513778	419591.5	605.3658
Kurtosis	-0.55726	-0.23191	0.032607	0.559473	-1.14069	2.888481	-1.22352
Skewness	-0.3032	0.926344	1.00545	1.284429	0.382241	1.179627	0.157565
Range	3359.2	833642.8	1366472	155	2	2715	77.43
Minimum	16726	62631.6	88102.4	64	12.май	5296	310.2
Maximum	20085.2	896274.4	1454574	219	14.май	8011	387.63
Sum	278484.1	5114170	7838994	1447	133.6	88583	4875.33
Count	15	15	15	14	10	14	14

From the table data, one can observe the average value, median, mode, maximum and minimum values for the dynamics of each factor. Additionally, one can see the indicators of standard deviation, variance, kurtosis, and skewness for each factor. To select factors for the multifactor econometric model, it is necessary to conduct a correlation analysis. For this purpose, partial and pair correlation coefficients are calculated between the factors.

Let's begin with a correlation analysis to determine the strength of interaction between each factor. For this, we will use the pair correlation coefficient.
The correlation coefficient takes values from -1 to +1, according to which the strength of the relationship is determined. A positive sign indicates a direct relationship, while a negative sign represents an inverse relationship. However, since the correlation coefficient only measures the degree of association between economic indicators, it cannot explain the causes of economic relationships. This purpose is served by a special method called regression analysis, which allows one to assess the degree of influence of factors affecting the resulting indicator.
Due to the different units of measurement for the variables and to better explain the interpretation of the multi-factor econometric model, we will construct a correlation matrix by logarithmizing the factor Y values in natural form (Table 2). The correlation matrix is a matrix that reflects the pairwise correlation coefficients between factors.

Table 2 . Correlation matrix

	ln_LR	ln_Inc~e	ln_GDP	ln_HEI	ln_edu~r	ln_Pol~s	ln_Doc~s
ln_LR	1.0000						
ln_Income	0.9952	1.0000					
ln_GDP	0.9941	0.9994	1.0000				
ln_HEI	0.9623	0.9769	0.9791	1.0000			
ln_eduyear	0.9601	0.9717	0.9666	0.9639	1.0000		
ln_Polycli~s	0.6056	0.5927	0.5984	0.7027	0.5965	1.0000	
ln_Doctors	0.9904	0.9972	0.9981	0.9768	0.9609	0.5813	1.0000

As can be seen from Table 2, the partial correlation coefficients indicate the strength of the relationship between the dependent variable (lny) and its influencing factors. The values of these partial correlation coefficients are greater than 0.5, allowing them to be included in the multifactor model. However, according to the calculation results, some values of the pairwise correlation coefficients (rx,y) between the aforementioned influencing factors exceed 0.7. This suggests that there is a multicollinearity problem among the selected factors. The crucial point here is that the influencing factors should not be closely interrelated. In other words, there should be no multicollinearity among the influencing factors. If the value of the pairwise correlation coefficient between two influencing factors is greater than 0.7, a multicollinearity problem arises.

For this, we first need to eliminate the problem of multicollinearity, and then move on to the next stages of econometric modeling. To do this, we must first remove the factors causing the multicollinearity problem from the model. From the data in Table 2, it can be seen that population income and expected years of education are significantly more closely related to other factors, and we should exclude them from the econometric model. Here, GDP from economic factors and the number of higher education institutions from education factors are included in the econometric model, as well as the sum of the number of medical institutions and medical workers is included as health factors. Secondly, another way to check for the absence of multicollinearity among the remaining influencing factors is to calculate the VIF (Variance Inflation Factors - multicollinearity effect) coefficients. VIF coefficients are calculated for each factor. If there is multicollinearity among the influencing factors, then VIF $>$ 10 (Table 3).

Table 3.
Measuring the effect of multicollinearity among influencing factors

Variable	VIF	1/VIF
ln_HEI	11,14	0.089746
ln_GDP	8,42	0.118772
ln_medicine	2,07	0.483693
Mean VIF	7	21

According to Table 3, the VIF coefficient is less than 10. Consequently, this indicates that the problem of multicollinearity among influencing factors is not very high or does not pose a serious threat to the model.
Additionally, to determine whether an important factor has been omitted in the model, we conduct the "Ramsey's Regression Equation Specification Error Test"(RESET) (Table 4).

Table 4. RESET test results

Ramsey RESET test using powers of the fitted values of ln_MR
H0: model has no omitted variables
F (3, 7) = 4.16

According to the data in Table 4, H0 - there are no specification errors in the model, all significant variables are included, and the regression equation has the required functional form; H1 - there is an error in the model (a specification error exists), i.e., important variables are missing or the functional form is incorrect.
According to this test, the null hypothesis was confirmed. To perform the analysis, we construct a multi-factor regression equation, where we assume that the relationship between y and x is linear

$$\ln y = a_0 + a_1 \ln x_1 + a_2 \ln x_2 + a_3 \ln x_3 + \varepsilon_i$$

a_0, a_1, a_2, a_3 we determine the parameters. $a_0 = 8.6$, $a_1 = 0.08$, $a_2 = -0.06$, $a_3 = 0.05$.

Table 4 Calculated parameters of the multifactor econometric model

Linear regression					Num. of obs	14
					F (3, 10) =	1476.50
					Prob > F =	0.0000
					R-squared =	0.9980
					Root MSE =	.0025
ln_LR	Coef.	Robust Std. Err.	t	P>t	[95% Conf. Interval]	
ln_GDP	.0830975	.0037629	22.08	0.000	.0747132	.0914818
ln_HEI	-.056988	.0074061	-7.69	0.000	-.0734899	-.0404863
ln_medicine	.0501529	.0109979	4.56	0.001	.025648	.0746577
_cons	8.581505	.1049001	81.81	0.000	8.347774	8.815237

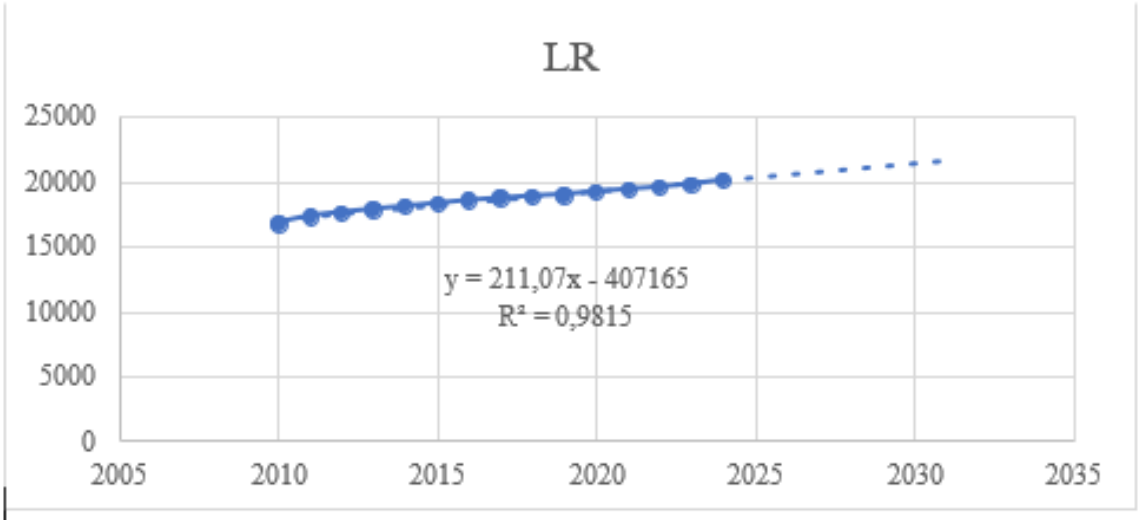
Thus, the linear regression equation of the correlation relationship takes the following form:

$$ln(Yx) = 8.6 + 0.08lnx1 - 0.06lnx2 + 0.05lnx3 \quad (1)$$

Here, the regression coefficients a_0, a_1, a_2, a_3 determine the relationship between the resulting variable (Y) and the influencing factor variables (x). In this econometric model, the dependent variable Y is the number of labor resources, and all independent variables are natural logarithms. That is, the economic meaning of this equation and its coefficients can be interpreted as follows: each percentage change in the independent variable Xi indicates a percentage change in Y. $a_0 \approx 8.6$ means that if $X_1, X_2, X_3 \to 0$, then the logarithm of the number of labor resources will be equal to 8.6, or e8.6=5332.1 thousand people. This value has no direct economic meaning because economic and social variables are never zero in real life. $a_1 = 0.08$, which means that every additional 1 percent increase in GDP leads to a 0.08 percent increase in labor resources. $a_2 = -0.06$. This means that a 1% increase in the number of higher education institutions will reduce the number of labor resources by 0.06%. The increase in the number of higher education institutions is slightly reducing the size of the workforce - this is usually due to temporary withdrawal from the labor market during the student period (engagement in studies)

or migration/structural changes. $a_3 = 0.05$. Thus, a 1% increase in the number of medical institutions and personnel will increase the number of labor resources by 0.05%, i.e., improvement of health infrastructure will have a positive impact on retaining the working-age population. According to the results of this regression equation, economic factors are more elastic than other factors in increasing the number of labor resources, confirming that economic growth has the most significant impact on the volume of labor resources. In other words, stimulating economic growth (through investments and job creation) is effective in increasing the volume of labor resources. However, education and healthcare policies also shape long-term quality and participation. Additionally, the multiple correlation coefficient between the resultant and all influencing factors was equal to R=0.99, indicating that there is a strong relationship between the resultant factor Y and the selected factors X_1, X_2, X_3. Furthermore, the accuracy of the selected factors' influence on the labor market is determined by the coefficient of determination R_2. Hence, R_2?0.99 , meaning that 99 percent of the change in the resulting factor is caused by X_1, X_2, X_3 factors (economic, health, and educational factors) (Table 4). It should be noted that regression and correlation indicators (parameters of the regression equation, coefficients of determination and correlation) may be distorted under the influence of random errors, since they are based on data from a quantitatively limited set. The influence of random errors on regression and correlation indicators should not be too significant; otherwise, such indicators will become meaningless. The significance, reliability, and applicability of the constructed econometric model for future forecasting are assessed based on several criteria: In this case, the F-criterion is equal to 1635.67. According to the Fisher (F) criterion, the hypothesis of the regression equation is statistically significant if it satisfies the condition F > Ftable. Here, F-table is determined from Fisher's F-criterion table with a significance level ? and degrees of freedom $k_1 = k, k_2 = n - k - 1$. The significance level α typically takes a value of 0.05, which corresponds to a probability of up to 5% for type I errors. According to the econometric model we constructed, the Fisher (F) criterion is 1635.67. Comparing this calculated value with the table value, we find that $1635.67 > 3.71$ under the condition of $\alpha = 0.05$. Therefore, the constructed econometric model is significant. The parameters of the econometric model are tested using Student's t-test and p-value. The calculated value of Student's t-statistic is compared with the critical value from the t-distribution table. If tcalculated>tcritical at $\alpha = 0.05$, the parameter is considered statistically significant; if tcalculated<tcritical at $\alpha = 0.05$, it is considered insignificant. In our example, for $\alpha = 0.05$, tcritical ≈ 2.15. Therefore, all the model parameters we have selected are significant.

Furthermore, in regression analysis, the calculated p-value for independent variables is used to assess the significance of each variable in the model or its relationship with the dependent variable. In other words, this p-value helps determine whether the independent variable has a significant effect on the dependent variable. ? If the p-value is < 0.05, the independent variable has a significant influence on the dependent variable. This means that the null hypothesis (the assumption that the independent variable has no effect on the dependent variable) is rejected, and the effect of this variable is considered statistically significant. If the p-value > 0.05, the influence of the independent variable on the dependent variable is not significant, i.e., the null hypothesis is not rejected, and the influence of this variable is considered insignificant. According to our analysis, all parameters are significant and have a substantial impact on the result.



Furthermore, considering the presence of heteroscedasticity in the regression model, it was addressed using robust standard errors (Table 4). This is because the robust operation allows for accurate estimation of standard errors even in the presence of heteroscedasticity. As a result, the t-statistics and p-values of the regression become more reliable. Additionally, such an econometric model enables us to make forecasts. For example, if we assume that labor resources increase only depending on the time factor, not under the influence of the selected factors, then this can be represented graphically as follows (Figure).

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Stochastic Processes in Asset Price Modeling

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DOI 10.5281/zenodo.18217272

Keywords: stochastic processes, Brownian motion, geometric Brownian motion, Ornstein–Uhlenbeck process, jump–diffusion models, asset price modeling, financial econometrics.

Abstract: This paper develops an integrated theoretical and econometric perspective on stochastic processes for asset price modeling in continuous time. Brownian motion andItocalculus are presented as the mathematical foundations of modern financial modeling, followed by analytical derivations of the Geometric Brownian Motion, the OrnsteinвЂ“Uhlenbeck mean-reverting process, and jump-diffusion models. Beyond formal solutions, the study emphasizes economic interpretability and empirical relevance by linking model structures to stylized facts observed in real financial markets. The analysis highlights how continuous-time stochastic models form a coherent and extensible framework for contemporary financial econometrics.

Introduction: Asset prices evolve under uncertainty and are naturally modeled using stochastic processes. Continuous-time models provide a rigorous mathematical framework for describing randomness, volatility, and risk. While classical econometric models are often formulated in discrete time, continuous-time stochastic differential equations allow for deeper theoretical insight and clearer probabilistic interpretation.

This study is intended as a conference contribution aimed at highlighting the connection between analytical stochastic modeling and empirical financial econometrics. Rather than focusing on extensive numerical estimation, the paper emphasizes analytical transparency and economic interpretation supported by real market data .

Brownian Motion and Ito Calculus An Ito process is defined as:

$$dX_t = a(X_t, t)\,dt + b(X_t, t)\,dW_t.$$

Here, X_t denotes a stochastic process observed at time t, W_t is a standard Brownian motion, $a(X_t, t)$ represents the drift coefficient determining the deterministic trend of the process, and $b(X_t, t)$ is the diffusion coefficient capturing random fluctuations. Itô's formula for a twice continuously differentiable function $f(t, X_t)$ is given by:

$$df = f_t\,dt + f_x\,dX_t + \frac{1}{2} f_{xx}\,b(t)^2\,dt.$$

In this expression, f_t, f_x, and f_{xx} denote partial derivatives of the function f with respect to time and the state variable X_t.

$$dX_t = a(X_t, t)\,dt + b(X_t, t)\,dW_t \tag{1}$$

X_t — stochastic process; t — continuous time; W_t — standard Brownian motion; $a(X_t, t)$ — drift function; $b(X_t, t)$ — diffusion coefficient.

$$df = \frac{\partial f}{\partial t}\,dt + \frac{\partial f}{\partial x}\,dX_t + \frac{1}{2}\,\frac{\partial^2 f}{\partial x^2}\,b^2(t)\,dt \tag{2}$$

$f(t, X_t)$ — twice continuously differentiable function; $\partial f/\partial t$, $\partial f/\partial x$, $\partial^2 f/\partial x^2$ — partial derivatives of f.

Geometric Brownian Motion. The Geometric Brownian Motion (GBM) model for an asset price S_t is defined by:

$$dS_t = \mu S_t\,dt + \sigma S_t\,dW_t$$

Here, S_t denotes the asset price at time t, μ is the drift parameter representing the expected rate of return, σ is the volatility parameter, and W_t is a Brownian motion. Applying Itô's formula to the logarithm of the price yields:

$$d(\ln S_t) = \left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma\,dW_t$$

The analytical solution of the GBM is:

$$S_t = S_0 \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right) t + \sigma W_t\right]$$

The expected value and variance of the asset price are:

$$\mathbb{E}[S_t] = S_0 \exp(\mu t)$$

$$\mathrm{Var}(S_t) = S_0^2 \exp(2\mu t)\left(\exp(\sigma^2 t) - 1\right)$$

$$dS_t = \mu S_t\,dt + \sigma S_t\,dW_t \tag{3}$$

S_t — asset price; μ — drift (expected return); σ — volatility; W_t — Brownian motion.

$$d(\ln S_t) = \left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma\,dW_t \tag{4}$$

$\ln S_t$ — logarithm of the asset price.

$$S_t = S_0 \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right) t + \sigma W_t\right] \tag{5}$$

S_0 — initial asset price; $\exp(\cdot)$ — exponential function.

$$\mathbb{E}[S_t] = S_0 e^{\mu t} \tag{6}$$

$\mathbb{E}[S_t]$ — mathematical expectation.

$$\mathrm{Var}(S_t) = S_0^2 e^{2\mu t} \left(e^{\sigma^2 t} - 1 \right) \tag{7}$$

$\mathrm{Var}(S_t)$ — variance measuring dispersion and risk.

Mean-Reverting Process: Ornstein–Uhlenbeck. A mean-reverting process is modeled by the Ornstein–Uhlenbeck equation:

$$dX_t = \kappa(\theta - X_t)\, dt + \eta\, dW_t \tag{8}$$

Here, κ denotes the speed of mean reversion and θ represents the long-run equilibrium level. The analytical solution is:

$$X_t = \theta + (X_0 - \theta)e^{-\kappa t} + \eta \int_0^t e^{-\kappa(t-s)}\, dW_s$$

In discrete time, the process corresponds to an AR(1) representation:

$$X_t - \theta = \phi(X_{t-1} - \theta) + u_t$$

κ — speed of mean reversion; θ — long-run equilibrium level; η — volatility parameter.

$$X_t = \theta + (X_0 - \theta)e^{-\kappa t} + \eta \int_0^t e^{-\kappa(t-s)}\, dW_s \tag{9}$$

X_0 — initial value of the process; $\int_0^t \cdot\, dW_s$ — stochastic integral.

$$X_t - \theta = \phi(X_{t-1} - \theta) + u_t, \qquad \phi = e^{-\kappa \Delta t} \tag{10}$$

ϕ — autoregressive coefficient; Δt — time step; u_t — error term.

Jump-Diffusion Models. To account for sudden price changes, a jump–diffusion model is specified as:

$$\frac{dS_t}{S_t} = \mu\, dt + \sigma\, dW_t + (J - 1)\, dN_t \tag{11}$$

Here, N_t is a Poisson process capturing the number of jumps. The corresponding log-price representation is:

$$\ln S_t = \ln S_0 + \left(\mu - \frac{1}{2}\sigma^2 \right) t + \sigma W_t + \sum_i Y_i \tag{12}$$

Y_i — logarithmic jump size; $\sum_i Y_i$ — cumulative jump effect. Empirical Illustration: German Stock Market (DAX) To demonstrate empirical relevance, daily closing prices of the German DAX stock index over the period 2016вЂ“2024 were considered. Logarithmic returns were computed and analyzed in terms of their distributional properties. The data exhibit volatility clustering and excess kurtosis, indicating deviations from the Gaussian assumptions of the GBM model. Periods of heightened market stress correspond to abrupt price movements, motivating the use of jump-diffusion specifications. Mean-reverting behavior is particularly relevant for volatility-related variables, supporting the

applicability of the OrnsteinвЂ“Uhlenbeck process.These developments indicate that contemporary research increasingly combines analytical clarity with empirically motivated extensions and modern estimation tools. From a modern econometric perspective, the empirical features observed in equity index returnsвЂ”namely volatility clustering, heavy-tailed distributions, and occasional extreme observationsвЂ”provide direct motivation for moving beyond the classical GBM framework. These stylized facts have been repeatedly documented in post-2020 empirical studies and justify the use of stochastic volatility, mean-reverting dynamics, and jump components as complementary modeling layers rather than competing specifications. In this sense, continuous-time stochastic models should be viewed as a flexible hierarchy, where increasing model complexity is guided by empirical diagnostics rather than purely theoretical considerations. First, rough volatility models (often formulated via fractional/Volterra-type dynamics) have been widely studied as they can reproduce short-maturity implied-volatility skews and the apparent pathwise roughness of realized volatility. At the same time, recent econometric work revisits the statistical evidence and proposes nonparametric diagnostics to test whether volatility is truly 'rough' in the data. Second, the integration of stochastic modeling with modern machine learning has led to neural stochastic differential equations (neural SDEs), where drift and diffusion coefficients are parameterized by neural networks. These models preserve the continuous-time structure while offering flexible functional forms, and they are increasingly used for time-series modeling, simulation, and calibration tasks. Third, high-frequency econometrics continues to refine identification and testing of jump components in the presence of microstructure noise and time-varying volatility. This line of work provides practical tools to decide when jump-diffusion specifications are empirically justified, and it informs model selection in applied asset-pricing studies.

Recent research in continuous-time finance increasingly focuses on extending classical diffusion and jump-diffusion specifications to better match empirical features of returns and volatility. Three directions are particularly influential. This result demonstrates that stochastic differential equations provide a coherent and flexible framework for modeling asset price dynamics. Analytical solutions offer clear economic interpretations, while empirical observations from the German stock market highlight the importance of extending classical models to capture real-world features such as volatility clustering and discontinuities. By integrating mathematical rigor with econometric reasoning and real market data, the study illustrates the continued relevance of stochastic processes in modern financial econometrics, particularly in the context of international research discussions. In particular, the results support the view that modern asset pricing is best approached through a data-driven hierarchy of stochastic models, where analytical tractability, econometric validation, and empirical realism jointly determine model choice.

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Stress Analysis of Inclined Edge Cracks in Bonded Planes

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Abstract: Inclined edge cracks originating at the interface of bonded planes and subjected to various stresses are studied. The crack problem is formulated into the singular integral equation (SIE) using the modified complex potential (MCP) together with the continuity conditions of traction and displacement. A semi-open quadrature rule is applied to solve the integrals of the SIE numerically. Stress intensity factors (SIFs) for both Modes I and II cracks in the upper plane, as well as in both the upper and lower parts of the planes are analysed. The crack configuration, crack inclination angle and elastic constant ratios of the planes have a significant effect on the behaviour of the nondimensional SIFs at the crack tip. These parameters also influence the stress concentration and distribution at the crack tips.

Keywords: Edge cracks; Modified complex potentials; Singular integral equation; Bonded planes; Stress intensity factors.

Numerical solution of a boundary value problem for a singularly perturbed equation by preliminary integration method

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DOI 10.5281/zenodo.18217304

Abstract. The aim of this paper is to develop a discrete variant of the preliminary integration method for solving boundary value problems of singularly perturbed differential equations. In this approach, the derivatives and the solution are expanded into finite series of Chebyshev polynomials of the first kind. Substitution into the governing equation and matching of coefficients lead to a reduced algebraic system, which is integrated using a discrete formula, with the boundary conditions incorporated in series form. The resulting algebraic system is solved to obtain an approximate solution. The numerical results presented in tables and figures demonstrate the high accuracy of the proposed method.

Keywords: Chebyshev polynomials, algebraic system, high accuracy, derivative.

In reference [2], a numerical solution of a singularly perturbed two-point boundary value problem on the interval $(0, 1)$ was obtained using the Liouville–Green transformation. In the present work, we consider the solution of this problem by applying a discrete variant of the preliminary integration method based on Chebyshev polynomials of the first kind.

It is required to find an approximate solution of the following differential equation using Chebyshev polynomials of the first kind:

$$4\varepsilon\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 0, \quad x \in (-1, +1), \tag{1}$$

under the boundary conditions:

$$y(-1) = 1, \quad y(+1) = 0 \tag{2}$$

here ε – small parameter.

The exact solution of problem (??)-(??) has the following form [3]:

$$y(x) = \frac{e^{\frac{x-1}{2\varepsilon}} - 1}{e^{-\frac{1}{\varepsilon}} - 1} \tag{3}$$

For the numerical solution of the posed problem, we apply a discrete variant of the preliminary integration method. The main idea of this method is as follows. The

highest derivative and the right-hand side of equation (1) are represented as finite series of Chebyshev polynomials of the first kind with unknown coefficients. Before solving the differential problem (1)-(2), the series representing the approximate solution is preliminarily "integrated" twice using a discrete formula designed to reduce the order of the derivative.

The main advantage of this approach is that, during the integration of a finite Chebyshev series, the smoothness of the approximating polynomials increases. For example, a zeroth-order polynomial is transformed into a second-order polynomial, and so on. This, in turn, positively affects the accuracy of computations. The boundary conditions of the problem are exactly satisfied by appropriately choosing the equations for the unknown coefficients. Thus, the discrete variant of the preliminary integration method allows one to obtain an approximate solution of the problem with sufficiently high accuracy for moderate values of the small parameter.

At the collocation points of the Chebyshev polynomials, the trial function (the exact solution) of the posed problem is compared with the approximate solution obtained using the discrete variant of the preliminary integration method. We now present the algorithm of the proposed method. The second derivative is represented in the form of the following series:

$$\frac{d^2y_a}{dx^2} = \sum_{i=0}^{N}{}'a_i^{(2x)}T_i(x), \tag{4}$$

where u_a denotes the approximate solution of the problem $u_a = u_{\text{approximate}}$, $T_i(y)$ - is the Chebyshev polynomial i– of order, and the prime over the summation indicates that a_i the coefficients of the series are taken with a $\frac{1}{2}$ factor when $i = 0$ – are the unknown expansion coefficients [1,4].

The series (4) representing the approximate solution is integrated twice, and all lower-order derivatives, as well as the approximate solution of the problem, are obtained in the form of finite series of Chebyshev polynomials of the first kind:

$$\frac{dy_a}{dx} = \sum_{i=0}^{N}{}'a_i^{(x)}T_i(x), \quad y_a(x) = \sum_{i=0}^{N}{}'a_iT_i(x). \tag{5}$$

Next, by substituting series (4) and the necessary series from (5) into the differential equation (1), we obtain

$$4\varepsilon\sum_{i=0}^{N}{}'a_i^{(2x)}T_i(x) + 2\sum_{i=0}^{N}{}'a_i^{(x)}T_i(x) = 0$$

Then, by equating the coefficients of like powers of the polynomials, we obtain an algebraic system of the following form:

$$4\varepsilon a_i^{(2x)} + 2a_i^{(x)} = 0, \quad i = 2, 3, \ldots, N. \tag{6}$$

For Chebyshev polynomials of the first kind, the following discrete formula of preliminary integration is available, which reduces the order of derivatives in series (6):

$$a_i^{((k-1)x)} = \frac{a_{i-1}^{(kx)} - a_{i+1}^{(kx)}}{2i} \tag{7}$$

Equation (6) is integrated twice using formula (??):

a) First integration

$$4\varepsilon a_i^{(y)} + 2a_i = 0;$$

b) After the second integration, we obtain the main algebraic equation with respect to the unknown coefficients a_i:

$$4\varepsilon a_i + \frac{a_{i-1} - a_{i+1}}{i} = 0, \quad i = 2, 3, \ldots, N \tag{8}$$

The series for the approximate solution in (5) contains $(N+1)$ unknown coefficients a_i, $(i = 0, 1, 2, \ldots N)$ the main algebraic system consists of $(N-1)$ equations, the remaining two equations are determined from the boundary conditions (2), expressed in terms of finite series of Chebyshev polynomials. Taking into account the following properties of Chebyshev polynomials $T_i(\pm 1) = (\pm 1)^i$, the boundary conditions (2) are written in the form [3]:

$$\begin{cases} u(-1) = \dfrac{1}{2}a_0 - a_1 + a_2 - a_3 + \ldots + a_N = 1, \\ u(+1) = \dfrac{1}{2}a_0 + a_1 + a_2 + a_3 \ldots + a_N = 0. \end{cases} \tag{9}$$

First, by adding and then subtracting the equations from system (9), we obtain

$$\begin{gathered} \frac{1}{2}a_0 + a_2 + a_4 + \ldots + a_N = \frac{1}{2}, \\ a_1 + a_3 + a_5 + \ldots + a_{N-1} = -\frac{1}{2}. \end{gathered} \tag{10}$$

The main algebraic equations (8), together with the boundary conditions (10), form a system $(N+1)$ of linear algebraic equations for determining $(N+1)$ the unknowns a_i, $(i = 1, 2, 3, \ldots, N)$. This algebraic system has the following form:

$$\begin{cases} -a_{i-1} + 4\varepsilon i a_i + a_{i+1} = 0, \\ \frac{1}{2}a_0 + a_2 + a_4 + \ldots + a_N = \frac{1}{2}, \\ a_1 + a_3 + a_5 + \ldots + a_{N-1} = -\frac{1}{2}, \quad i = \overline{2, \ldots, N}. \end{cases} \tag{11}$$

It is convenient to represent system (11) in matrix form

$$Aa = b \tag{12}$$

where A – is a square matrix of order $M \times M$, with $M = (N+1)$ consisting of the coefficients of system (12), $a^T = (a_0, a_1, \ldots a_N)$ is the vector of unknowns, and b– is the right-hand side of system (12). By solving system (11), the coefficients are determined; then, using formulas (3), the values of the exact solution are calculated, and using formula (5), the values of the approximate solution at the collocation points of the Chebyshev polynomials $y_l = \cos\frac{\pi l}{N}$, $l = 0, 1, 2, \ldots N$ are obtained.

The differential problem (1)-(2) was solved using the discrete preliminary integration method for values of the small parameter $\varepsilon = 10^{-2}$ and when the number of approximating polynomials is $N = 10, 20$. Table 1 presents the computational results for the given value of the small parameter $\varepsilon = 10^{-2}$. Table 1 also shows a comparison of the exact and approximate solutions at the collocation points of the Chebyshev polynomials for $N = 10$.

Table 1. Comparison of the exact and approximate solutions

ε	N	l	$u_e(y_l)$- exact solution	$u_a(y_l)$- approximate solution	$\lvert u_e - u_a \rvert$ - error
10^{-2}	10	1	1.0	1.136	0.136
		3	1.0	1.171	0.171
		5	1.0	1.175	0.175
		7	0.999999999	1.170374337	0.1703743377
		9	0.9134622189	1.030226917	0.1167646985

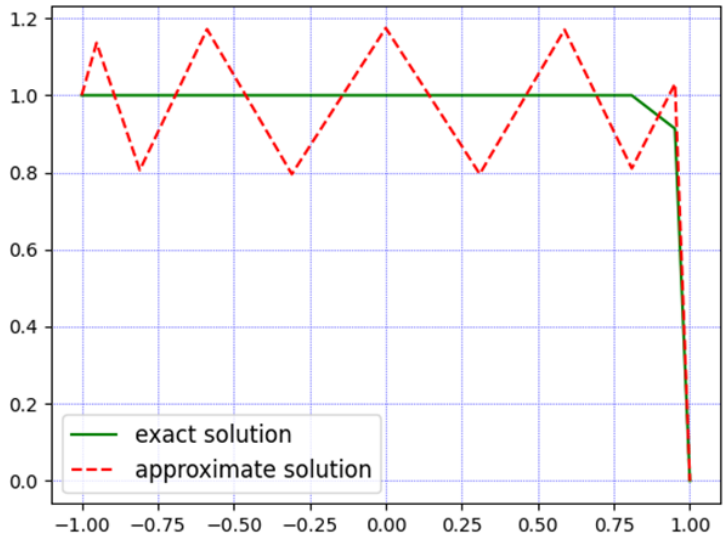

Fig 1. Comparison of the exact and approximate solutions for $\varepsilon = 10^{-2}$ and $N = 10$.

The results of the comparison between the exact and approximate solutions for $N = 20$ and the same value of the small parameter $\varepsilon = 10^{-2}$ are presented in Table 2.

As the number of approximating polynomials increases, the absolute error between the exact and approximate solutions decreases.

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Table 2. Comparison of the exact and approximate solutions

ε	N	l	$u_e(y_l)$- exact solution	$u_a(y_l)$- approximate solution	$\lvert u_e - u_a \rvert$ - error
10^{-2}	20	5	1.0	1.0032	0.0032
		10	1.0	0.994	0.006
		15	1.0	1.003	0.003

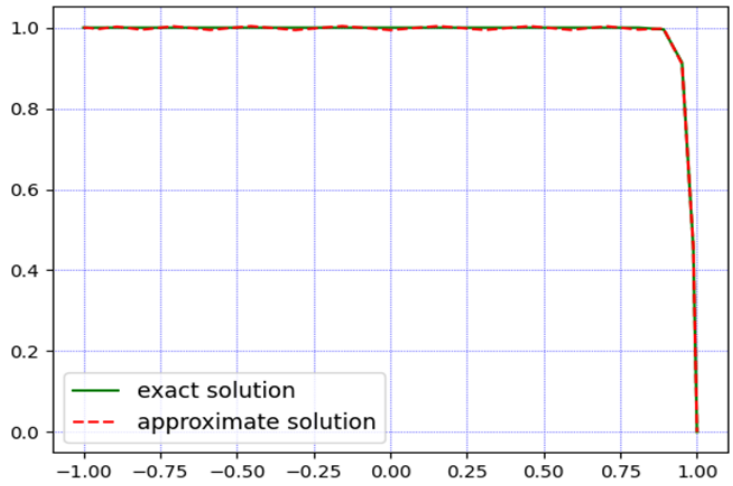

Fig 2. Comparison of the exact and approximate solutions for $\varepsilon = 10^{-2}$ and $N = 20$.

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History of Political relations between Germany and Kazakhstan in 1991-2025

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DOI 10.5281/zenodo.18217321

Abstract. The following article demonstrated history of political relations of Germany and Kazakhstan in 1991-2025. There is a strong partnership between Kazakhstan and Germany based on trust, mutual support, and shared democratic values. political cooperation between the countries includes official visits and consultations, international forums, interparliamentary cooperation and security cooperation. Germany and Kazakhstan expressed satisfaction with cooperation within international organizations and agreed to continue to utilize opportunities for mutual support of initiatives and candidacies within the United Nations (UN), the Organization for Security and Cooperation in Europe (OSCE), and other international organizations. In this scientific article, such methods as content analysis, historical-analytical method were used, which allowed to analyze the events, documents and sources of political relations between Kazakhstan and Germany.

Key words: Political, politics, democratic values, partnership, mutual support, trust, official visits, consultations, international forums

There is a strong partnership between Kazakhstan and Germany based on trust, mutual support, and shared democratic values.[1]

If to talk about some aspects of political cooperation between the countries:

- Official visits and consultations - Regular high-level official visits and consultations;
- International forums – two countries actively cooperate within international forums and organizations such as the UN, OSCE, and the Shanghai Cooperation Organization;
- Interparliamentary cooperation – There are parliamentary friendship groups in Kazakhstan Parliament (Kazakhstan-Germany) and the German Bundestag (Germany-Central Asia);
- Security cooperation – Kazakhstan and Germany maintain cooperation through the exchange of experience and practical measures between law enforcement agencies in the areas of security and the fight against terrorism and extremism.[2]

In 2024, 21 projects involving German capital were implemented in Kazakhstan, with a total investment of approximately 1 billion euro, covering sectors such as the production of building materials and valves.

If to talk about Joint Declaration on Cooperation between the Republic of Kazakhstan and the Federal Republic of Germany, several aspects can be mentioned as:

- Political Dialogue
- Parliamentary Diplomacy
- Legal Cooperation
- Central Asia + Germany
- Security, Counterterrorism, and Drug Trafficking
- Economic Cooperation
- Energy
- Industry
- Climate, Environment, and Water Management
- Transport and Logistics
- Food and Agriculture
- Innovation, New Technologies, and Digitalization
- Finance
- Healthcare
- Science and Education
- Culture
- Visa
- Fundamentals of Technical Cooperation
- Final Provisions

Let's talk about political aspects deeper.

- Germany and Kazakhstan expressed satisfaction with cooperation within international organizations and agreed to continue to utilize opportunities for mutual support of initiatives and candidacies within the United Nations (UN),

the Organization for Security and Cooperation in Europe (OSCE), and other international organizations. They reaffirmed their commitment to the purposes and principles of the UN Charter, the Helsinki Final Act, and other documents adopted within the OSCE, as well as other universally recognized norms of international law.

- The Parties emphasize that universally recognized principles and norms of international law, democracy, human rights, and the rule of law as common values underlie bilateral relations. The Parties share the opinion that an active civil society, free media, and an independent judiciary make a significant contribution to the preservation of these values.[3]

If to demonstrate relations of Kazakhstan and Germany graphically it can be clearly seen that the connection between two parties has positive growth:



Graph1. Punctuation of historical relation between Kazakhstan and Germany[4]

So, from the graph it can be clearly seen that relation of two sides has positive growth. If the correlation point was around 6 in 2022, in 2025 it reached 7.

If to talk about the **strengths and weaknesses** of each country, in terms of population it can be seen that the population of Germany is 4.46 bigger than the population of Kazakhstan (population of Germany: 83 783 942, population of Kazakhstan: 18 776 707). The land area of Kazakhstan is 7.74 bigger than the land area of Germany (land area (sq km) of Kazakhstan: 2 699 700, land area (sq km) of Germany: 348 672). Moreover, the position in the international influence ranking of Germany is 8, while Kazakhstan's is 70. So, 62 places lower. Additionally, Germany holds contact with 161 countries, while Kazakhstan has contact with 45 countries. Germany maintains positive relation with 46 countries, while Kazakhstan with 12.
If to talk about the history of relations between two sides, it can be seen that:

- On December 1, 1991, Germany recognized the state sovereignty of Kazakhstan.
- On February 11, 1992, diplomatic relations were established.
- In December 1992, the Embassy of Germany was opened in Almaty, which was moved to Astana in 2007.

- In September 1993, the Embassy Kazakhstan was opened in Bonn, and in 1999, it was relocated to Berlin.
- Since July 2022, N. Onzhanov has been the Ambassador of Kazakhstan to the Federal Republic of Germany.
- Since August 2021, M. Iversen has been the Ambassador of the Federal Republic of Germany to the Republic of Kazakhstan.[5]

If to talk about interparliamentary cooperation:

- **In 2015, 2017, 2018, 2019** Chairman of the Senate of the Parliament of the Republic of Kazakhstan K. Tokayev to participate in the Munich Security Conference in Germany.
- **In 2012** Chairman of the Mazhilis of the Parliament of the Republic of Kazakhstan N. Nigmatulin made an official visit to Germany.
- **In 2025** Deputy Chairman of the Mazhilis of the Parliament of the Republic of Kazakhstan – Special Representative of the President of the Republic of Kazakhstan on the promotion of the candidacy of the Republic of Kazakhstan for non-permanent membership in the UN Security Council D. Nazarbayev made an official visit to Germany.
- **In 2010** President of the Bundestag N. Lammert made an official visit to Kazakhstan.
- **2015** Vice Presidents of the Bundestag J. Singhammer made an official visit to Kazakhstan.
- **In September, 2019** Vice Presidents of the Bundestag H.P. Friedrich made an official visit to Kazakhstan.
- **On February 19-21, 2023,** a delegation of the parliamentary group "Germany - Central Asia"visited the Republic of Kazakhstan.

There are parliamentary friendship groups in the Majilis of the Parliament of the Republic of Kazakhstan (“Kazakhstan-Germany”) and the German Bundestag (“Germany-Central Asia”).

And now let’s talk about bilateral and multilateral cooperation of two sides.

- Kazakhstan and Germany actively cooperate within international organizations, including the UN, the UN Human Rights Council, the OSCE, and others.
- Kazakhstan co-sponsored the resolution on the Arms Trade Treaty, initiated by Germany.
- In December 2021, Germany abstained for the first time on the resolution "On the Universal Declaration on Achieving a Nuclear-Weapon-Free World,"initiated by Kazakhstan.
- In 2007, Germany initiated the development of the EU Partnership Strategy for Central Asia and actively participated in the development of the new Strategy "The EU and Central Asia: New Opportunities for a Stronger Partnership."
- Kazakhstan is a member of the Bonn Initiative for a peaceful settlement in Afghanistan, launched by Germany in 2001.
- Germany initiated the Water Initiative for Central Asia (Berlin Process, 2008-2019).
- In January 2020, a presentation of the new German initiative, Green Central Asia, took place.[6]

Conclusion

There is a strong partnership between Kazakhstan and Germany based on trust, mutual support, and shared democratic values. political cooperation between the countries includes official visits and consultations - Regular high-level official visits and consultations; International forums – two countries actively cooperate within international forums and organizations such as the UN, OSCE, and the Shanghai Cooperation Organization; Interparliamentary cooperation – There are parliamentary friendship groups in Kazakhstan Parliament (Kazakhstan-Germany) and the German Bundestag (Germany-Central Asia); Security cooperation – Kazakhstan and Germany maintain cooperation through the exchange of experience and practical measures between law enforcement agencies in the areas of security and the fight against terrorism and extremism.

Germany and Kazakhstan expressed satisfaction with cooperation within international organizations and agreed to continue to utilize opportunities for mutual support of initiatives and candidacies within the United Nations (UN), the Organization for Security and Cooperation in Europe (OSCE), and other international organizations.

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Optimal quadrature formulas in the space $W_2^{(m,m-1,m-2)}$ of periodic functions

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DOI 10.5281/zenodo.18217347

1. Introduction Numerical calculation of integrals of the highly oscillating integrals is one of the more critical problems on numerical analysis because such integrals are widely used in science and technology. The following types of the Fourier integrals are also examples of strongly oscillating integrals for sufficiently large ω:

$$I(\omega, \varphi) = \int_0^1 e^{2\pi i\omega x}\varphi(x)dx \tag{4}$$

It is known that the Fourier transforms are very important tools and they are used widely, in particular, in the problems of Computed Tomography (see, for instance [9]). Since in practice we have finite discrete values of a function, we have to approximately calculate the Fourier transforms. Therefore, one has to consider the problem of approximate calculation of the integral with the weight function $\exp(2\pi i\omega x)$. Initially, a formula for the numerical integration of the strongly oscillatory functions one of the non standard numerical integration methods was given by Filon [4].

Subsequently for integrals with different types of highly oscillating functions many special effective methods such as the Filon-type method, Clenshaw-Curtis-Filon type method, modified Clenshaw-Curtis method, Levin type methods, generalized quadrature rule, and Gauss-Laguerre quadrature are developed (see [2] for full details, for instance, [8,] and references therein).

Recently, in the work [7] author constructed optimal quadrature formulas for numerical calculation of Fourier integrals in the Sobolev space $L_2^{(m)}$ and in the Hilbert space $W_2^{m,m-1,m-2}$, and these formulas were applied to approximate reconstruction of Computed Tomography images. Compared with the optimal quadrature formulas in non-periodic case constructed in [6], the approximation formula for the periodic case constructed in the work [7] is much simpler, therefore it is easy to implement and it costs less computation even though both provide similar performances.

In various Hilbert and Banach spaces of periodic and non-periodic functions, optimal quadrature formulas have been constructed by many researchers for integral (1) with $\omega = 0$. The results for this case can be found, for instance, in the books [6, 9] and the literature in them. In particular, some recent results on optimal quadrature and interpolation formulas are obtained in the work [3].

In the work [5, pp. 119-142] weighted optimal quadrature and cubature for-mulas in the Sobolev space of periodic functions were constructed. In particular, one can get from these results the optimal quadrature formulas for numerical calculation of the integral (1). For example, the practical use of these formulas which were constructed in the Hilbert spaces of non-periodic functions created difficulties in computational work.

Therefore, construction of new optimal quadrature formulas, which are simple in implementation, in various Hilbert spaces of periodic functions is very important. We note that we have constructed the optimal quadrature formula in the case $m = 2$ for integral (1) with $\omega = 0$ in the paper [10]. In the present work we get the results for $m \geq 3$.

Here, we solve the problem of construction of optimal quadrature formula in the Hilbert space $\widetilde{W_2^{m,m-1,m-2}}$ of periodic functions. In the book [9], the optimal quadrature formulas were constructed in the general case in the space of periodic functions, and it was proved that the coefficients of the formulas have the form $\mathring{C}_k = h$. Nevertheless the estimation of the error for the optimal formulas was not given. The main goal of the present work is to find the sharp upper bound for the error of the optimal quadrature formula constructed in the space $W_2^{m,m-1,m-2}(0,1]$ of periodic, real-valued functions.

We consider the Hilbert space $W_2^{m,m-1,m-2}(0,1]$, $m \geq 3$ of non-periodic, real-valued functions $\varphi(x)$, $0 \leq x \leq 1$, that $(m-2)$ and order derivative is absolute continuous and mth order derivative (in the generalized sense) are square integrable, with the inner product

$$\langle \varphi, \psi \rangle_{W_2^{m,m-1,m-2}} = \int_0^1 \left(\varphi^{(m)}(x)\psi^{(m)}(x) + 2\varphi^{(m-1)}(x)\psi^{(m-1)}(x) + \varphi^{(m-2)}(x)\psi^{(m-2)}(x) \right) dx \tag{5}$$

where $\varphi, \psi \in W_2^{(m,m-1,m-2)}(0,1]$, and the corresponding norm of the function φ is defined as follows

$$\|\varphi\|_{W_2^{(m,m-1,m-2)}} = \left(\int_0^1 \left((\varphi^{(m)}(x))^2 + 2(\varphi^{(m-1)}(x))^2 + (\varphi^{(m-2)}(x))^2 \right)^2 dx \right)^{\frac{1}{2}}.$$

Let we denote by $\widetilde{W_2^{(m,m-1,m-2)}}(0,1]$ the subspace of the space $W_2^{(m,m-1,m-2)}(0,1]$ consisting of real-valued, 1-periodic functions $\varphi(x)$, $x \in (0,1]$.

In the present paper, we consider the Hilbert space $W_2^{(m,m-1,m-2)}(0,1]$ of 1-periodic, real-valued functions. Notice that every element of the space satisfies the following condition of 1-periodicity

$$\varphi(x+\beta) = \varphi(x) \qquad \text{for} \qquad x \in \mathbb{R} \quad \text{and} \quad \beta \in \mathbb{Z}.$$

We consider a quadrature formula of the following form

$$\int_0^1 \varphi(x)dx \cong \sum_{k=1}^{N} C_k \varphi(x_k) \tag{6}$$

where $\varphi \in W_2^{\widetilde{(m,m-1,m-2)}}$, C_k are the coefficients of the quadrature formula and N is the number of nodes, $h = \frac{1}{N}$ and x_k $(0 < x_1 < x_2 < ... < x_N \leq 1)$ are nodes. Since quadrature formula of the form (1) with equidistant nodes is optimal for the periodic functions class, we choose the nodes as $x_k = hk$ (see [9]).

The error of the quadrature formula (1) is given as follows

$$(\ell, \varphi) = \int_0^1 \varphi(x)dx - \sum_{k=1}^{N} C_k \varphi(hk) = \int_0^1 \left[\left(\varepsilon_{(0,1]}(x) - \sum_{k=1}^{N} C_k \delta(x-hk) \right) * \phi_0(x) \right] \varphi(x)dx, \tag{7}$$

where $h = \frac{1}{N}$, $\varepsilon_{(0,1]}(x)$ is the characteristic function of the interval $(0,1]$, δ is the Dirac delta-function, $\phi_0(x) = \sum\limits_{-\infty}^{\infty} \delta(x - h\beta)$, $*$ is the convolution operation and

$$\ell(x) = \left(\varepsilon_{(0,1]}(x) - \sum_{k=1}^{N} C_k \delta(x - hk) \right) * \phi_0(x) \tag{8}$$

and it is the periodic error functional of the quadrature formula (1).

The error (2) of the quadr ature formula (1) is a linear functional in $W_2^{\widetilde{(m,m-1,m-2)^*}}$. It should be noted that $W_2^{\widetilde{(m,m-1,m-2)^*}}$ is the conjugate space to the space $W_2^{\widetilde{(m,m-1,m-2)}}$ and the conjugate space consists of all periodic functionals which are orthogonal to the unity [11], i.e.,

$$(\ell, 1) = 0 \tag{9}$$

Simplifying the right-hand side of the above equality and taking into account the condition (6) we obtain the following analytical expression

$$\|\ell\|_{W_2^{\widetilde{(m,m-1,m-2)}}} = (-1)^m \sum_{k=1}^{N} \sum_{k'=1}^{N} C_k C_{k'} \sum_{\beta \neq 0} \frac{e^{-2\pi i \beta (hk - hk')}}{((2\pi i\beta)^m - (2\pi i\beta)^{m-2})^2}$$

Conclusion

This paper has addressed the problem of estimating the upper bound of the absolute error for an optimal quadrature formula in the space of real-valued periodic functions. The analysis is based on the use of the extremal function associated with the quadrature formula, which allows for an effective evaluation of the error. It is shown that the norm of the error functional for the optimal quadrature formula constructed in this space is smaller than the corresponding norm in the Sobolev space. These results demonstrate the advantage of the proposed approach for numerical integration of periodic functions. Future research will focus on a deeper investigation of the error functional, including the explicit computation of its norm and the determination of the optimal coefficients of the quadrature formula, with the aim of further improving the accuracy and efficiency of optimal quadrature methods.

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Tomato Consumption in the CIS Countries: Dynamics and Prospects

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DOI 10.5281/zenodo.18217364

Tomato Consumption Trends

Tomatoes are one of the most popular vegetables in the CIS countries, playing a key role both in the daily diet of the population and in processing for the food industry. They are

used not only in fresh form, but also for the production of sauces, pastas, and canned food, which makes the market especially significant on a regional scale.

The "Values/Volume"field shows the consumption of tomatoes in the CIS countries in the period from 2018 to 2030 (million tons). According to data from Statista Market Insights (June 2025), the dynamics of consumption in the CIS is characterized by relative stability. In the period from 2018 to 2021, there were small fluctuations, after which the market entered a smooth growth trajectory. By 2030, consumption is projected to increase from 9.5–10 million tons to 11–11.5 million tons.

This growth is due to both population growth and the preservation of traditional culinary habits, in which the tomato occupies a central place.

Average Consumption per Capita

Tomatoes are one of the most popular vegetables in the CIS countries, playing a key role both in the daily diet of the population and in processing for the food industry. They are used not only in fresh form, but also for the production of sauces, pastas, canned food, which makes the market especially significant on a regional scale.

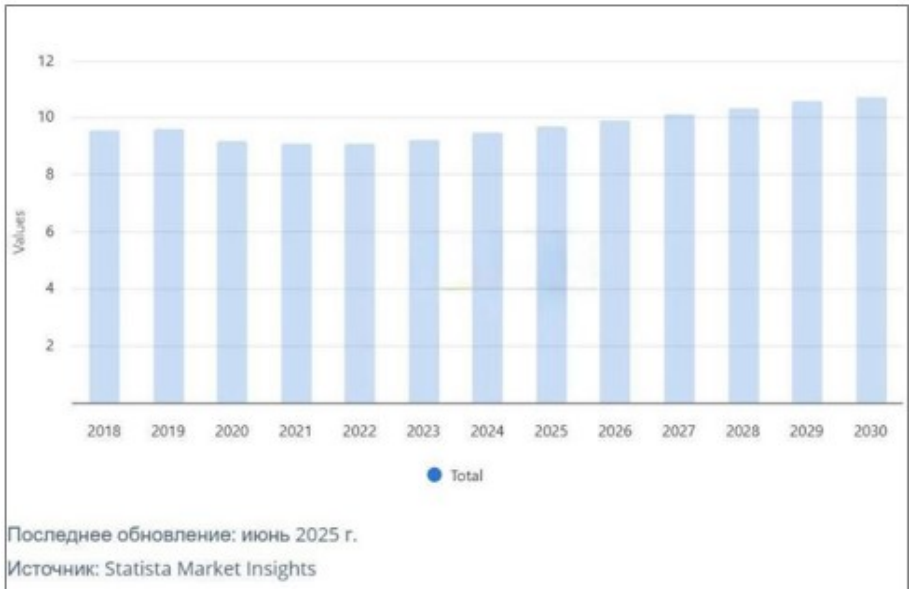


Fig 1. The "Values/Volume"field shows the consumption of fertilizers in the CIS countries in the period from 2018-2030

According to data from Statista Market Insights (June 2025), the dynamics of consumption in the CIS is characterized by relative stability. In the period from 2018 to 2021, there were small fluctuations, after which the market entered a smooth growth trajectory. By 2030, consumption is projected to increase from 9.5–10 million tons to 11-11.5 million tons. This growth is due to both population growth and the preservation of traditional culinary habits, in which the tomato occupies a central place.1

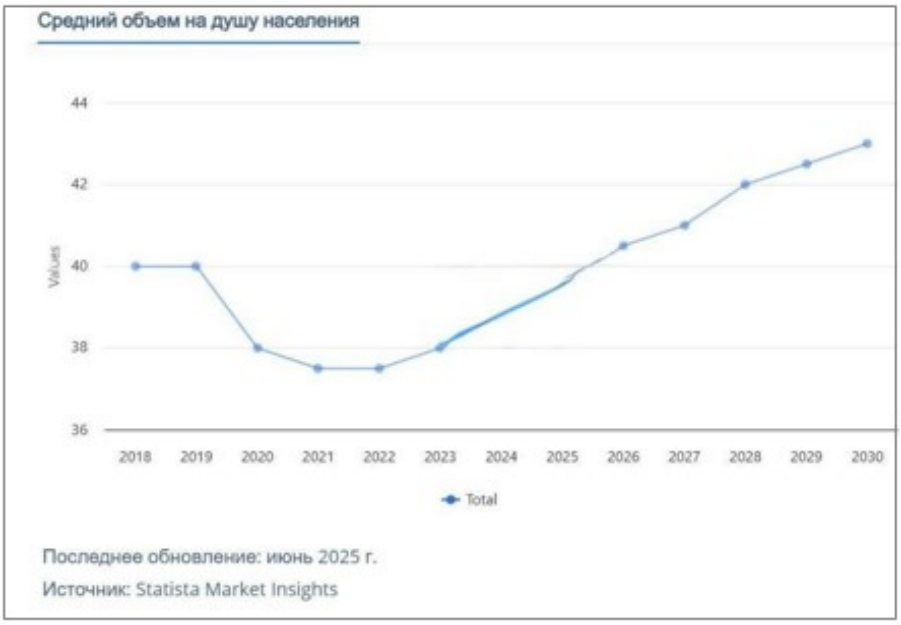


Fig 1. The field "Average consumption per capita"shows the apparent consumption of tomatoes in the CIS countries in the period from 2018-2030

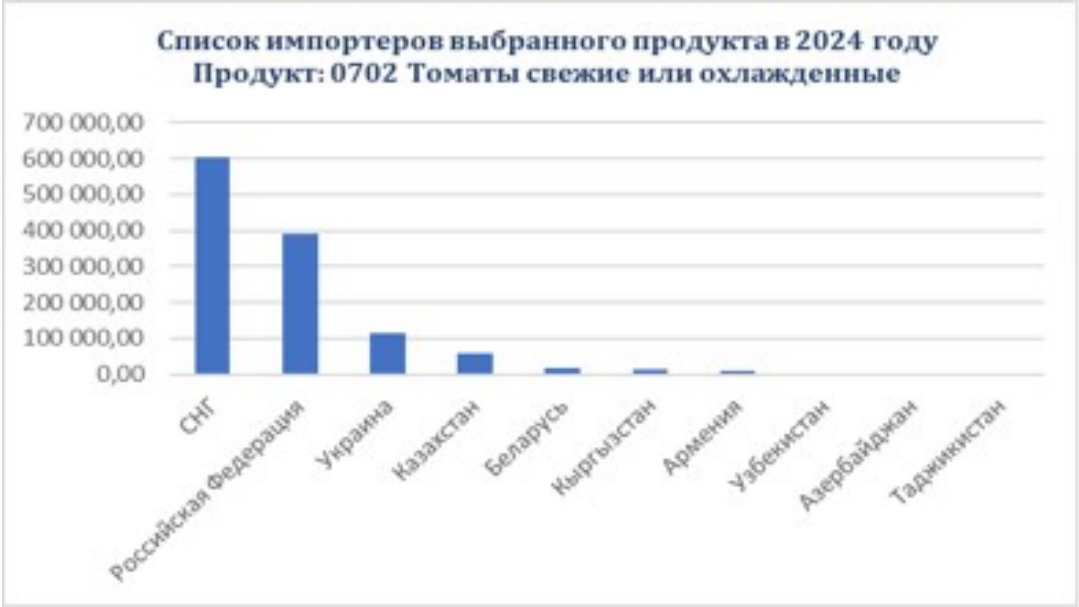
In 2018-2019, the average figure was kept at about 40 kg per year per person, but in 2021-2022 there was a noticeable decline to 37 kg. This decrease is due to price fluctuations and economic factors that limit the purchasing power of the population. Nevertheless, starting in 2023, the situation is gradually changing: consumption is growing, and by 2030 it is projected to reach the level of 43-44 kg per person per year. This result indicates a recovery in consumer demand and a strengthening of the role of tomatoes in the diet.

In the future, the tomato market in the CIS will develop towards an increase in production and consumption. The key drivers of growth will be the improvement of greenhouse technologies, the expansion of the range of processed products and the growing popularity of environmentally friendly and organic products. At the same time, given the high elasticity of demand for vegetables, the market will remain sensitive to macroeconomic factors and price fluctuations.

Thus, tomatoes in the CIS continue to be a basic necessity product, the demand for which is stable even in times of crisis. In the long term, we can expect not only an increase in domestic consumption, but also an expansion of export potential, especially to the countries of the Middle East and Asia.

Forein Trade in Tomatoes in the CIS: analysis based on Trademark data (2024)

The tomato market in the CIS countries is characterized not only by domestic production and consumption, but also by active participation in foreign trade. According to Trademark data for 2024, the total import of tomatoes by the countries of the region amounted to about $ 604 million, which is only a small part of the global figure of $ 12.2 billion. Exports, on the contrary, reached $354 million, which is also a moderate share of the global total of $12.4 billion. Thus, the region remains rather a net importer, but individual countries demonstrate competitiveness in foreign markets.

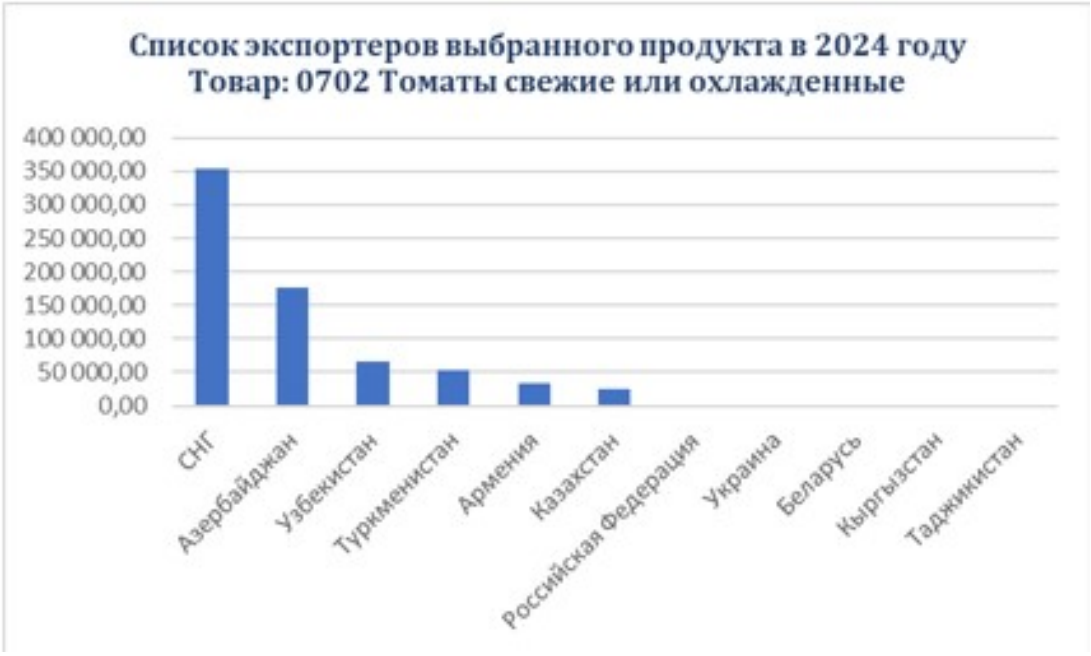


The largest importer in the CIS is Russia, which accounts for over $391 million. This is more than half of the region's total imports and almost two thirds of its volume. Thus, the Russian market remains a key driver of demand, forming the basis for export supplies both inside and outside the CIS. The second largest importer is Ukraine with an indicator of 113.6 million dollars, followed by Kazakhstan (57.1 million) and Belarus (16 million). Imports to Armenia, Kyrgyzstan and Uzbekistan are significantly lower, while Tajikistan occupies a marginal position вЂ“ only 151 thousand dollars.

If we consider exports, Azerbaijan is the leader in the region, supplying tomatoes worth over 175 million dollars. This is half of all CIS exports, reflecting the successful development of the greenhouse industry and its geographical proximity to high-demand markets. Uzbekistan ranks second ($66.5 million), which is actively developing export channels, primarily to Russia and Kazakhstan. This is followed by Turkmenistan (52.9 million) and Armenia (33.2 million). Against this background, Russia's exports look symbolic of only 856 thousand dollars, which confirms its dependence on imports.

Interestingly, a number of countries in the region (for example, Ukraine, Belarus and Kazakhstan) simultaneously appear as importers and exporters. Their export figures are relatively small, from $289,000 for Belarus to $24.6 million for Kazakhstan, which indicates re-export or niche supplies rather than a large-scale presence on the world market.

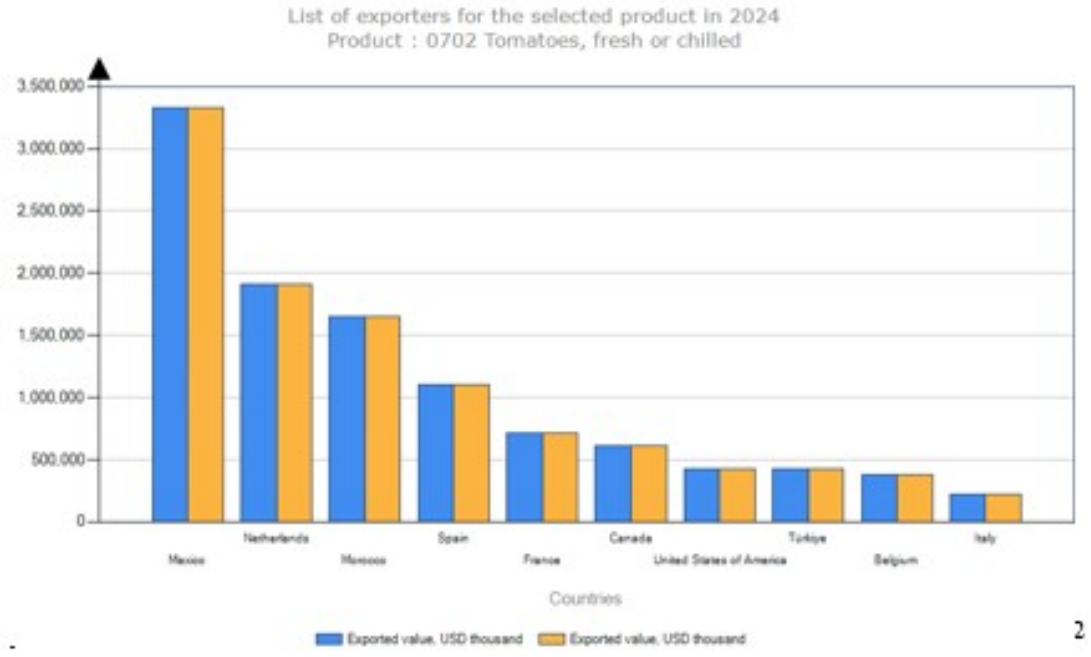
Thus, the structure of foreign trade in tomatoes in the CIS demonstrates a pronounced asymmetry: Russia is the largest importer and consumer, while Azerbaijan and Uzbekistan are key exporters. This complementarity forms a kind of "CIS internal market where commodity flows are distributed depending on climatic conditions and investment activity in greenhouse technologies.



In the future, we can expect further growth in exports from the countries of the South Caucasus and Central Asia, while maintaining steady demand in the Russian market. This creates opportunities for expanding interregional cooperation, as well as stimulating competition with traditional tomato suppliers to Europe and the Middle East.

THE WORLD'S LARGEST TOMATO EXPORTERS IN 2024

The market for fresh and chilled tomatoes in 2024 demonstrated a steady concentration of exports in the hands of a limited number of countries. Mexico remains the leader by a huge margin, with exports totaling more than $3.3 billion. This result is due to its proximity to the world's largest market, the United States of America, as well as large-scale investments in greenhouse production and modern logistic



The second place is occupied by the Netherlands (about $ 1.9 billion), which is not only a producer, but also a major logistics hub for the re-export of tomatoes within the European Union. Morocco occupies the third position (about $ 1.7 billion), which has significantly strengthened its position in recent years due to favorable climatic conditions and the diversification of sales markets in Europe.

This is followed by Spain (about 1.1 billion dollars) and France (700 million), which traditionally act as important suppliers for the European market. Canada and the United States occupy average positions with exports in the range of $ 600 million and $400-500 million, respectively, due to high domestic demand and limited volumes for foreign trade.

Among other countries, Turkey and Belgium are notable, each of which exported about $ 400 million, as well as Italy with a more modest result of about $ 250 million. Thus, the global structure of tomato exports remains quite concentrated: the three leading players Mexico, the Netherlands and Morocco control more than half of the global market. At the same time, the dynamics of recent years shows the growing role of the countries of North Africa and the Eastern Mediterranean, which are gradually increasing competition with traditional European producers.

Conclusion If we compare the CIS with the global picture, the difference in scale becomes obvious. In 2024, global exports of fresh and chilled tomatoes reached $12.4 billion. Thus, the CIS remains a relatively modest player in the global tomato trade. The region is more focused on domestic consumption, while world leaders are building their strategy on large-scale exports. Nevertheless, the CIS countries, especially Azerbaijan and Uzbekistan, have the potential to strengthen their positions due to the growing demand in Europe and the Middle East, which makes the development of greenhouse production and logistics key areas for future growth.

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Correlation and Regression Analysis of the Laws of Change in Water Consumption in Economic Sectors

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DOI 10.5281/zenodo.18217374

Annotation.

In this work, the state of water resource use in the conditions of the region and their distribution in the main sectors of the economy were analyzed. Using correlation and regression models, the degree of interdependence and forces of influence of water consumption in the context of economic sectors were assessed. The obtained results make it possible to effectively manage water resources, their optimal distribution, and determine promising directions. The research results can be used to analyze the current state of rational use of water resources and water consumption by industry, as well as to assess the possibilities of optimal distribution in the future.

Keywords: regression analysis, regression methods, correlation index, determination index.

Introduction

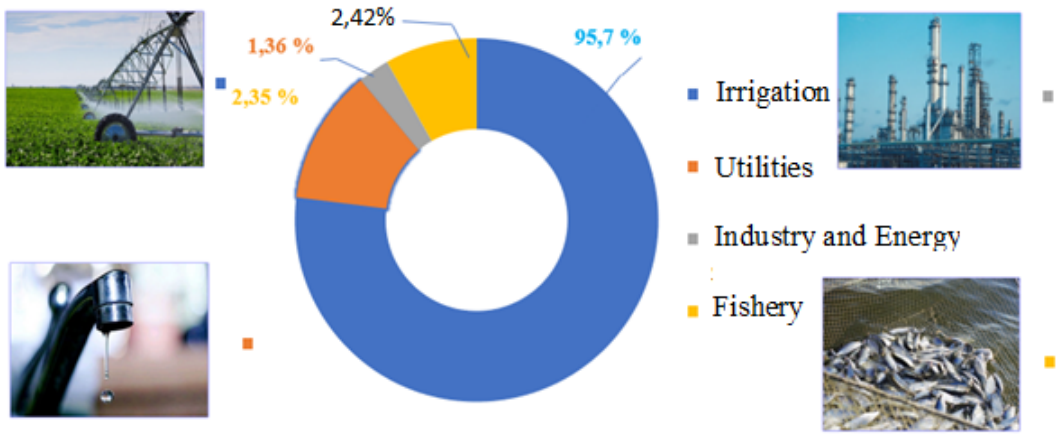
The efficient and rational use of water resources is closely linked to agriculture, industry, energy, and the standard of living of the population. Therefore, the development of models for the effective distribution and management of water consumption in the context of economic sectors is relevant. This is especially important for long-term planning,

increasing production efficiency, and reducing environmental risks in conditions of water scarcity.

In several scientific sources, correlation and regression methods are widely used to determine the relationship between water consumption and economic growth indicators. The relationship between such indicators as water consumption, gross domestic product, population, and production volume is analyzed, and the possibility of forecasting trends in the use of water resources in the future is created [?, ?, ?]. The possibility of using correlation and regression analysis for forecasting the gross regional product was studied in [4]. Mathematical and informational methods of effective forecasting of water resource consumption were studied in [5].

Literature analysis and methodology

In this work [6], the application of correlation methods for analyzing the patterns of changes in water consumption and water use in economic sectors is considered. Indicators of water resource use in the sectors of the economy of the Republic of Karakalpakstan are shown in Figure 1.



1) Irrigation

Irrigation accounts for 95.73% of the total volume of water resources in the republic. Based on the data from sources [7], [8], [9], and in accordance with the adopted methodology, a correlation dependence model is developed. The correlation analysis is conducted using statistical data for the period 2007–2023, as presented below.

As a result of calculations, the linear regression equation is

$$y \approx -21.28x + 6943.28,$$

with the correlation index $R \approx -0.0582$ and the determination index $R^2 \approx 0.003$. The graph of the regression equation is shown in Figure 1.2a.

The quadratic regression equation is

$$y = -38.0433x^2 + 663.4989x + 4774.8204,$$

with the correlation index $R \approx 0.4569$ and the determination index $R^2 \approx 0.2087$. The graph of the regression equation is shown in Figure 1.2b.

The cubic regression equation is

$$y = -3.6139x^3 + 59.5324x^2 - 59.2838x + 6010.7788,$$

with the correlation index $R \approx 0.492$ and the determination index $R^2 \approx 0.2421$. The graph of the regression equation is shown in Figure 1.2c.

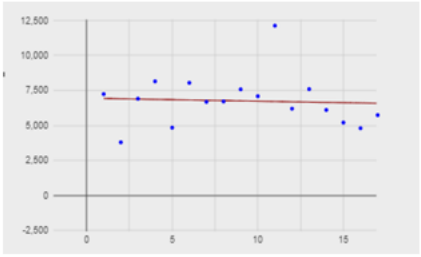


Figure 1.2.a. Graph of the equation of linear regression of changes in water use in irrigation

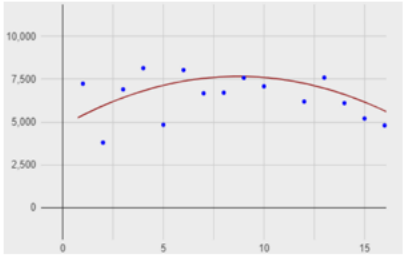


Figure 1.2.v. Quadratic regression equation graph of changes in water use in irrigation

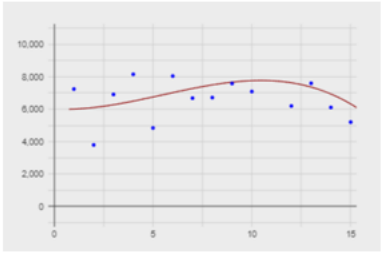


Figure 1.2.c. Graph of the cubic regression equation of changes in water use in irrigation

2) Industry

Industry accounts for **0.42%** of the total volume of water resources of the republic. According to Table 1 [4:1-25], and in accordance with the adopted methodology, a correlation dependence model is developed. The correlation analysis is carried out using statistical data for the period **2007–2023**, as follows:

As a result of calculations, the linear regression equation is

$$y \approx 0.0368x + 5.1868,$$

with the correlation index $R \approx 0.559$ and the determination index $R^2 \approx 0.3125$. The graph of the regression equation is shown in Figure 1.3a.

The quadratic regression equation is

$$y = 0.0084x^2 - 0.1142x + 5.6647,$$

with the correlation index $R \approx 0.7883$ and the determination index $R^2 \approx 0.6213$. The graph of the regression equation is shown in Figure 1.3b.

The cubic regression equation is

$$y = 0.0015x^3 - 0.0322x^2 + 0.1868x + 5.15,$$

with the correlation index $R \approx 0.8947$ and the determination index $R^2 \approx 0.8004$. The graph of the regression equation is shown in Figure 1.3c.

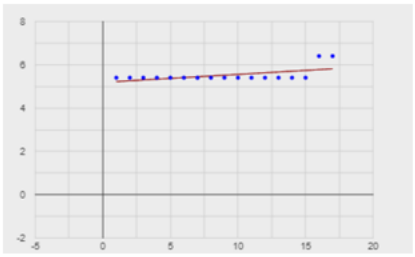


Figure 1.3.a. Graph of the linear regression equation of changes in industrial water use

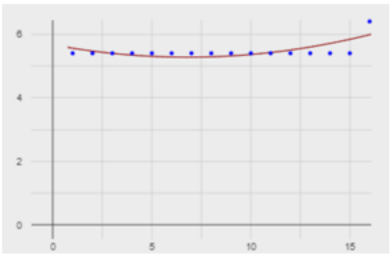


Figure 1.3.v. Quadratic regression equation graph of changes in industrial water

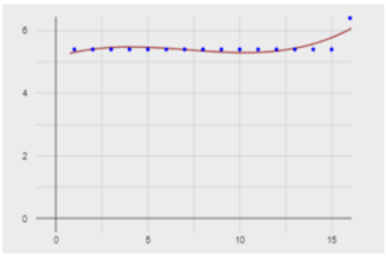


Figure 1.3.c. Graph of the cubic regression equation of changes in industrial water use

3) Utilities

Utilities account for **2.35%** of the total volume of water resources of the republic. According to Table 1 [4:1-25], and in accordance with the adopted methodology, a correlation dependence model is developed. The correlation analysis is carried out using statistical data for the period **2007–2023**, as follows:

As a result of calculations, the linear regression equation is

$$y = 1.4282x + 149.9185,$$

with the correlation index $R \approx 0.8021$ and the determination index $R^2 \approx 0.6434$. The graph of the regression equation is shown in Figure 1.4a.

The quadratic regression equation is

$$y = 0.0057x^2 + 1.3265x + 150.2406,$$

with the correlation index $R \approx 0.8022$ and the determination index $R^2 \approx 0.6436$. The graph of the regression equation is shown in Figure 1.4b.

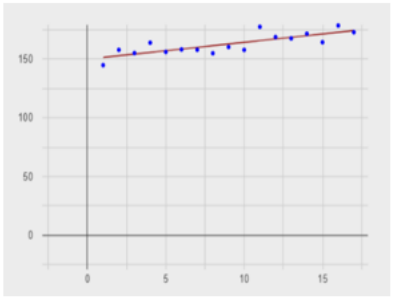


Figure 1.4.a. Graph of the equation of linear regression of changes in the use of municipal water

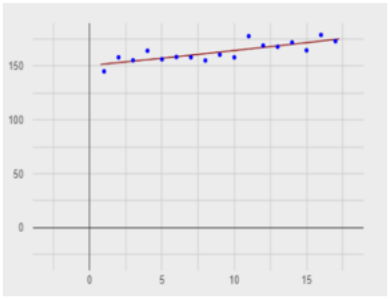


Figure 1.4.c. Graph of the quadratic regression equation of changes in water use in the utilities sector

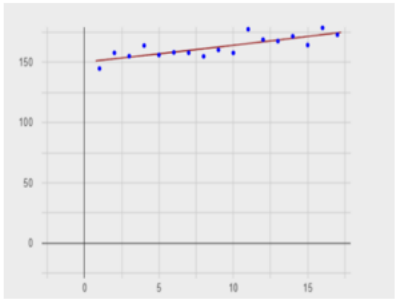


Figure 1.4.c. Graph of the cubic regression equation of changes in water use in the utilities sector

The cubic regression equation is

$$y = 0.0037x^3 - 0.0955x^2 + 2.0759x + 148.9591,$$

with the correlation index $R \approx 0.8032$ and the determination index $R^2 \approx 0.6451$. The graph of the regression equation is shown in Figure 1.4c.

4) Energy

The energy sector accounts for **3.85%** of the total volume of water resources of the republic. According to Table 1 [4:1-25], and in accordance with the adopted methodology, a correlation dependence model is developed. The correlation analysis is carried out using statistical data for the period **2007–2023**, as follows:

As a result of calculations, the linear regression equation is

$$y \approx -1.9945x + 289.7804,$$

with the correlation index $R \approx -0.1604$ and the determination index $R^2 \approx 0.0257$. The graph of the regression equation is shown in Figure 1.5a.

The quadratic regression equation is

$$y = -0.8119x^2 + 12.6199x + 243.5016,$$

with the correlation index $R \approx 0.3268$ and the determination index $R^2 \approx 0.1068$. The graph of the regression equation is shown in Figure 1.5b.

The cubic regression equation is

$$y = 0.055x^3 - 2.2971x^2 + 23.6216x + 224.6887,$$

with the correlation index $R \approx 0.3369$ and the determination index $R^2 \approx 0.1135$. The graph of the regression equation is shown in Figure 1.5c.

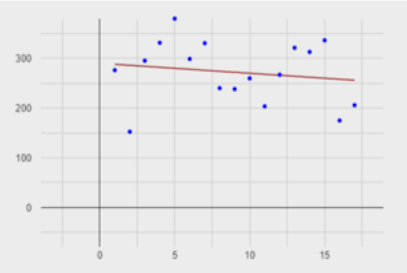


Figure 1.5.a. Graph of the linear regression equation of changes in water use in the energy sector

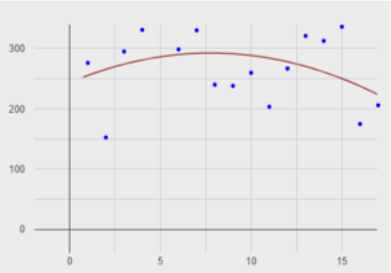


Figure 1.5.v. Quadratic regression equation graph of changes in energy water use

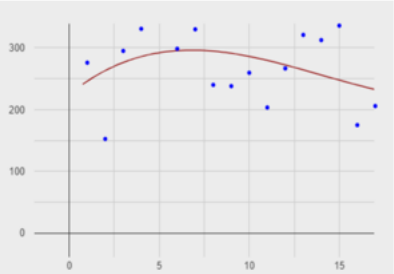


Figure 1.5.c. Graph of cubic regression equation of changes in energy water use

5) Fishery

Fishery accounts for **1.36%** of the total volume of water resources of the republic. According to Table 1 (4)[4:-25], and in accordance with the adopted methodology, a correlation dependence model is developed. The correlation analysis is carried out using statistical data for the period **2007–2023**, as follows:

As a result of calculations, the linear regression equation is

$$y = 4.9265x + 51.8121,$$

with the correlation index $R \approx 0.2102$ and the determination index $R^2 \approx 0.0442$. The graph of the regression equation is shown in Figure 1.6a.

The quadratic regression equation is

$$y = 1.04x^2 - 13.7943x + 111.0947,$$

with the correlation index $R \approx 0.2857$ and the determination index $R^2 \approx 0.0816$. The graph of the regression equation is shown in Figure 1.6b.

The cubic regression equation is

$$y = 0.5793x^3 - 14.601x^2 + 102.0656x - 87.0257,$$

with the correlation index $R \approx 0.3806$ and the determination index $R^2 \approx 0.2906$. The graph of the regression equation is shown in Figure 1.6c.

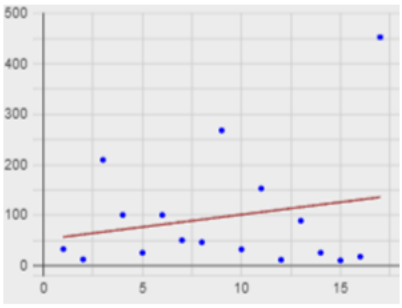


Figure 1.6.a. Graph of the linear regression equation of changes in water use in fisheries

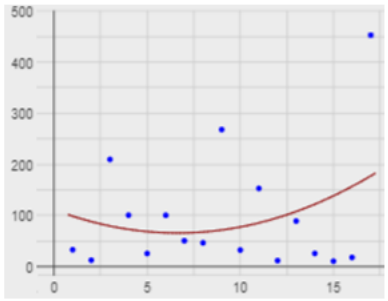


Figure 1.6.c. Graph of the quadratic regression equation of changes in water use in fisheries

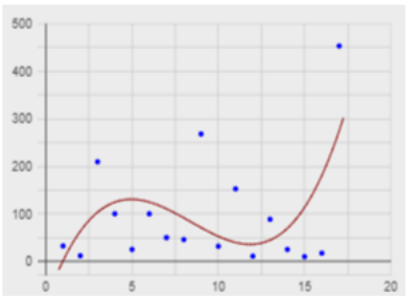


Figure 1.6.c. Graph of the cubic regression equation of changes in water use in fisheries

Consideration

Based on these calculations, the following discussion is conducted on the assessment of the correlation index and determination index: -sectors of the economy - housing and communal services and industry. It is proven that water consumption and water use are constant in relation to the water level of the year, and the regression equation calculated for them has a correlation index $R \geq 0.5$ and $R^2 \geq 0.5$ a determination index.

As can be seen from Table 1, R has negative and positive values. Negative values represent a decrease in water consumption and water use in the republic, while positive values represent an increase. It can be seen that the total water consumption of the republic in irrigation decreases, while in other sectors of the economy it increases. For further confirmation of this data, it will be necessary to organize regular measurements of accurate calculations of water resources used by rivers, reservoirs, and other water bodies that provide water resources to economic sectors. The presence of a correlation index of the linear regression equation, calculated during the analysis of changes in water consumption and water use in the irrigation and energy sectors and regression assessments, represents a decrease $R < 0$ in water consumption and water use.

Results

The results of regression calculations for these sectors of the economy are presented in Table 1.

Table 1. Results of regression calculations by sectors of the economy (Correlation index *R* and determination index R^2)

Fields of economy	Linear regression		Quadratic regression		Cubic regression		Conclusion
	R	R^2	*R*	R^2	*R*	R^2	
Irrigation	-0.058	0.003	0.456	0.208	0.492	0.242	Decrease in water consumption has a tendency
Industry	0.559	0.3125	0.7883	0.6213	0.890	0.800	Increase in water consumption There is a tendency
Public utilities	0.802	0.643	0.802	0.643	0.803	0.645	Sustainable growth trend
Energy	-0.160	0.025	0.320	0.106	0.330	0.110	No intrinsic connection and water consumption has a downward trend
Fishing	0.210	0.040	0.280	0.080	0.380	0.290	Weak bond

Conclusion

The following conclusions can be drawn from this research:

- The largest share of water consumption is attributed to irrigation; however, in recent years, a downward trend in water consumption has been observed.
- Water consumption in the industrial and utilities sectors demonstrates a clear upward trend.
- No stable relationship is identified in water consumption within the energy sector and fisheries; consequently, sharp fluctuations may occur depending on the level of water availability.

In general, the efficient allocation of water resources requires reliance on scientifically grounded statistical and economic models.

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Monte Carlo method for the Spatially Homogenous Boltzmann equation

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DOI 10.5281/zenodo.18217382

Abstract.

This work presents a Monte Carlo computational approach for solving the spatially homogeneous nonlinear Boltzmann equation through a modifiedвЂќ particle-in-cellsвЂќ model. Building upon the BelotserkovskiвЂ“Yanitski and Kac models, the paper introduces a stochastic simulation scheme that synthesizes the principle of splitting operators with branching Markov processes. The method constructs unbiased estimators with small variance to compute molecular velocities and particle distributions, providing higher numerical accuracy and physical realism compared to traditional linear models. The developed algorithm enables efficient simulation of rarefied gas dynamics and demonstrates stable results across varying particle densities. Computational experiments, including shock wave structure modeling, show that this approach achieves improved convergence and fidelity in approximating spatially heterogeneous Boltzmann behavior.

Keywords: *Spatially Homogeneous Boltzmann equation, Particle-in-Cell Model, Branching Markov process, Monte Carlo Method, Unbiased Estimator, Conjugated Computational Schemes.*

Introdution

This paper studies and proposes statistical algorithms for solving the nonlinear integral Boltzmann equation in measures, which describes the physical processes of particle collisions and coalescence. The equations under consideration are one of the integral forms of the kinetic equations of rarefied gas dynamics. Markov processes with simple interactions are used to construct unbiased estimates for the solution of the kinetic equation of rarefied gas dynamics. The well-known relationship between nonlinear integral equations and branching Markov processes is exploited [1].

Recently, many studies have explored the application of the Monte Carlo method to solving such problems. The proposed computational scheme is analogous to the adjoint scheme in the linear case. The proposed scheme is examined in detail for the case of "rigid"molecules. To describe the scheme, it is convenient to switch from an equation in measures to an equation in densities. Various computational schemes are considered and their effectiveness compared. The resulting computational schemes are tested in solving a problem of shock wave structure. A statistical algorithm is proposed using the nonlinear spatially homogeneous Boltzmann equation for the numerical implementation of the "particle-in-cell"statistical model, and methods for constructing unbiased, minimal-variance estimates for calculating the particle velocity in each cell are investigated.

Description of the Problem

Let us consider a nonlinear integral equation

$$F_t(dr,du) = F_0(dr-ut,du)\exp\{-H(r,u)t\} + \int_0^t \Big[\exp\{-H(r,u)(t-s)\}\times \\ \times \iint_D T_{t-s}(dr,du;r_1,u_1,r_2,u_2)\,H(r_1,u_1)F_s(dr_1,du_1)\,F_s(dr_2,du_2)\Big]\,. \tag{1}$$

where $(r,u)\in\mathbb{R}\times\mathbb{R}^3$, $(r_i,u_i)\in D$, $i=1,2$, $t\in\mathbb{R}^1$. F_t and F_0 are measures in D, $H(r,u)$

is a non-negative function, $T_{t-s}(B, r_1, u_1, r_2, u_2)$ is a measurable function of the arguments (r_1, u_1, r_2, u_2) at fixed B, and at fixed (r_1, u_1, r_2, u_2) is a measure in D. The measures F_t, F_0, T_{t-s} are considered non-negative.

Many works are dedicated to studying this kind of equation (for example [2]-[3]), where probabilistic features of solutions of this type of equation were studied. Equations of the type (1) represent one of the integral forms of kinetic equations for rarefied gas dynamics. The questions of existence and uniqueness of the solution of equation (1) were studied in [3,4]. It is known [3] that if F_0 and T_{t-s} are probabilistic measures and $H(r, u) < c = \text{const}$, then equation (1) has a unique nonnegative solution, which will be a probability measure. Existence and sub-stochasticity of the minimal nonnegative solution of equation (1) were also proven in [3].

In [2] authors constructively built a random process whose one-dimensional distribution satisfied equation (1). The advantages of this (unlike to problems mentioned above) allowed using a Monte Carlo method. This method of constructing unbiased estimators is a method of "direct modeling". The advantage of the constructed computational scheme is that it permits estimating several functionals of the solution to equation (1) at one time. The disadvantage of this method is the difficulty constructing the random process that gives the minimal variance. Currently, this problem is not solved for that case. In the theory of Monte Carlo methods, there are methods (like "importance sampling") that allow one to avoid these difficulties when solving concrete problems.

Below, for equation (1) we present construction methods for unbiased estimators for functionals of the type (h, f), where $h(r, u, t)$ is a given function and $f(r, u, t)$ is a density of the measure F_t. The computational scheme that we derive here is an analogue of the conjugated scheme in the linear case. Furthermore, we will call this scheme a "conjugated scheme". It is shown that the application of the results of Ermakov and others [2] allow the construction of a computational scheme with small variance, in particular, when prior information on the solution of equation (1) is known. When applying the above scheme to real physical problems, we must also show the statistical conservation of mass, impulse, and energy.

Construction of unbiased estimators

As is known, [5] among the statistical methods, which use Monte Carlo directly for modeling the flow of rarefied gas, the most efficient one is the statistical method of direct modeling of the nonstationary flow. In [6], they described a statistical "particle-in-cell" model, which was constructed based on the synthesis of ideas of splitting the physical processes in each Δt. This method was used by Harlow [7], and in statistical interpretations by Berd [8] and Kac [9]. The specific of the "particle-in-cell" method are as the following. The state of every particle is characterized by vectors of coordinates and speeds u at a time $k\Delta t$, where $k = 0, 1, \ldots$. A discrete random process

$$\{r(t), u(t)\} = \{r_1(t), u_1(t), \ldots, r_N(t), u_N(t)\}$$

should be constructed in the heterogeneous coordinate space at each time $k\Delta t$. Here N is the number of molecules of monatomic gas in some volume D. The principle of splitting operator is used for constructing the random process $\{r(t), u(t)\}$.

The sampling divides into two steps. Monte Carlo method is used both for the numerical simulation of collisions of the particles in cells (the first step), as well as for the collision-free motion (free-molecular) of particles (the second step). For the numerical realization of the second step in [10] an algorithm is given, which does not require saving the corresponding spatial coordinates of the particles. For the numerical realization of the first step, spatially homogeneous model of Kac has been used. Thus, the synthesis of the ideas of splitting, and model of Kac has occurred. That gives rise to the Markov model, which approximates the Boltzmann equation with accuracy that depends statistically (condition of molecular chaos) on the number of particles. In [11], Yanitski explained the value of the principle of splitting.

The computational approach of Belotserkovski–Yanitski [6] is as efficient as the method of Berd, but has higher accuracy in approximating the Boltzmann equation. It is important to emphasize that "particle-in-cell" the statistical model of Belotserkovski–Yanitski is physically closer to the real problem.

As we propose yet another computational scheme, which directly uses the nonlinear spatially homogeneous Boltzmann equation for the numerical realization of the first step in the "particles-in-cell" model of Belotserkovski–Yanitski. The method of constructing unbiased estimators with small variance is next given for calculating of a velocity of a single molecule $u(k\Delta t)$ for $k = 0, 1, 2, \ldots$.

To find the velocity of the particle in each cell, we will use equation (1) in the spatially homogeneous case. In this case, equation (1) has the form

$$\begin{aligned} F_{\Delta t}(du) = F_0(du)\exp\{-H(u)\Delta t\} + \int_0^{\Delta t} ds\, \exp\{-H(u)(\Delta t - s)\} \\ \times \iint_D T_{\Delta t - s}(du; u_1, u_2)\, H(u_1)\, F_s(du_1)\, F_s(du_2). \end{aligned} \tag{2}$$

where $F_0(du)$, $F_{\Delta t}(du)$ are distribution particle velocity at time 0 and Δt respectively. Let $H(u) = c = \text{const}$. As is known [4], this equation has a unique positive solution that is a probabilistic measure, and the iterative method starting with $F_0(du)\exp\{-c\Delta t\}$ converges to the solution of equation (2). This is why we can apply the approach that was developed in [12] to equation (2). In [4], a direct computational scheme for solving equation (2) with certain side conditions was given. The results of [4] cannot be applied in our case, since the direct scheme, as given in [4], cannot estimate the value at the point (cell). In [4] a functional is estimated, not a point value.

Let us suppose that when the initial distribution, $F_0(du)$, has a density, the distribution $F_{\Delta t}(du)$ also has a density, $f(u, \Delta t)$. Making the transformation, which is analogous to the transformations [13], we obtain the following equation

$$\begin{aligned} f(u, \Delta t) = f(u, 0)\exp\{-ct\} + \int_0^{\Delta t} ds \int d\bar{\omega} \int du_c \\ \times\big(1 - \exp\{-c\Delta t\}\big) f_1(s)\, \frac{|u - u_c|}{c}\, f(u_c - |u - u_c|\bar{\omega}, s)\, f(u_c + |u - u_c|\bar{\omega}, s) \\ \times \int_0^{\Delta t} ds \int_{\mathbb{R}^3} \big(1 - \exp\{-c\Delta t\}\big) f_1(s) \left(1 - \frac{|u - u_c|}{c}\right) f(u, s)\, f(u_2, s)\, du_2. \end{aligned} \tag{3}$$

For equation (3), we construct a branching Markov process, and thus we can derive various "conjugated" computational schemes for calculating an unbiased estimator of the functional (h, f). Since $u = (u_x, u_y, u_z)$, we suppose $h(u,t) = u_x$, i.e. at time Δt, the velocity is computed by the following formula

$$U(\Delta t) = \int u_x f(u, \Delta t)\, du.$$

Now we describe the computational scheme. At time Δt, one particle is born with initial density $p_0(u)$ at the point $(u, \Delta t) = (u_x, u_y, u_z, \Delta t)$. Here, the estimator has the form

$$\zeta_{\Delta t}(u) = \frac{u_x}{p_0(u)}.$$

Further, with probability $\alpha_0(\Delta t) = \exp\{-c\Delta t\}$, the particle is absorbed at the point $(u, \Delta t)$; here the estimator is computed by the formula

$$\zeta_{\Delta t}(u) = \frac{u_x}{f(u,0)}.$$

If the particle has not been absorbed, then the time $s_1 \in [0, \Delta t]$ is simulated with the probability density

$$f_1(s) = \frac{c\exp\{-c(\Delta t - s)\}}{1 - \exp\{-c\Delta t\}}.$$

Then, with probability $\alpha_1(\Delta t) = \beta\big(1 - \exp\{-c\Delta t\}\big)$, where $\beta \in [0,1]$, two particles will appear at the points $(u_c - |u - u_c|\bar{\omega}, s_1)$ and $(u_c + |u - u_c|\bar{\omega}, s_1)$. Here ω is an isotropic vector. The transition from u to u_c is done with the transition density $\psi_1(u_c)$. The density distribution at the point (u_c, s_1), under the condition that collision has taken place (velocity changes), we write in the form of

$$P_1(u, \Delta t \to u_c, s_1) = f_1(s)\psi_1(u_c).$$

Here the estimator is computed by the formula

$$\zeta_{\Delta t}(u) \;\to\; \zeta_{\Delta t}(u)\,\frac{2|u - u_c|}{\beta c}\,\psi_1(u_c).$$

Further, with probability $\alpha_2(\Delta t) = \big(1 - \exp\{-c\Delta t\}\big)(1 - \beta)$ two particles are created at the points (u, s_1) and (u_2, s_1). Transition from u to u_2 is done with transition density $\psi_1(u_2)$. The density distribution of the point (u_2, s) with velocity remaining constant can be written in the form

$$P_2(u, \Delta t \to u_2, s_1) = f_1(s)\psi_1(u_2).$$

In this case the estimator is computed by the formula

$$\zeta_{\Delta t}(u) \;\leftarrow\; \zeta_{\Delta t}(u)\left(1 - \frac{|u - u_2|}{c}\right)(1 - \beta)^{-1}\big(\psi_1(u_2)\big)^{-1}.$$

Furthermore, new particles behave the same as the initial particle. Moreover, $s_0 = 0$, $s_k \in [0, t - \sum_{i=0}^{k-1} s_i]$, $t < \sum_{i=0}^{k} s_i$, the moments of time s_k are chosen with probability

density $f_1(s)$. At each step, the times $s_1, s_2, \ldots$ are chosen and the process finishes, when all particles are absorbed, or the condition

$$t - \sum s_i \leq \varepsilon,$$

where ε is a small number which was given beforehand, is satisfied.

Applying the results of [13] to equation (3), we derive the following assertion. If a process is given with an initial density

$$P_0(u, \Delta t) = \frac{|u_x f(u, \Delta t)|}{\big(|u_x|, |f(u, \Delta t)|\big)}.$$

the transition density

$$P_1(u, \Delta t \to u_c, s_1) = f_1(s)\, \frac{2|u - u_c|\, c}{c}\, f(u_c - |u - u_c|, s)\, \frac{f(u_c + |u - u_c|, s)}{f(u, \Delta u)},$$

$$P_2(u, \Delta t \to u_2, s_1) = f_1(s) \left(1 - \frac{|u - u_c|\, c}{c}\right) \frac{f(u, s)\, f(u_2, s)}{f(u, \Delta t)}.$$

and absorption probability

$$g(u, \Delta t) = \frac{f(u, 0)}{f(u, t)}\, \exp\{-c\Delta t\}.$$

then the variance of estimator $\zeta_{\Delta t}(u)$ will is the solution to equation (3).

Now, let us study the construction of unbiased estimators with small variance, using [14] and known data on the solution at $t \to \infty$. It's known [15] that as $t \to \infty$ the gas approaches the equilibrium state. Then features of the gas do not depend on spatial coordinates or time. In this case, the solution of equation (3) has the form (Maxwellian)

$$f(u) = \frac{1}{(2\pi)^{3/2}} \exp\left(-\frac{u^2}{2}\right). \tag{4}$$

For large t, the solution of our problem is close to (4). If we put (4) into expressions for P_1 and P_2, this determines a Markov process. And gives us to use the small variance estimator $\zeta_{\Delta t}(u)$. The density distribution of the point (u_c, s_1), when collision has taken place. When particle velocities change, has the form

$$\tilde{P}_1(u, \Delta t \to u_c, s) = f_1(s)\, \frac{1}{(2\pi)^{3/2}} \exp\left(-u_c^2 + |u - u_c|^2 + \frac{u^2}{2}\right).$$

If we make a change of variables in the following way: $\tilde{v} = 2v = 2(u_c - u)$, then we get for

$$\tilde{P}_1\left(u, \Delta t \to \frac{\tilde{v}}{2} + u, s_1\right) = f_1(s)\, \frac{1}{(2\pi)^{3/2}} \exp\left(-\frac{(\tilde{v} - u)^2}{2}\right). \tag{5}$$

and for the point (u_2, s_1), the collision has taken place (case of constant velocity) and the expression for the density distribution $\tilde{P}_2$ can be written as

$$\tilde{P}_2(u, \Delta t \to u_2, s_1) = f_1(s)\, \frac{1}{(2\pi)^{3/2}} \exp\left(-\frac{u_2^2}{2}\right). \tag{6}$$

here

$$g(u,\Delta t)=\exp\{-c\Delta t\}. \tag{7}$$

Thus, velocities $\tilde v$ and u_2 are distributed according to a normal law with densities

$$\psi_1(\tilde v)=\frac{1}{(2\pi)^{3/2}}\exp\left(-\frac{(\tilde v-u)^2}{2}\right),\qquad \psi_2(u_2)=\frac{1}{(2\pi)^{3/2}}\exp\left(-\frac{u_2^2}{2}\right),$$

and the optimal transition densities have the form (5), (6), (7).

For the construction of this algorithm, which is connected with densities (5), (6), (7), it is more convenient to apply equation (3), after the following change of variables

$$f(u,t)=\frac{1}{2}\,\psi(u)\,\varphi(u,\Delta t).$$

Here $\psi(u)$ is a known function, which is defined by (4). The computational scheme is simple. Because in computing the estimators there is no need to use weights that are larger than ours. Thus for (3) we derive

$$\begin{aligned}\varphi(u,\Delta t)=\varphi(u,0)\exp\{-c\Delta t\}&+\int_0^{\Delta t}ds\int d\bar\omega\int_{\mathbb{R}^3}du_c\left(1-\exp\{-c\Delta t\}\right)\\&\times f_1(s)\,\frac{2|u-u_c|}{2c}\,\frac{\psi(u_c-|u-u_c|\bar\omega)\,\psi(u_c+|u-u_c|\bar\omega)}{\psi(u)}\\&\times\varphi(u_c-|u-u_c|\bar\omega,s)\,\varphi(u_c+|u-u_c|\bar\omega,s)\\&+\int_0^{\Delta t}ds\int_{\mathbb{R}^3}du_2\left(1-\exp\{-c\Delta t\}\right)f_1(s)\left(1-\frac{|u-u_c|}{c}\right)\\&\times\frac{1}{2}\,\frac{\psi(u)\psi(u_2)}{\psi(u)}\,\varphi(u,s)\,\varphi(u_2,s).\end{aligned}\tag{10}$$

where $u_c=u+\dfrac{\tilde v}{2}$. The algorithm for constructing the unbiased estimator

$$U(\Delta t)=\int u_x f(u,\Delta t)\,du$$

is described as follows.

At time Δt, one particle with initial density $p_0(u)$ at the point $(u,\Delta t)=(u_x,u_y,u_z,\Delta t)$ appears. The particle at the point $(u,\Delta t)$ has a statistical "weight", which is equal to

$$\zeta_{\Delta t}(u)=\frac{u_x}{p_0(u)}.$$

Furthermore, at the point $(u,\Delta t)$ the unbiased estimator $\zeta_{\Delta t}(u)$ is constructed for equation (6.26) in the following way. With probability

$$g(u,\Delta t)=\exp\{-c\Delta t\},$$

the particle is absorbed at the point $(u,\Delta t)$. The estimator is computed by the formula

$$\zeta_{\Delta t}(u)\leftarrow\varphi(u,0)\,\zeta_{\Delta t}(u).$$

If the particle has not been absorbed, the time $s_1 \in [0, \Delta t]$ is simulated with probability density $f_1(s)$. Then, with probability

$$\frac{1 - g(u, \Delta t)}{2},$$

two particles appear at points

$$\left(\frac{\tilde{v}}{2} + u + \frac{\tilde{v}}{2}\bar{\omega},\, s_1\right) \quad \text{and} \quad \left(\frac{\tilde{v}}{2} + u - \frac{\tilde{v}}{2}\bar{\omega},\, s_1\right).$$

Here the vector $\tilde{v} = u + \eta$, where η is sampled from a standard normal distribution. Here the estimator is computed by

$$\zeta_{\Delta t}(u) \leftarrow \frac{\zeta_{\Delta t}(u)\,|\tilde{v}|}{8c}.$$

Furthermore, with probability

$$\frac{1 - g(u, \Delta t)}{2},$$

two particles appear at the points (u, s_1) and (u_2, s_1). Here the vector u_2 is sampled from a standard normal distribution. In this case, the estimator is computed as

$$\zeta_{\Delta t}(u) \leftarrow \zeta_{\Delta t}(u)\left(1 - \frac{|u - u_c|}{c}\right).$$

New particles behave the same as the original particles. The process finishes when all particles are absorbed, or when

$$\Delta t - \sum s_i \le \varepsilon.$$

When the process completes, the estimator $\zeta_{\Delta t}(u)$ is multiplied by a value of function $\frac{1}{2}\psi(u)$ at the point $(u, \Delta t)$, i.e.

$$\zeta_{\Delta t}(u) \leftarrow \frac{\zeta_{\Delta t}(u)\psi(u)}{2}.$$

Thus, in each cell we compute the velocity at each time $k\Delta t$, where $k = 0, 1, 2, \ldots$.

Computational experiment and conclusions

As is known, the nonlinear Boltzmann equation describes the behavior of rarefied gas much better than the linear Kac model. This is why we can expect that application of the nonlinear spatially homogeneous Boltzmann equation in the "particles-in-cell" model allows us to do a computation, which gives a more exact approximation to the spatially heterogeneous Boltzmann equation.

We underline that boundary conditions at infinity can be taken into consideration by the method [6]. There computations show that the results of modeling are similar with known [5] solutions of the Boltzmann equation. As an example, computations for the wave shock structure have been done.

In the figure the Mach number is $M = 2$. The confidence level is 97%. The calculation were made with the following parameters. The number of particles in cells is $N_1 = 8$.

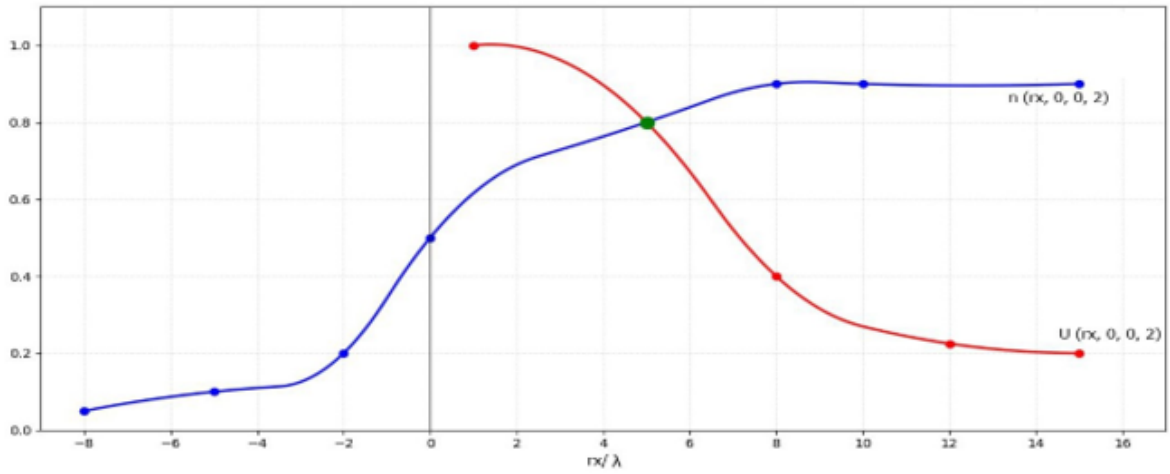

Figure 1. The calculated profile of density and velocity

The ratio $\Delta t/\Delta r_x$, where $\Delta r = (\Delta r_x, \Delta r_y, \Delta r_z)$, is chosen for numerical stability. The parameters for the problem are the following:

$$t = 2, \quad \Delta r_x = \Delta r_y = \Delta r_z = 0.2, \quad \lambda = 1.$$

We note that in interval, where the Boltzmann equations "work" (intermediate interval), the "particle-in-cell" statistical model approximates the spatially heterogeneous Boltzmann equation better. At the same time, this approximation is independent of the number of particles in D (even if a volume will only have one particle).

As a conclusion, first we note that for equation (3) we could apply an iterative process. For the computation of the integrals, which are obtained in the first and the second iteration, we can directly apply the Monte Carlo method. When comparing the density profiles at the first and the second iterations, we noticed very little bias. This can be explained by the fact that the iteration process to (3) converges slowly.

The second, gas dynamics problems can serve as examples of problems whose solution requires the use of modern multiprocessor computers. In particular, the multidimensional Boltzmann equation requires such technology for its solution. The complex nature of the kernel of this equation and the dependence of the solution on many variables necessitated the use of the Monte Carlo method. This method, as is well known, is particularly convenient for multiprocessor computers and allows for natural parallelization. Above, we describe an algorithm for solving the initial-boundary value problem for the Boltzmann equation, which could be effectively implemented on parallel computing systems.

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On the properties of solutions of a nonlinear filtration problem with a source

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We will discuss the next parabolic equation

$$x^{-n}\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left(\left|\frac{\partial u^m}{\partial x}\right|^{p-2}\frac{\partial u^m}{\partial x}\right)+x^{-n}u^{\beta},\quad (x,t)\in R_+\times(0,+\infty) \tag{1}$$

with nonlinear boundary condition

$$-\left|\frac{\partial u^m}{\partial x}\right|^{p-2}\frac{\partial u^m}{\partial x}(0,t)=u^q(0,t),\ \ t>0 \tag{2}$$

and initial value condition

$$u(x,0)=u_0(x)\geq 0,\qquad x\in R_+ \tag{3}$$

where $1<p<1+1/m,\ \ \beta\,,\ q>0,\ \ \ n\in R,\ \ u_0(x)$ – bounded, continuous, non-negative and non-trivial initial data.

Equation (1) occurs in various areas of natural science [1, 3-5]. For example, equation (1) is considered in mathematical modeling of the thermal conductivity of nanofluids, in the study of problems of fluid flow through porous media, in problems of the dynamics of biological populations, polytropic filtration, structure formation in synergetics and nanotechnologies, and in a number of other areas [1–4].

Equation (1) is called a parabolic equation with variable density [1] and in case $m(p-1)-1>0$ corresponds to the equation of slow filtration [2-3]. Problem (1)–(3)has been intensively studied by many authors (see [2, 5-7] and references therein) for various values of numerical parameters.

In work [5], the condition of global unsolvability in time of the solution of the Cauchy problem for equation (1) at was obtained that $m=1,\ \ \rho(x)=1$ and the critical exponent of the Fujita type $\beta=2p-1$ was established.

In [7], the unboundedness of the solution of the following reaction-filtration model with a nonlinear boundary condition was studied

$$\begin{aligned}&u_t=\Delta u^m+u^{\beta},\quad (x,t)\in\Omega\times(0,T),\\ &\frac{\partial u}{\partial \eta}=u^q,\quad (x,t)\in\partial\Omega\times(0,T),\\ &u(x,0)=u_0(x),\quad x\in\Omega,\end{aligned}$$

where $\Omega\in R^N$ is the bounded area. The authors showed that all positive solutions exist globally in the case $m>1$ if and only if $\beta\,,\ q\leq 1$, and in the case $m\leq 1$ when $\beta\leq 1,\ \ q\leq 2m/(m+1)$.

Main results. Below, we will determine the condition of solvability and unsolvability in general in terms of time for solving problem (1)–(3) in the case of fast diffusion. It is assumed that $1<p<1+1/m$.

Theorem 1. *If* $q\leq\frac{m(1-n)+1}{p-n}(p-1)$ *and* $0<\beta\leq 1$ *, then any solution to problem (1)-(3) is global.*

Theorem 2. *If* $\beta<1$ *and* $q\geq\dfrac{(m(1-n)+1)(p-1)}{(p-n)}$*, then the solution of problem (1)-(3) is unbounded in a finite time.*

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Heat Transfer Optimization in Boundary Layer Flow of Hybrid Nanofluids

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Abstract: Hybrid nanofluids have become popular working fluids due to their improved thermal conductivity and enhanced thermophysical properties, making them highly effective for many engineering applications. In this study, the flow and heat transfer characteristics of a ternary hybrid nanofluid in stagnation-point flow over a radially shrinking disk are investigated. The effects of suction, magnetic field, thermal radiation, viscous dissipation, and convective boundary conditions are considered.

A mathematical model based on governing partial differential equations is developed to describe the flow and thermal fields. These equations are transformed into a system

of nonlinear ordinary differential equations using suitable similarity transformations and solved numerically using the finite difference method implemented in MATLAB. Two possible solutions are identified, and the influences of key physical parameters on velocity and temperature distributions, skin friction coefficient, and local Nusselt number are analysed.

The results indicate that the ternary hybrid nanofluid exhibits significantly higher values of the local skin friction coefficient and local Nusselt number compared to hybrid nanofluids and conventional nanofluids. To achieve optimal thermal and flow performance, two complementary optimization techniques are employed: Response Surface Methodology (RSM) and Artificial Neural Networks (ANN). RSM is used to examine the combined effects and interactions of controlling parameters and to determine optimal operating conditions, while ANN serves as a data-driven predictive model capable of capturing nonlinear relationships between parameters and responses.

The optimization results demonstrate that both RSM and ANN effectively identify parameter ranges that enhance the local Nusselt number and improve flow behaviour. Comparative analysis confirms the reliability and effectiveness of both methods in optimizing fluid flow and heat transfer performance. The highest heat transfer rate for the ternary hybrid nanofluid is achieved when the Eckert number is minimized and the Biot number, radiation parameter, and magnetic parameter attain their maximum values. Under these optimal conditions, the ternary hybrid nanofluid outperforms both hybrid and conventional nanofluids in terms of heat transfer efficiency.

Keywords: Hybrid nanofluids; Boundary layer flow; Heat transfer optimization; Response surface methodology; Artificial neural networks; Nusselt number.

An AI-based hybrid NER and sentiment analysis model for uzbek texts in the system analysis of economic and international relations discourse

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DOI 10.5281/zenodo.18217389

Abstract

This article discusses an AI-based hybrid model combining sentiment analysis and named entity extraction (NER) for Uzbek-language discourse in economics and international relations. The relevance of this work stems from the need for automated analysis of public opinion and expert commentary on economic reforms, price dynamics, employment, and foreign policy initiatives, given that the Uzbek language remains resource-poor and has a limited set of annotated corpora and tools. Using a multi-source dataset

including annotated Uzbek sentences, emoji lexicons, named entity dictionaries, and normalization resources, a hybrid architecture is developed and tested. The proposed model integrates features of a transformer sentiment classifier with information on the types and distribution of named entities, enabling more accurate identification of which actors (states, organizations, individuals) are subject to positive or negative assessments. The results of computational experiments demonstrate that the NER-enhanced AI model outperforms text-oriented baseline approaches and provides a significant increase in the macroaverage F1 score. Potential applications of the developed tool for public opinion monitoring and decision support in economics and international relations are discussed.

Keywords: artificial intelligence; sentiment analysis; named entity extraction; Uzbek language; economic discourse; international relations; systems analysis.

I. Introduction

With the digitalization of the economy and the strengthening of international ties, a large share of information about economic processes and foreign policy events now appears in textual form: news, analytical reports, expert commentary, and social media posts. For government agencies and research centers, it is important not only to collect such data but also to systematically analyze public sentiment and the attitudes of different groups toward reforms, economic policy, and international initiatives. Traditional content analysis methods are labor-intensive and do not scale well, which makes AI-based approaches to automatic sentiment analysis and named entity extraction increasingly relevant [1-2]. For Uzbek, this task is complicated by the lack of large annotated corpora and specialized models, which limits the application of modern machine learning methods to the analysis of economic and international relations discourse. It is crucial to take into account not only the overall "positive"or "negative"tone of a text, but also the specific entitiesвЂ”states, organizations, and individualsвЂ”to which these assessments are directed. In this context, the development of a hybrid AI model combining sentiment analysis and named entity recognition (NER) on Uzbek-language data is particularly timely. The aim of this study is to develop and test such a hybrid AI model for Uzbek texts, focused on systemic discourse analysis in the fields of economics and international relations. The research involves creating a multi-source corpus and associated lexical resources, designing a model architecture that integrates sentiment and NER features, and conducting computational experiments with comparisons to text-only baselines [3-4]. Previous work on sentiment analysis in economics and international relations has concentrated mainly on English and other major languages, where machine learning and Transformer-based models, often combined with NER, are used to assess attitudes toward specific entities. For Uzbek and other low-resource languages, however, studies are largely limited to basic sentiment analysis or smallscale NER experiments, and there is a shortage of large annotated corpora and domainspecific AI models. This gap motivates the present paper, which proposes a hybrid AIbased model and a multi-source Uzbek corpus adapted to economic and international relations discourse [5].

II. Data and Subject Area.

The AI model being developed is focused on analyzing Uzbek-language texts related to economic processes and international relations. For this purpose, a multisource corpus was created, including fragments of news reports, analytical materials, official commentaries, as well as social media posts and reviews. The texts were selected based on topics related to economic reforms, price and currency dynamics, employment, migration, foreign

policy initiatives, and interactions with other countries and international organizations. Before tagging, the data was cleared of duplicates, technical noise, and obvious personal identifiers. The corpus is annotated with sentence-level sentiment. Each fragment is manually assigned one of three labels: positive, negative, or neutral. The tagging was performed by native Uzbek speakers according to pre-developed instructions, including examples of typical cases and borderline situations. In disputed cases, an annotator consensus procedure was used, which improved the consistency and reliability of the target annotation. The resulting sentiment labels are used as the target variable for training and evaluating the models [6]. In addition to sentiment, the texts are annotated by named entities reflecting key actors in economic and international discourse. Specifically, mentions of states and cities, international and national organizations, companies, as well as individuals (politicians, experts, business representatives), and, where necessary, other relevant categories are highlighted. This annotation allows us to track the specific entities targeted by specific assessments and serves as the basis for integrating NER information into a hybrid AI sentiment analysis model. To support text processing, additional lexical resources are used: dictionaries of named entities (lists of countries, cities, organizations, and people), an emoji lexicon indicating their polarity and intensity, and a set of pairs for normalizing informal Uzbek writing. The latter takes into account spelling variations, the mixing of Latin and Cyrillic, colloquial abbreviations, and common typos. Together, the multi-source corpus and its accompanying lexical resources form the data base on which the proposed hybrid AI model is trained and tested [7-8].

III. Hybrid AI model for NER-boosted sentiment analysis.

The hybrid AI model being developed is designed as a pipeline, in which the original Uzbek-language text sequentially passes through several modules: preprocessing, basic sentiment analysis, named entity extraction, and integration of the resulting features into the final classifier. The system's input consists of individual sentences or short text fragments related to economic and foreign policy events. After cleaning and normalization, the text is tokenized and fed to a transformer sentiment classifier, and simultaneously to the NER module. In the final stage, the features extracted by both modules are combined and used to calculate the final sentiment score. A pre-trained multilingual transformer model, adapted to the Uzbek language using a pre-built annotated corpus, is used as the core component of the sentiment analysis. The upper layers of the model contain a classification "head,"which associates each fragment with one of three labels: positive, negative, or neutral sentiment. This module is trained using a standard classification problem formulation, splitting the corpus into training, validation, and test sets and optimizing the cross entropy loss function. The resulting latent representations and probabilistic estimates by class are then used as part of the feature description of the text. The Named Entity Extraction (NER) module is designed to detect and categorize mentions of key actors in economic and international discourse [9-10]. This is achieved using a sequential tagging model, also based on a pre-trained language representation and further trained on Uzbek data with entity annotation. The tagging identifies entities such as "state,city,organization,company,"and "person,"along with additional categories if necessary. The NER module outputs a list of entities indicating their boundaries, types, and, if necessary, frequencies and positions in the text. This data allows for explicit consideration of which objects a given assessment fragment belongs to. The integration of sentiment analysis and NER results occurs at the feature space level. For each text fragment, aggregated features are generated that describe: (1) the probability

distribution across sentiment classes obtained by the base model; (2) the number and types of detected entities; (3) additional metrics, if necessary, such as the proportion of mentions of individual actor categories or the presence of emoji with pronounced polarity. The combined feature vector is fed to the final classifier (e.g., a linear classifier or a small neural network), which produces a refined sentiment score taking into account information about the named entities [8]. All components are technically implemented using modern deep learning libraries and process data in a unified format, simplifying training and subsequent deployment of the model. This construction allows, on the one hand, to utilize the strengths of transformer representations for the Uzbek language, and on the other, to explicitly include in the analysis the structure of economic and international discourse by taking into account named entities, which constitutes the key idea of the proposed hybrid AI model.

IV. Experiments and Results.

To evaluate the effectiveness of the developed hybrid AI model, computational experiments were conducted on the multi-source corpus of Uzbek-language texts described above. The data was randomly divided into training, validation, and test sets, preserving the proportions of sentiment classes. The primary metrics used were accuracy, recall, precision, and the macro-averaged F1 score, allowing for a balanced performance across all three sentiment classes. Model settings were selected based on the validation results, and the final evaluation was conducted on a deferred test set. The baseline was a text-based sentiment analysis model using only the latent representations of the transformer, without considering information about named entities or additional lexical resources. Several advanced variants were then considered: a model integrating features associated with emoji and normalized informal writing, and finally, a full hybrid AI model combining the results of sentiment analysis and NER. The comparison revealed that adding only emoji features yielded a moderate performance improvement, but the inclusion of named entities provided the greatest contribution to the improvement. The experimental results demonstrate that the hybrid NER-enhanced AI model significantly outperforms the text-only baseline approach in terms of the macro-average F1 score, achieving an increase of approximately 7-10% depending on the configuration and data subsample. The greatest improvement is observed for texts containing multiple actors and mixed evaluations, as well as for messages where the overall context appears neutral but a clearly expressed negative or positive assessment of a specific country, organization, or individual is present. This confirms the hypothesis that explicitly accounting for named entities and their distribution in the text allows for a more accurate interpretation of sentiment direction [11]. A qualitative error analysis revealed that the remaining difficulties are primarily related to ironic statements, complex contexts where positive and negative elements are intentionally mixed, and rare or ambiguous named entities that are underrepresented in the training corpus and lexical resources. Nevertheless, the obtained results demonstrate the potential of the hybrid AI approach for systemic analysis of Uzbek-language economic and international discourse and justify the need for further expansion of the corpus and refinement of the models.

V. Application of the Model in Economics and International Relations.

The developed hybrid AI model can be used as a system analysis support tool in a number of practical problems related to economics and international relations. First and foremost, it enables automated public opinion monitoring on key economic policy

issues: reforms, taxes, subsidies, price and exchange rate dynamics, and the labor market. Regular processing of news feeds and social media publications makes it possible to track changes in sentiment towards specific decisions and actors, identify periods of increased negative or positive sentiment, and compare them with the event calendar. A second important area of application relates to the analysis of foreign policy discourse. By identifying named entities, the model can assess attitudes toward individual countries, international organizations, and bilateral initiatives reflected in Uzbek-language sources. This makes it possible to build more detailed pictures of the perception of foreign policy moves, identify asymmetries in assessments of different partners, and track how the tone of discussions changes in the context of specific events. Finally, the hybrid AI model can be integrated into analytical platforms used by government agencies, research centers, and the private sector. Within such systems, sentiment and NER analysis results can be visualized as dashboards, time series, and actor maps, facilitating data interpretation for non-AI experts. Thanks to its modular architecture, the model can be further trained on additional domain data or adapted for related tasks, expanding its range of practical applications.

VI. Conclusion and Future Research Directions.

In this paper we implemented and evaluated a hybrid AI model that combines sentiment analysis and NER for Uzbek texts. The experiments confirm that incorporating named entities and domain-specific features improves performance compared to text-only baselines. Experiments confirmed that the hybrid NER-enhanced AI model provides a significant increase in the macro-average F1 score (approximately 7-10%) and is particularly effective in cases where a single text fragment contains coexisting assessments of different actors or when the overall context of the message conceals explicit attitudes toward individual states, organizations, or individuals. This makes the developed approach a promising tool for systematically monitoring public opinion and providing analytical support for decisions in the socioeconomic and foreign policy spheres related to Uzbek-language sources. At the same time, the study has several limitations. Despite its multi-source nature, the text corpus is still inferior in volume and thematic coverage to larger resources for global languages; certain entity types and rare domain terms are only partially represented. Future plans include expanding the dataset with new sources and time periods, refining the sentiment and NER tagging schemes, and exploring more complex feature integration schemes, including joint training of sentiment and entity extraction modules within a single architecture. Adapting the developed approach to other resource-poor languages in the region and integrating the model into full-featured analytical platforms for working with text data streams in near-real time also appears promising.

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Mathematical models in the process of water resources distribution in main canals of irrigation systems taking into account boundary conditions

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DOI 10.5281/zenodo.18217404

Keywords: effective water management, ntermittent water delivery, agricultural water systems, recursive optimization technique, system regulation theories, finite-variable optimization methods, oscillatory phenomena, computational modeling

Abstract: The article discusses mathematical modeling for optimizing water distribution in primary irrigation canals, considering boundary conditions. It uses direct, kinematic, and convection-diffuse wave equations to depict unsteady water movement. An analytical solution for optimal water distribution using the direct wave equation is provided. Numerical solutions for kinematic and convection-diffusion equations are also offered. The article explains optimality conditions for these models, integrating mathematical frameworks with irrigation system dynamics for effective water management. These models are valuable for optimizing water distribution, ensuring efficient use in irrigation networks. By combining analytical and numerical approaches, the article enhances understanding of water flow dynamics, aiding decision-making in agricultural water management. Additionally, it sheds light on

essential optimality conditions, enriching comprehension of water flow dynamics. The integration of mathematical frameworks with practical irrigation system dynamics offers valuable insights into water resource management strategies. These models are indispensable tools for optimizing water distribution, thereby enhancing the efficiency of irrigation networks. Through analytical and numerical methods, the article contributes significantly to informed decision-making in agricultural water management, facilitating sustainable practices and resource conservation. Furthermore, by elucidating the essential optimality conditions for these models, the article enhances comprehension of water flow dynamics. Integrating mathematical frameworks with practical irrigation system dynamics provides valuable insights into water resource management strategies. These models are indispensable tools for optimizing water distribution, thus improving the efficiency of irrigation networks. Through analytical and numerical methods, the article significantly contributes to informed decision-making in agricultural water management, fostering sustainable practices and resource conservation.

1. INTRODUCTION

The current theory of optimal water distribution in irrigation systems takes into account various aspects such as water efficiency, meeting the needs of different water users, and environmental constraints. However, there are additional challenges when considering the discreteness of water supply, as it is necessary to take into account possible interruptions in the water supply and their impact on the system management processes. Dynamic programming methods, control theories, and other mathematical approaches can be used to formulate optimality conditions under such conditions[5-12]. In particular, discrete optimization methods can be used to find optimal strategies for water distribution when the resource is available [1-8]. Understanding that accounting for discreteness conditions can lead to discontinuities in both spatial and temporal parameters is key to effectively modeling and managing water flow in irrigation systems. It is important to take into account that both the parameters of the equations of motion and the functions on the right sides of these equations can be discrete in time[21-33]. When developing optimal conditions for the distribution of water among water consumers, it is necessary to take into account these features.[3-40] This may require the use of specialized optimization and control techniques that are able to take into account the discreteness over time of the system's parameters. In addition, wave phenomena that occur during the movement of flow in canals due to the discreteness of water supply should also be taken into account in the development of control models and algorithms [3-5]. This may include analyzing the characteristics of wave processes and developing control strategies aimed at minimizing their impact on the system. Comprehending the efficient allocation of water within irrigation canal networks under the discrete nature of water delivery to consumers as a problem of optimal regulation of systems with distributed parameters [6–9] represents a significant methodology that considers all aspects of the process. This method allows the formulation of an optimal control problem that integrates both the differential equations describing water flow and the discrete features of time and space, along with the wave processes that occur in the system [41–48].

METHODS AND RESULTS

For such a one-dimensional object with distributed parameters, described by one-dimensional first-order differential equations for time and second-order for a spatial variable, we can represent the equation as follows:

$$\frac{\partial Q}{\partial t} = f\left(Q, \frac{\partial Q}{\partial x}, \frac{\partial^2 Q}{\partial x^2}, u\right), \qquad 0 < x < 1;\ 0 < t < T; \tag{1}$$

where $Q(x,t)$ is the distributed state of objects, $u(x,t)$ is distributed control, and f is a known function differentiated by a set of arguments.

The boundary conditions of the objects are written as follows:

$$\frac{\partial Q}{\partial t}(0,t) = g_1\big(Q(0,t),\, v_1(t)\big), \tag{2}$$

$$\frac{\partial Q}{\partial t}(1,t) = g_2\big(Q(1,t),\, v_2(t)\big), \tag{3}$$

where $v_1(t)$ is the boundary control action applied at $x = 0$, $v_2(t)$ is the boundary control action applied at $x = 1$; g_1, g_2 are known functions differentiated by a set of arguments[10–15].

The initial conditions of the objects are set as follows:

$$Q(x,0) = Q_0(x), \tag{4}$$

where $Q_0(x)$ is a known given function.

Under conditions of discreteness of water supply, the control functions are of the form:

$$u(x,t) = \sum_{i=1}^{I}\sum_{k=1}^{K} u_{ik}\,\delta(x-a_i)\,1(t-t_k), v_1(t) = \sum_{k=1}^{K} v_k^1\,1(t-t_{k1}), v_2(t) = \sum_{k=1}^{K} v_k^2\,1(t-t_{k2}), \tag{5}$$

where u_{ik}, v_{1k}, and v_{2k} are the discrete values of the control actions at time t_k, t_{k1}, and t_{k2}. $\delta(x-a_i)$ is the delta Dirac function of x, and $1(t-t_{ki})$ are the unit functions of time t.

The criterion of optimal control of the object under consideration is characterized by the following quality functionality:

$$I\big(u(x,t),v_1(t),v_2(t),Q_0(x)\big)=\int_0^{l_1}\int_0^T G\left(Q,\frac{\partial Q}{\partial x},\frac{\partial^2 Q}{\partial x^2},u\right)dx\,dt + \int_0^{l_1} G_0\big(Q(x,T),Q_0(x)\big)\,dx + \int_0^T\Big[G_1\big(Q(0,t),v_1(t)\big)+G_2\big(Q(l,t),v_2(t)\big)\Big]\,dt \quad (6)$$

where G, G_0, G_1, G_2 are known functions differentiated by the set of arguments.

Control constraints are as follows:

$$u_*\le u(x,t)\le u^* v_{1*}\le v_1(t)\le v_1^* v_{2*}\le v_2(t)\le v_2^* Q_*\le Q_0(x)\le Q^* \quad (7)$$

where $u^*, u_*, v_1^*, v_{1*}, v_2^*, v_{2*}, Q^*, Q_*$ are the specified maximum and minimum values of control functions.

Introducing a Hamiltonian with a conjugate function:

$$H=G+\lambda f. \quad (8)$$

On the basis of the methods of the calculus of variations and Lagrange factors, we obtain explicit expressions for the variation of the functional in the neighborhood of the optimal trajectory:

$$\delta I=\int_0^T\int_0^l\left(\frac{\partial H}{\partial u}\right)_0\delta u\,dx\,dt+ \quad (11)$$

$$+\int_0^l\left[\left(\frac{\partial G_1}{\partial Q_0(x)}\right)_0+\lambda\right]\delta Q_0(x)\,dx+ \quad (12)$$

$$+\int_0^T\left[\left(\frac{\partial H}{\partial Q'}(0,t)\right)_0\left(\frac{\partial g_1}{\partial v_1(t)}\right)_0\delta v_1(t)+\left(\frac{\partial H}{\partial Q'}(l,t)\right)_0\left(\frac{\partial g_2}{\partial v_2(t)}\right)_0\delta v_2(t)\right]dt. \quad (9)$$

In this case, the conjugate function satisfies the following differential equation:

$$\frac{\partial\lambda}{\partial t}=-\left[\left(\frac{\partial H}{\partial Q}\right)_\circ-\frac{\partial}{\partial x}\left(\frac{\partial H}{\partial Q'}\right)_\circ+\frac{\partial^2}{\partial x^2}\left(\frac{\partial H}{\partial Q''}\right)_\circ\right], \quad (10)$$

with boundary conditions:

$$\left(\frac{\partial G_1}{\partial Q(0,t)}\right)_\circ-\left(\frac{\partial H}{\partial Q''}(0,t)\right)_\circ\left(\frac{\partial g_1}{\partial Q(0,t)}\right)_\circ-\left(\frac{\partial H}{\partial Q'}(0,t)\right)_\circ-\frac{\partial}{\partial x}\left(\frac{\partial H}{\partial Q''}(0,t)\right)_\circ=0, \quad (11)$$

$$\left(\frac{\partial G_2}{\partial Q(l,t)}\right)_\circ - \left(\frac{\partial H}{\partial Q''}(l,t)\right)_\circ \left(\frac{\partial g_2}{\partial Q(l,t)}\right)_\circ - \left(\frac{\partial H}{\partial Q'}(l,t)\right)_\circ - \frac{\partial}{\partial x}\left(\frac{\partial H}{\partial Q''}(l,t)\right)_\circ = 0, \quad (12)$$

conditions at a finite point in time:

$$\lambda(x,T) = \left(\frac{\partial G_0}{\partial Q(x,T)}\right)_\circ, \quad (13)$$

where the symbol $(\cdot)_\circ$ means that the corresponding quantity is computed along the optimal trajectory, $Q_i' = \partial Q_i/\partial x_i, \qquad Q_i'' = \partial^2 Q_i/\partial x_i^2$.

In (9), the effect of the variations $\delta * u$, $\delta * v_1$, $\delta * v_2$, and $\delta * Q_0$ on the variations $\delta * I$ is clearly expressed.

Further, equating the coefficients for variations in control to zero, in the expression of the variation of the functional, we obtain the rest of the necessary conditions for the optimality of the problem under consideration.

$$\left(\frac{\partial H}{\partial u}\right)_\circ = 0, \qquad \left(\frac{\partial G_1}{\partial Q_0(x)} + \lambda\right)_\circ = 0,$$

$$\left(\frac{\partial H}{\partial Q''}(0,t)\right)_\circ \left(\frac{\partial g_1}{\partial v_1(t)}\right)_\circ = 0, \quad (14)$$

$$\left(\frac{\partial H}{\partial Q''}(l,t)\right)_\circ \left(\frac{\partial g_2}{\partial v_2(t)}\right)_\circ = 0.$$

Thus, the necessary conditions for the optimality of the problem (1–6) are determined by the conditions given in (10–14).

Let us examine the necessary conditions for achieving optimal water distribution in irrigation canal networks under discrete water supply conditions, employing simplified models of water flow dynamics within canal segments [17–20].

3. DISCUSSION

Straight Wave Model. The objective of optimal water distribution in the straight wave model is to minimize variations in water discharge along the canal length and at its endpoint, while maintaining a discrete water supply with a specified flow rate to the respective lateral water intakes, i.e., the points of water withdrawal [17–19].

$$I = \int_0^l \left[Q(x,T) - Q^*\right]^2 dx + \int_0^T Q^2(l,t)\, dt, \quad (15)$$

The direct wave model has the following partial differential equation:

$$\frac{\partial Q}{\partial t} + v\frac{\partial Q}{\partial x} = q, \tag{16}$$

where $Q(x,t)$ denotes the flow rate of water along the canal length, and $q(x,t)$ represents the rate of lateral water withdrawal. In the case of discrete water supply to five concentrated intake points located at positions a_i, the expression takes the following form:

$$q(x,t) = -\sum_{i=1}^{N} q_i\,\delta(x-a_i)\,I(t-T). \tag{17}$$

Initial, Boundary Conditions, and Variable Definition Domains:

$$Q(x,0) = Q_0(x), \qquad Q(0,t) = u(t), x \geq 0, \qquad t \geq 0, \qquad v > 0. \tag{18}$$

In this case, introducing the conjugate function $\lambda(x,t)$, we give expressions for the functional variation in the neighborhood of the optimal trajectory without intermediate transformations:

$$\delta I = \int_0^l\int_0^T \left(\frac{\partial \lambda}{\partial t} - v\,\frac{\partial \lambda}{\partial x}\right)_0 \delta Q\,dx\,dt + \int_0^l \left\{2\big[Q(x,T) - Q^*\big] + \lambda(x,T)\right\}_0 \delta Q(x,T)\,dx + \tag{13}$$

$$+ \int_0^T \left[2Q(l,t) + v\,\lambda(l,t)\right]_0 \delta Q(l,t) - v\,\lambda(0,t)_0\,\delta u(t)\,dt. \tag{19}$$

Hence, the necessary conditions for the optimality of this problem are of the form:

$$\frac{\partial \lambda}{\partial t} - v\,\frac{\partial \lambda}{\partial x} = 0, \lambda(x,T) = -2\big[Q(x,T) - Q^*\big], \lambda(l,t) = \frac{2}{v}\,Q(l,t), \qquad \lambda(0,t) = 0. \tag{20}$$

The analytic solution of boundary value problems (16)-(18) and (19) on the basis of their Green's function according to [5] is written as follows:

$$Q(x,t) = \int_0^t\int_0^l G_1(x,\xi,t-\tau)\,w_1(\xi,\tau)\,d\xi\,d\tau, \tag{21}$$

$$\lambda(x,t) = \int_0^t\int_0^l G_2(x,\xi,t-\tau)\,w_2(\xi,\tau)\,d\xi\,d\tau. \tag{22}$$

where $G_1(x,\xi,t-\tau)$ and $G_2(x,\xi,t-\tau)$, $w_1(\xi,\tau)$ and $w_2(\xi,\tau)$ – Green's functions and standardizing functions, which are of the form

$$G_1(x,\xi,t-\tau) = l(x-\xi)\,\delta\big(v(t-\tau) - (x-\xi)\big),$$

$$G_2(x,\xi,t-\tau) = l(x-\xi)\,\delta\big(-v(t-\tau) - (x-\xi)\big),$$

$$w_1(x,t) = q(x,t) + Q_0(x)\,\delta(t) - v\,u(t)\,\delta(x),$$

$$w_2(x,t) = 2\big[Q(x,T) - Q^*\big]\delta(t-T) - 2v\,Q(l,t)\,\delta(x-l).$$

With the help of fundamental solutions of the fundamental and conjugate equations at $Q_0(x) = 0$, considering the function $u(t)$ as a step function and on the basis of the necessary conditions of optimality, the minimum of the functional is achieved by controlling

$$u_1(t) = \sum_{i=1}^{N} q_{N-i}\, l\left(t - \sum_{k=1}^{i} \tau_{N-k}\right), \tag{23}$$

where $\tau_i = (a_i - a_{i-1})/v$.

In this case, the solution of equation (17) will be written in analytical form

$$Q(x,t) = -\sum_{i=1}^{N} q_i\, l(x-a_i)\, l(v(t-T) - (x-a_i)) + l(t-vx) \sum_{i=1}^{N} q_{N-i}\, l\left(t - \sum_{k=1}^{i} \tau_{N-k}\right). \tag{24}$$

This approach assumes that, to control the water discharge at the lateral intake points located in various sections of the canal, it is essential to adjust the discharge at the canal's initial section in advance [14]. A graphical illustration of this concept is presented in Figure 1, shown in the lower diagram (25).

As illustrated in the figure, the control function $u(t)$ - representing the flow rate at the canal's inlet at time $t_o = T - \tau_5 - \tau_4 - \tau_3 - \tau_2 - \tau_1$ - is adjusted to match the specified discharge value of the final water intake, denoted as q_5. Then, after the passage of time τ_5, the control value increases by q_4, after the time τ_4 the control increases by q_4, after the time τ_3 the control increases by q_3, and so on to the value q_1, i.e. with optimal water distribution under conditions of discrete water supply in the canal, first of all, water intakes located in series from the end of the canal are provided.
Fig. 2 shows the water flow distribution function $Q(x,t)$ (24) in isometric.

Kinematic wave model. The criterion for the optimal distribution of water under the conditions of the kinematic wave model corresponds to the shape represented in equation (15). The kinematic wave model is based on the equation of conservation of mass of the flow[2,7,20].

$$\frac{\partial \omega}{\partial t} + \frac{\partial Q}{\partial x} = q, Q = \omega C\sqrt{Ri}, \qquad C = \frac{1}{n}R^y, \qquad y = 2.5\sqrt{n} - 0.13 - 0.75\sqrt{R}\,(\sqrt{n} - 0.1), \tag{25}$$

where $Q(x,t)$ represents the variation in water discharge within the canal section, ω denotes the cross-sectional area of the flow, R is the hydraulic radius, C refers to the Chezy coefficient, n is the roughness coefficient, and i indicates the slope of the canal bed [23].

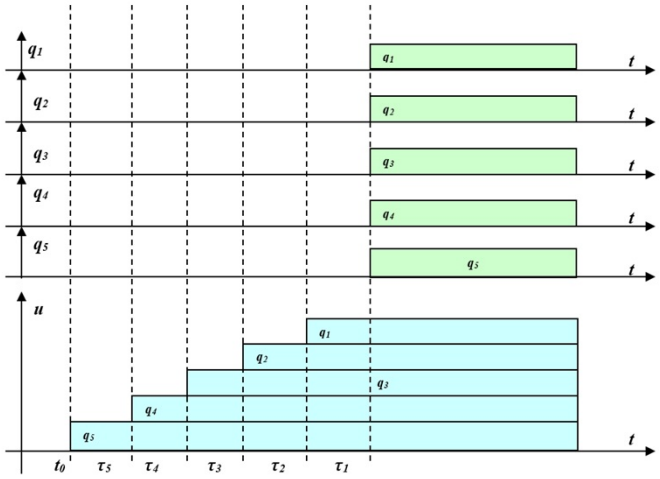

Рис. 1: Fig. 1: Change the water flow rate at the beginning of the canal to ensure optimal water distribution over the canal section.

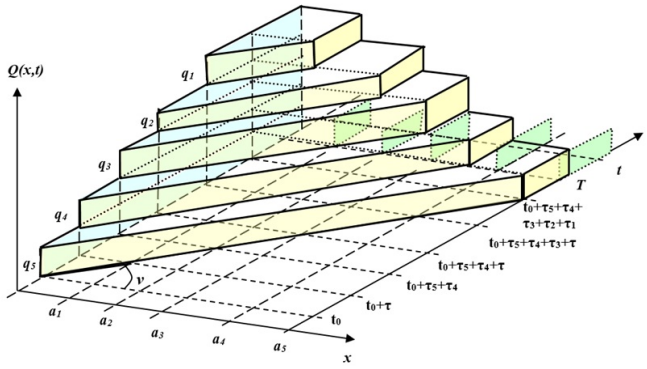

Рис. 2: Fig. 2. Change the water flow rate at the beginning of the canal to ensure optimal water distribution on the canal section.

Starting conditions

$$Q(x,0) = Q_0(x), \qquad \omega(x,0) = \omega_0(x) \tag{26}$$

where $Q_0(x)$ and $\omega_0(x)$ denote the initial distributions of the flow rate and the cross-sectional area of the water flow, respectively.

The boundary conditions and the ranges of variable definitions are analogous to those in the previous problem.

The water discharges at the intake points of the canal section, $q(x,t)$, under discrete water distribution conditions, are determined as expressed in equation (17).

Likewise, the necessary conditions of optimality are defined as

$$\frac{\partial \lambda}{\partial t} - v\frac{\partial \lambda}{\partial x} = 0, \qquad \lambda(x,T) = -2\big[Q(x,T) - Q^*\big], \qquad \lambda(l,t) = \frac{2}{v}Q(l,t), \qquad \lambda(0,t) = 0. \tag{27}$$

where

$$v = \left(\frac{\partial Q}{\partial \omega}\right)_0.$$

In the kinematic wave model, the equations describing the main variables are nonlinear, and the equations for the conjugate variables are linear equations of the hyperbolic inverse wave type. The velocity of the reverse wave is defined as the derivative of the water flow rate over the cross-sectional area of the flow and is a variable function [41].

Convection-diffusion model. The criterion for optimal water distribution under the conditions of the convection-diffuse model is (15). The convection-diffusion model is based on the consideration of the drag force and the neglect of the inertial force [2]

$$\frac{\partial Q}{\partial t} = f\left(Q, \frac{\partial Q}{\partial x}, \frac{\partial^2 Q}{\partial x^2}, q\right) \tag{28}$$

where $f = -\left(\frac{\partial Q}{\partial K}\frac{\partial K}{\partial h}\right)\frac{\partial Q}{\partial x} + \frac{K^2}{2B|Q|}\frac{\partial^2 Q}{\partial x^2} + q$, K is the flow modulus, B is the top flow width.

The flow modulus K characterizes the magnitude of the frictional forces and is determined by the following formula:

$$K = \omega \cdot C\sqrt{R}. \tag{30}$$

The rest of the parameters, the initial and boundary conditions, as well as the domains of variable definition, have the same appearance as in the kinematic wave model.

On the basis of the above methodology, the necessary conditions for optimality in the case of the convection-diffuse model are determined

$$\frac{\partial \lambda}{\partial t} - v_1\frac{\partial \lambda}{\partial x} - v_2\frac{\partial^2 \lambda}{\partial x^2} = 0, \tag{31}$$

where

$$v_1 = \left(\frac{\partial f}{\partial Q'}\right)_0 = \left(\frac{\partial Q}{\partial K}\frac{\partial K}{\partial h}\right)_0, \qquad v_2 = \left(\frac{\partial f}{\partial Q''}\right)_0 = \left(\frac{K^2}{2B|Q|}\right)_0. \tag{32}$$

Boundary conditions

$$v_2\frac{\partial \lambda}{\partial x}(0,t) = (v_2 - v_1)\lambda(0,t), v_2\frac{\partial \lambda}{\partial x}(l,t) = (v_1 - v_2)\lambda(l,t), \tag{33}$$

conditions by the end of the process $\lambda(x,T) = -2\big[Q(x,T) - Q^*\big]$.

In the convection-diffusion model, equation (29) for the principal variable is nonlinear, and equation (31) for the conjugate variables is a linear parabolic equation with a convective term. The coefficients in these equations are defined as derivatives of the functions on the right-hand side of equation (32) for the variables $Q^{'}$ and $Q^{''}$respectively.

Boundary conditions are expressed as conditions of the third kind.

For both models, kinematic and convective-diffusion waves, the optimal control can be determined using numerical methods for modeling the movement of water in channels and the necessary optimality conditions for an approximate solution of the problems of optimal water distribution under conditions of discreteness of water supply.

4. CONCLUSIONS

The article presents the development of mathematical models and the necessary optimality conditions for solving the problem of optimal water resource distribution in the main canals of irrigation systems, taking into account the boundary conditions. As a mathematical representation of the unsteady water flow within the canal sections, simplified models—such as the equations of linear (straight) waves, kinematic waves, and convection–diffusion waves—were employed. An analytical solution to the optimal water distribution problem in main canals, irrigation systems, and related facilities was derived using the straight wave equation.Additionally, numerical solutions for specific cases based on the kinematic and convection–diffusion models of unsteady water flow in irrigation system components were obtained, and the corresponding optimality conditions for these models were established.

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Modeling of unsteady water movement in open canals during the construction of transport facilities

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DOI 10.5281/zenodo.18217416

Keywords: mathematical model, model, tansport systems, analys system, canals, unsteady water movement.

Abstract: The article considers the simulation of the planned unsteady movement of water in open channels using the finite element method in the construction of bridges and tunnels in the transport system. A system of two-dimensional equations, boundary conditions of unsteady water movement in isoparametric spatial coordinates, as well as results of computer simulation of unsteady water movement in two-dimensional simulated areas of open channels are presented. The processes occurring in water management objects occur in a multidimensional spatial domain and in time. The complexity of these processes does not allow researchers, designers and operators to quickly assess the qualitative and quantitative parameters of the operation of sections of river beds, canals and hydraulic structures to implement the required water resource management regime [5]. Currently, with the widespread development of numerical methods for solving complex mathematical problems using modern computer technologies, there is a real opportunity to obtain specific qualitative and quantitative characteristics of various complex dynamic processes. Numerical finite difference and finite element methods are used to model fluid dynamics problems. Finite-difference methods for modeling two-dimensional water motion were considered in [1-4].

INTRODUCTION In the Republic of Uzbekistan, the Karshi Main Canal (KMC) is a large and important hydraulic structure, which, together with the Talimarjan Reservoir, provides water to more than 350 thousand hectares of cultivated area in the region. It should be noted that the Talimarjan reservoir provides drinking water to large industrial enterprises, as well as a population of more than 400 thousand people in four districts of the Kashkadarya region. Water intake into the Karshi cascade of pumping stations develops the complex hydraulic process of erosion of the river banks by dump currents, which creates difficult conditions for supplying the Karshi cascade of pumping stations. The above allows us to conclude that the problem of delivering water to consumers in the required volume is in demand; accuracy of water supply is required through optimal control of water distribution in the canals of irrigation systems [6-40].

Mathematical methods, algorithms and corresponding software, obtained adequate modeling results will ensure a reduction in unproductive losses of water resources in canals, which is of great economic importance for the republic. The use of modern high-performance computers and information technologies will make it possible to obtain timely

information about water resources, to use modeling results for the purpose of managing water resources in all water management systems and objects, which include river sections, reservoirs and canals (gravity flow and with mechanical water lifting systems) [8, 41-56]. Computer modeling allows you to simulate the behavior of various elements of engineering structures, devices, including individual water management facilities, their interaction, taking into account all influencing factors in conditions close to real ones, during their operation

2 LITERATURE REVIEW The planned unsteady movement of water in open channels is described by the two-dimensional system of Saint-Venant equations [5]

$$\begin{aligned} \frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} + \frac{\partial (vh)}{\partial y} + i = 0, \\ \frac{\partial (uh)}{\partial t} + \frac{\partial (u^2 h)}{\partial x} + \frac{\partial (uvh)}{\partial y} + g \frac{\partial}{\partial x}\left(\frac{h^2}{2}\right) = gh(S_{ax} - S_{fx}), \\ \frac{\partial (vh)}{\partial t} + \frac{\partial (v^2 h)}{\partial y} + \frac{\partial (uvh)}{\partial x} + g \frac{\partial}{\partial y}\left(\frac{h^2}{2}\right) = gh(S_{ay} - S_{fy}) \end{aligned} \tag{1}$$

Where is $h = h(x, y, t)$ – water surface depth; $u = u(x, y, t)$ – longitudinal component of water flow velocity; $v = v(x, y, t)$ – transverse component of water flow velocity; v_t – fluid viscosity, S_{ax} – bottom slope along the axis $x, S_a y$ – bottom slope along the axis y, S_{fx} – slope of the free water surface along the axis x, S_{fy} – slope of the free water surface along the axis y, g – acceleration due to gravity; $i(x, y, t)$ – water flow rate, x – axis coordinate along length; y – width axis coordinate; t – time.

The ordinate of the channel bottom is given by the function $z_0(x, y)$, then the bottom slopes at the corresponding coordinates are determined

$$S_{ax} = \frac{\partial z_0}{\partial x}, S_{ay} = \frac{\partial z_0}{\partial y} \tag{2}$$

Using Manning's formula, it can be obtained the slopes of free surfaces along the ordinates

$$S_{fx} = \frac{n^2 \, u \sqrt{u^2 + v^2}}{h^{4/3}}, \qquad S_{fy} = \frac{n^2 \, v \sqrt{u^2 + v^2}}{h^{4/3}}. \tag{3}$$

Let us introduce the change of variables $p = uh$, $q = vh$. Then equation (1) has the form

$$\frac{\partial h}{\partial t} + \frac{\partial p}{\partial x} + \frac{\partial q}{\partial y} + i = 0$$

$$\frac{\partial p}{\partial t} + \frac{\partial}{\partial x}\left(\frac{p^2}{h} + \frac{g \, h^2}{2}\right) + \frac{\partial}{\partial y}\left(\frac{p \, q}{h}\right) + g \, h \, \frac{\partial z_0}{\partial x} + \frac{g \, n^2 \, p \sqrt{p^2 + q^2}}{h^{7/3}} = 0$$

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{p\,q}{h}\right) + \frac{\partial}{\partial y}\left(\frac{q^2}{h} + \frac{g\,h^2}{2}\right) + g\,h\,\frac{\partial z_0}{\partial y} + \frac{g\,n^2\,q\,\sqrt{p^2+q^2}}{h^{7/3}} = 0 \tag{4}$$

Writing these equations in vector form, we get

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} + D = 0 \tag{5}$$

where U, F, G *and* D are function vectors

$$U = \begin{pmatrix} h \\ p \\ q \end{pmatrix}, \qquad F = \begin{pmatrix} p \\ \frac{p^2}{h} + \frac{gh^2}{2} \\ \frac{pq}{h} \end{pmatrix}, \qquad G = \begin{pmatrix} q \\ \frac{pq}{h} \\ \frac{q^2}{h} + \frac{gh^2}{2} \end{pmatrix} \tag{6}$$

Since the functions F(U) and G(U) depend on the function U, we write equation (5) in the following form

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial U}\frac{\partial U}{\partial x} + \frac{\partial G}{\partial U}\frac{\partial U}{\partial y} + D = 0 \tag{7}$$

Let us finally write equation (8) in vector-matrix form.

$$\frac{\partial U}{\partial t} + A\frac{\partial U}{\partial x} + B\frac{\partial U}{\partial y} + D = 0 \tag{8}$$

where are the matrices

$$A(p,q,h) = \frac{\partial F}{\partial U} = \begin{pmatrix} 0 & 1 & 0 \\ -\frac{p^2}{h^2} + gh & \frac{2p}{h} & 0 \\ -\frac{pq}{h^2} & \frac{q}{h} & \frac{p}{h} \end{pmatrix} \tag{9}$$

$$B(p,q,h) = \frac{\partial G}{\partial U} = \begin{pmatrix} 0 & 0 & 1 \\ -\frac{pq}{h^2} & \frac{q}{h} & \frac{p}{h} \\ -\frac{q^2}{h^2} + gh & 0 & \frac{2q}{h} \end{pmatrix} \tag{10}$$

To make the solution unique, it is necessary to set the initial and boundary conditions. Initial conditions characterize the entire movement at some point in time t0, taken as the initial.

$$U(x,y,t_0) = U_0(x,y), (x,y) \in \Omega \tag{11}$$

Where is $U_0(x,y)$ – given distribution functions of flow depth and flow rates, Ω – two-dimensional region. The domain of definition of variables where the movement of water flow occurs has a complex geometric shape in the case of sections of rivers and natural canals [6]. A significant difficulty in formulating a two-dimensional problem lies in specifying the boundary conditions. We will consider the border $d\Omega$ areas Ω , consisting of liquid and impermeable solid parts, i.e. $d\Omega = d\Omega_j, d\Omega_\tau$, moreover, these parts of the

border can consist of several parts [8]. On the liquid part of the boundary, a change in depth or a change in water flow is specified

$$h_i(x,y,t) = H_i(t), q_i(x,y,t)\cos\alpha + p_i(x,y,t)\sin\alpha = Q_i(t), \tag{12}$$

$$\alpha = (n,\hat{Ox}), \quad (x,y) \in d\Omega_{\text{ж}i}, \quad i = 1, n_{\text{ж}}$$

Where is $q_i(x,y,t)$ and $p_i(x,y,t)$ unknown longitudinal and transverse components of flow rates, $H_i(t)$ and $Q_i(t)$ - specified functions for changing the depth and flow rates on the corresponding liquid part, α – angle between the normal to the boundary and the axis x [4].

On the solid part of the boundary is given in the form

$$q_i(x,y,t)\cos\alpha + p_i(x,y,t)\sin\alpha = 0, \alpha = (n,\hat{Ox}), \quad (x,y) \in d\Omega_{\text{ж}i}, \quad i = 1, n_{\text{ж}} \tag{13}$$

The physical meaning of (12) means that the total component of the longitudinal and transverse water flow rates normal to the boundary will be equal to zero.

Then the common boundary of the region is the union of the solid and liquid parts of the boundary [20].

$$d\Omega = d\Omega_{\text{ж}} \cup d\Omega_m, \quad d\Omega_{\text{ж}} = \bigcup_{i=1}^{n_{\text{ж}}} d\Omega_{\text{ж}i}, \quad d\Omega_m = \bigcup_{i=1}^{n_m} d\Omega_{mi} \tag{14}$$

Thus, to simulate two-dimensional water flow in open channels, it is necessary to solve the system of equations (9) with boundary conditions (12)-(13).

System of equations (9) refers to quasilinear partial differential equations with complex boundary conditions of the domains of definition of the variables. An exact solution to the formulated problem cannot be obtained, so it is necessary to use various numerical methods for an approximate solution based on the finite element method, which uses isoparametric transformation of the equations and the used (triangular and quadrangular) elements. The domain of definition of ? variables is divided into N finite subdomains consisting of triangular or quadrangular elements [5].

3 DISCUSSION

Enter curvilinear coordinates $x = x(\xi,\eta), y = y(\xi,\eta)$ and determine the elements of the isoparametric transformation. The relationship between the derivatives of the transformation is determined [6]

$$\frac{\partial}{\partial x} = \frac{\partial\xi}{\partial x}\frac{\partial}{\partial\xi} + \frac{\partial\eta}{\partial x}\frac{\partial}{\partial\eta} \tag{15}$$

$$\frac{\partial}{\partial y} = \frac{\partial\xi}{\partial y}\frac{\partial}{\partial\xi} + \frac{\partial\eta}{\partial y}\frac{\partial}{\partial\eta} \tag{15}$$

or in vector-matrix form

$$\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix} = \begin{pmatrix} \xi_x & \eta_x \\ \xi_y & \eta_y \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial\xi} \\ \frac{\partial}{\partial\eta} \end{pmatrix} \tag{16}$$

Where is

$$\begin{aligned}
\xi_x &= \frac{\partial \xi}{\partial x}, \qquad \xi_y = \frac{\partial \xi}{\partial y} \\
\eta_x &= \frac{\partial \eta}{\partial x}, \qquad \eta_y = \frac{\partial \eta}{\partial y} \\
0 &\le \xi \le 1, \qquad 0 \le \eta \le 1
\end{aligned} \tag{17}$$

Likewise

$$\begin{aligned}
\frac{\partial}{\partial \xi} &= \frac{\partial x}{\partial \xi}\frac{\partial}{\partial x} + \frac{\partial y}{\partial \xi}\frac{\partial}{\partial y} \\
\frac{\partial}{\partial \eta} &= \frac{\partial x}{\partial \eta}\frac{\partial}{\partial x} + \frac{\partial y}{\partial \eta}\frac{\partial}{\partial y}
\end{aligned} \tag{18}$$

or in vector-matrix form

$$\begin{pmatrix} \frac{\partial}{\partial \xi} \\ \frac{\partial}{\partial \eta} \end{pmatrix} = \begin{pmatrix} x_\xi & y_\xi \\ x_\eta & y_\eta \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix} \tag{19}$$

$$x_\xi = \frac{\partial x}{\partial \xi}, \quad x_\eta = \frac{\partial x}{\partial \eta}, \quad y_\xi = \frac{\partial y}{\partial \xi}, \quad y_\eta = \frac{\partial y}{\partial \eta} \tag{20}$$

$$\begin{pmatrix} \frac{\partial}{\partial \xi} \\ \frac{\partial}{\partial \eta} \end{pmatrix} = \frac{1}{\xi_x \eta_y - \xi_y \eta_x} \begin{pmatrix} \eta_y & -\xi_y \\ -\eta_x & \xi_x \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix} = \begin{pmatrix} x_\xi & y_\xi \\ x_\eta & y_\eta \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix} \tag{21}$$

Introducing the Jacobian notation $J = \xi_x \eta_y - \xi_y \eta_x$, let's write the connections in a different form

$$\frac{1}{J} \begin{pmatrix} \eta_y & -\eta_x \\ -\xi_y & \xi_x \end{pmatrix} = \begin{pmatrix} x_\xi & y_\xi \\ x_\eta & y_\eta \end{pmatrix} \tag{22}$$

Relationship between derivatives of spatial variables with fundamental and isoparametric coordinates

$$\begin{aligned}
x_\xi &= \frac{1}{J}\,\eta_y, \qquad y_\xi = -\frac{1}{J}\,\eta_x \\
x_\eta &= -\frac{1}{J}\,\xi_y, \qquad y_\eta = \frac{1}{J}\,\xi_x
\end{aligned} \tag{23}$$

$$\begin{aligned}
\eta_y &= J x_\xi, \qquad \eta_x = -J y_\xi \\
\xi_y &= -J x_\eta, \qquad \xi_x = J y_\eta \\
J &= \xi_x \eta_y - \xi_y \eta_x = J^2 \big(x_\xi y_\eta - x_\eta y_\xi \big) \\
J &= \frac{1}{x_\xi y_\eta - x_\eta y_\xi}
\end{aligned} \tag{24}$$

The relationship with the main and isoparametric variables will be written as follows

$$\begin{aligned} p^{\xi} &= \xi_x p + \xi_y q \\ p^{\eta} &= \eta_x p + \eta_y q \\ p &= \frac{1}{J}\left(\eta_y p^{\xi} - \xi_y p^{\eta}\right) \\ q &= \frac{1}{J}\left(-\eta_x p^{\xi} + \xi_x p^{\eta}\right) \end{aligned} \tag{25}$$

or in vector-matrix form

$$\begin{aligned} \begin{pmatrix} p^{\xi} \\ p^{\eta} \end{pmatrix} &= \begin{pmatrix} \xi_x & \xi_y \\ \eta_x & \eta_y \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} \\ \begin{pmatrix} p \\ q \end{pmatrix} &= \frac{1}{J} \begin{pmatrix} \eta_y & -\xi_y \\ -\eta_x & \xi_x \end{pmatrix} \begin{pmatrix} p^{\xi} \\ p^{\eta} \end{pmatrix} \end{aligned} \tag{26}$$

We substitute the elements of the isoparametric transformation into the main equation, and after simple algebraic transformations we obtain the following vector-matrix equation in curvilinear coordinates [18].

$$\frac{\partial V}{\partial t} + A^{\xi\eta}(V)\frac{\partial V}{\partial \xi} + B^{\xi\eta}(V)\frac{\partial V}{\partial \eta} + D^{\xi\eta}(V) = 0 \tag{27}$$

Next, the initial (11) and boundary conditions (12)-(13) are also translated into curvilinear coordinates $x = x(\xi, \eta), y = y(\xi, \eta)$.

Transforming quadrilaterals. For a quadrilateral with vertices $(x_i, y_i, i = 1, 2, 3, 4)$ we will carry out the transformation elements according to the formulas

$$\begin{aligned} x &= x(\xi, \eta) = x_1 + (x_2 - x_1)\xi + (x_3 - x_1)\eta + A\xi\eta \\ y &= y(\xi, \eta) = y_1 + (y_2 - y_1)\xi + (y_3 - y_1)\eta + B\xi\eta \end{aligned} \tag{28}$$

Where is

$$A = x_4 - x_3 - x_2 + x_1, \qquad B = y_4 - y_3 - y_2 + y_1,$$

$$J(\xi, \eta) = \det \begin{bmatrix} x_2 - x_1 + A\eta & x_3 - x_1 + A\xi \\ y_2 - y_1 + B\eta & y_3 - y_1 + B\xi \end{bmatrix}.$$

Derivatives are determined by the formula

$$\frac{\partial x}{\partial \xi} = x_\xi = x_2 - x_1 + A\eta, \qquad \frac{\partial x}{\partial \eta} = x_\eta = x_3 - x_1 + A\xi$$
$$\frac{\partial y}{\partial \xi} = y_\xi = y_2 - y_1 + B\eta, \qquad \frac{\partial y}{\partial \eta} = y_\eta = y_3 - y_1 + B\xi \tag{29}$$

The inverse transformation for quadrilaterals can be constructed based on formula (1), but during implementation you can do without the explicit form of the inverse transformation. Transforming triangles. For a triangle with vertices $(x_i, y_i, i = 1, 2, 3)$ transformation elements using the bilinear transformation method have the form

$$x = x(\xi, \eta) = x_1 + (x_2 - x_1)\xi + (x_3 - x_1)\eta$$
$$y = y(\xi, \eta) = y_1 + (y_2 - y_1)\xi + (y_3 - y_1)\eta \tag{30}$$

$$J(\xi, \eta) = \det \begin{bmatrix} x_2 - x_1 & x_3 - x_1 \\ y_2 - y_1 & y_3 - y_1 \end{bmatrix}$$
$$= (x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1) \tag{31}$$

$$x_\xi = x_2 - x_1, \quad x_\eta = x_3 - x_1$$
$$y_\xi = y_2 - y_1, \quad y_\eta = y_3 - y_1 \tag{32}$$

The inverse transformation to (2) has the form

$$\xi = \frac{(y_3 - y_1)(x - x_1) - (x_3 - x_1)(y - y_1)}{(x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)}$$
$$\eta = \frac{(y_2 - y_1)(x - x_1) - (x_2 - x_1)(y - y_1)}{(y_2 - y_1)(x_3 - x_1) - (x_2 - x_1)(y_3 - y_1)} \tag{33}$$

Numerical solution method. As an algorithm for the numerical solution of equation (26) with boundary conditions, we use the finite element method based on the Galerkin-Petrov scheme [6, 7]:

1. Domain of definition ? variables is divided into N finite subareas

$\Omega_i (i = 1, 2, ..., N)$ (for example, irregular triangles and quadrilaterals having areas of the same order) so that $U_{i=1}^{N} \Omega_i, \Omega i \cap \Omega_j = 0$ at i?j and go to isoparametric coordinates using the bilinear transformation; 2. Choosing bases $\varphi_i(\xi, \eta)$ and $\psi_i(\xi, \eta)$ - for sub-areas ?i.; 3. The bases for the elements are selected from the approximation conditions, for example, for a triangular element - linear approximation; 4. Second basis $\psi_i(\xi, \eta)$ we choose as the characteristic function of the region Ωi:

$$\psi_i(\xi, \eta) = \begin{cases} 1, & (x, y) \in \Omega_i \\ 0, & (x, y) \notin \Omega_i \end{cases} \tag{34}$$

5. An approximate solution V(?,?, t) is sought in the form

$$V^j(\xi,\eta,t)=\sum_{i=1}^{n_i^j}Q_i^j(t)\phi_i^j(\xi,\eta) \tag{35}$$

Where is

$$Q_i^j(t)=\begin{pmatrix} h_{i,1}^j(t) & \cdots & h_{i,n_i^j}^j(t) \\ p_{i,1}^{\xi j}(t) & \cdots & p_{i,n_i^j}^{\xi j}(t) \\ p_{i,1}^{\eta j}(t) & \cdots & p_{i,n_i^j}^{\eta j}(t) \end{pmatrix}$$

vector-matrix of unknown coefficients and number of functions in approximations of elements of the domain of definition of variables and $\varphi_i(\xi,\eta)$ – linear or quadratic basis functions on a triangle and quadrilateral [3,4].

6. To determine the coefficients $Q_i^j(t)$ at internal points of the domain of definition of variables, a system of equations is used

$$\left(\frac{\partial V^j}{\partial t}+A^{\xi\eta}(V^j)\frac{\partial V^j}{\partial \xi}+B^{\xi\eta}(V^j)\frac{\partial V^j}{\partial \eta}+D^{\xi\eta}(V^j),\psi_j\right), \qquad j=1,2,\ldots,N \tag{36}$$

or

$$\iint_{\Omega_j}\left(\frac{\partial V^j}{\partial t}+A^{\xi\eta}(V^j)\frac{\partial V^j}{\partial \xi}+B^{\xi\eta}(V^j)\frac{\partial V^j}{\partial \eta}+D^{\xi\eta}(V^j)\right)\partial\omega_j=0, \qquad j=1,\ldots,N_j \tag{37}$$

Where is scalar product (u,v). 7. Based on the given boundary conditions (12) – (13), additional equations are compiled for unknown boundary points;

8. Substituting solutions into the equation and calculating the integrals in (41), we obtain a matrix system of nonlinear ordinary differential equations at each node of the element, using the quasi-linearization method, we obtain

$$\frac{dQ_i^j}{dt}+G_i^jF_i^j(Q_i^j)=U_i^j \tag{38}$$

$$j=1,\ldots,N_j$$

Where is G^j and U^j - vectors of coefficients obtained as a result of numerical integration.

9. Solving the resulting system of matrix differential equations based on the finite-difference method, we obtain the functions $Q_i^j(t)$ for all grid nodes, then passing to the main variables using the inverse isoparametric transformation, we obtain the final solution [8].

10. Next, points 6-9 of the algorithm are repeated cyclically for subsequent time steps[8].

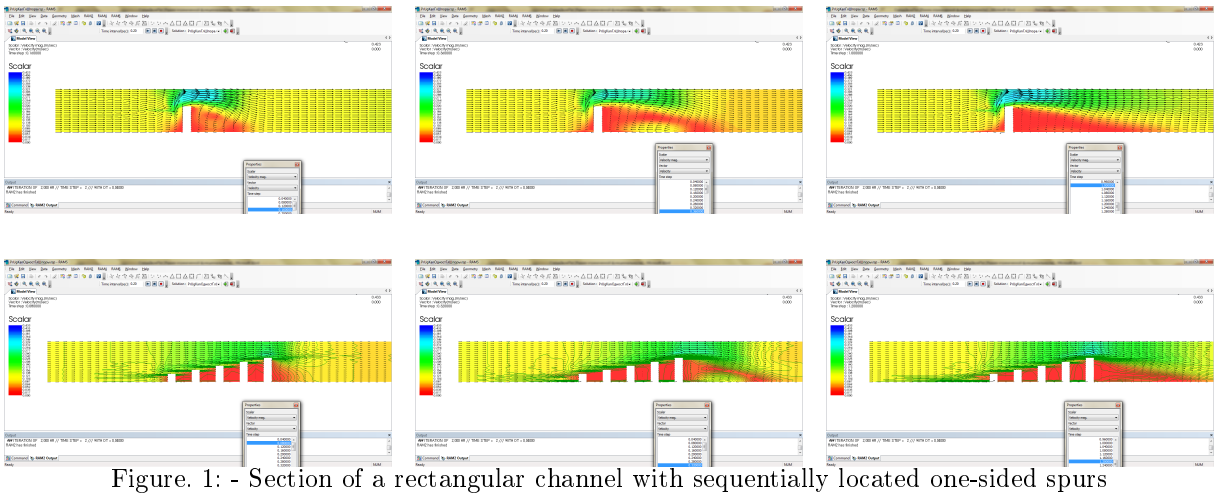

Figure. 1: - Section of a rectangular channel with sequentially located one-sided spurs

The above algorithm is implemented in the form of a software package and modeling of unsteady water movement in open channels is carried out using model examples [8].

4 RESULTS A rectangular section of the canal bed (Fig. 1) with a spur on the right side along the flow. The parameters of the rectangular channel are 50.0 m in width and 1000.0 m in length. The channel is divided into 1960 quadrangular elements, the approximate area of each quadrangle is 25 square meters. m, channel slope 0.00001, bottom mark at the end of the channel was 4.0 m.

The initial conditions were assumed to be equal to zero along the entire channel and the water surface elevation at the end of the channel was equal to 5.0 m, i.e. there was standing water in the riverbed. At the solid upper and lower boundaries, the conditions for zero water flow normal to the canal side are accepted.

As boundary conditions, a liquid boundary with a constant water flow rate of 6 m3/s, was adopted on the left side of the rectangular channel, and a constant water horizon of 5.0 m was selected on the right side of the section.

From the moment the numerical experiment began, water began to flow from the right boundary evenly across its width at a water flow rate of 6 м3/s and changes in the parameters of the water flow began. In Fig. 1 a, b and c show velocity diagrams at various times after the start of the process 120 seconds later. On the left side of the figures are scales for water flow rates. The figures show how the velocity diagrams of the water flow along the length and width change over time and how the whirlpool zone behind the spur changes, which changes along the length over time.

A rectangular section of the channel (Fig. 2) with successively located one-sided spurs on the right side along the flow. The parameters of the rectangular channel, initial, boundary and other conditions are similar to Example 1. In Fig. 2 a, b and c show velocity diagrams at various times after the start of the process 120 seconds later. On the left side of the figures are scales for water flow rates. The figures show how the diagrams of water flow velocities along the length and width change over time. The whirlpool zone behind the spur is almost absent in this case [13-15].

A zigzag rectangular river bed (Fig. 3) 30 m wide and 400 m long. The bed is divided into 600 quadrangular elements, the approximate area of each quadrangle is 15 to 30 square meters. m, channel slope 0.0001, bottom mark at the end of the channel was 5.0

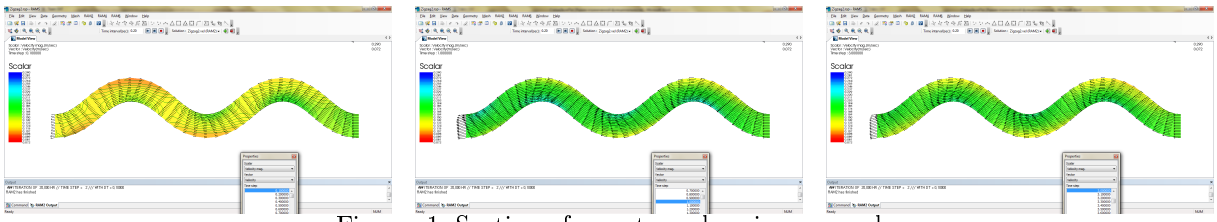

Figure. 1: Section of a rectangular zigzag canal

m.

The initial conditions along the entire channel are such that the water flow is zero and the elevation at the end of the section is 6.0 m. Changes in water flow are specified at the right liquid boundary 10 м3/s, and on the left liquid boundary there is a change in the elevation of the free water surface equal to 6.0 m.

From the moment the numerical experiment began, water began to flow from the right boundary evenly across the width at the rate of water flow 10 m3/s and changes in water flow parameters began. In Fig. 1 a, b and c show velocity diagrams at various times after the start of the process 100 seconds later. On the left side of the figures are scales for water flow rates. The figures show how the velocity diagrams of the water flow along the length and width change over time and how the maximum values of velocities occur at the bends of the zigzag channel, and at the humps of the zigzag channel they have minimum velocities [16-17].

5 CONCLUSION Thus, based on modeling the planned unsteady movement of water in open channels using the algorithm and software package described above, it is possible to determine changes in the main parameters of water flow in space and time and the associated dynamics of transient processes of water resources.

The development of a modeling system based on finite element methods makes it possible to evaluate qualitative and quantitative changes in water flow parameters along the length and width of the channel, and also makes it possible to determine the design parameters of protective, regulating water flow structures and improve the operational parameters of existing hydraulic structures.

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Approximate solution of the I kind Fredholm singular integral equations

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DOI 10.5281/zenodo.18217429

Keywords: Optimal quadrature formulas, the extremal function, Sobolev space, optimal coefficients, Hadamard type singular integral.

Problems are frequently solved using differential and integral equations in fields such as mechanics, hydrodynamics, aerodynamics, electrodynamics, quantum mechanics, and

the theory of elasticity, as well as various other areas of technology. These equations are studied in the course of mathematical physics in science. It is well known that the main boundary value problems for the Laplace equation, namely the Dirichlet and Neumann problems, are solved using the potentials of simple and double layers. These boundary value problems lead to singular integral equations with Cauchy kernel, Hilbert kernel and singularity of a higher order. Analytical solutions of singular integral equations are represented by singular integrals [1-3]. The antiderivative function of the singular integral in the analytical solution can be found in some cases. Obtaining approximate methods for calculating such singular integrals with high accuracy is one of the important tasks of computational mathematics.

In the work [4] is considered a singular integral equation

$$\frac{1}{\pi}\int\limits_{-1}^{1}\frac{\phi(x)}{x-t}\mathtt{d}x=\varphi(t),\quad t\in(-1,1). \tag{1}$$

Let us recall the definition of singular integrals in the sense of principal value of Cauchy.

The principal Cauchy value of the special integral $\int\limits_a^b \frac{\phi(x)}{x-t}\mathtt{d}x,\ a<t<b$ is the limit

$$\lim_{\varepsilon\to 0}\left[\int\limits_a^{t-\varepsilon}\frac{\phi(x)}{x-t}\mathtt{d}x+\int\limits_{t+\varepsilon}^{b}\frac{\phi(x)}{x-t}\mathtt{d}x\right],$$

if it exists.

It is known that if a function ϕ on the interval $[a,b]$ satisfies the Hölder condition with exponent α $(0<\alpha\leq 1)$ and coefficient A, i.e. if

$$|\phi(x_1)-\phi(x_2)|\leq A|x_1-x_2|^{\alpha},$$

then there exists the integral $\int\limits_a^b \frac{\phi(x)}{x-t}\mathtt{d}x,\ a<t<b.$

Equation (1) has four distinct solutions expressed as follows:

Case I: The function $\phi(t)$ is bounded within a certain range at the two endpoints of t, which are $t=-1$ and $t=1$, then a solution of (1) is

$$\phi_1(t)=-\frac{\sqrt{1-t^2}}{\pi}\int\limits_{-1}^{1}\frac{\varphi(x)}{\sqrt{1-x^2}(x-t)}\mathtt{d}x, \tag{2}$$

provided that

$$\int\limits_{-1}^{1}\frac{\varphi(x)}{\sqrt{1-x^2}}\mathtt{d}x=0.$$

Case II: The function $\phi(t)$ is bounded at the end $t=1$, but unbounded at the end $t=-1$, then a solution of (1) is

$$\phi_2(t) = -\frac{1}{\pi}\sqrt{\frac{1-t}{1+t}}\int\limits_{-1}^{1}\sqrt{\frac{1+x}{1-x}}\frac{\varphi(x)}{x-t}\mathtt{d}x, \tag{3}$$

Case III: The function $\phi(t)$ is bounded at the end $t=-1$, but unbounded at the end $t=1$, then

$$\phi_3(t) = -\frac{1}{\pi}\sqrt{\frac{1+t}{1-t}}\int\limits_{-1}^{1}\sqrt{\frac{1-x}{1+x}}\frac{\varphi(x)}{x-t}\mathtt{d}x, \tag{4}$$

Case IV: The function $\phi(t)$ is unbounded at both the endpoints $t=\pm 1$,

$$\phi_4(t) = -\frac{1}{\pi\sqrt{1-t^2}}\int\limits_{-1}^{1}\frac{\sqrt{1-x^2}\varphi(x)}{x-t}\mathtt{d}x + \frac{C}{\sqrt{1-t^2}}, \tag{5}$$

where

$$\int\limits_{-1}^{1}\phi(t)\mathtt{d}x = C.$$

For this purpose, the approximate calculation of singular integrals (2)-(5) is sufficient.

Several methods have been developed for the calculation of the singular integrals (2)-(5). They are: Discrete vortex method, Interpolation methods, Piecewise interpolation method and other numerical quadrature methods. We consider the following quadrature formula

$$\int\limits_{-1}^{1}\frac{\omega_i(x)\varphi(x)}{x-t}\mathtt{d}x \cong \sum_{\alpha=0}^{m-1}\sum_{\beta=0}^{N}C_{\alpha,i}[\beta]\varphi^{(\alpha)}(x_\beta - 1), \quad i=1,2,3,4, \tag{6}$$

where

$$\omega_1(x) = \frac{1}{\sqrt{1-x^2}}, \quad \omega_2(x) = \sqrt{\frac{1+x}{1-x}}, \quad \omega_3(x) = \sqrt{\frac{1-x}{1+x}}, \quad \omega_4(x) = \sqrt{1-x^2},$$

with the error functional

$$\ell_i(x) = \frac{\omega_i(x)\varepsilon_{[-1,1]}(x)}{x-t} - \sum_{\alpha=0}^{m-1}\sum_{\beta=0}^{N}(-1)^\alpha C_{\alpha,i}[\beta]\delta^{(\alpha)}(x-x_\beta+1), \ i=1,2,3,4, \tag{7}$$

where $-1<t<1$, $C_{\alpha,i}[\beta]$ are the coefficients, $x_\beta - 1$ ($\in[-1,1]$) are the nodes, N is a natural number, $\varepsilon_{[-1,1]}(x)$ is the characteristic function of the interval $[-1,1]$, δ is the Dirac delta function, φ is a function of the space $L_2^{(m)}(-1,1)$. Here $L_2^{(m)}(-1,1)$ is the Sobolev space of functions with a square integrable mth generalized derivative and equipped with the norm

$$\|\varphi|L_2^{(m)}(-1,1)\| = \left\{\int\limits_{-1}^{1}(\varphi^{(m)}(x))^2\mathtt{d}x\right\}^{1/2}$$

and $\int\limits_{-1}^{1}(\varphi^{(m)}(x))^2\mathtt{d}x<\infty$.

Since the functional ℓ of the form (7) is defined on the space $L_2^{(m)}(-1,1)$ it is necessary to impose the following conditions (see [5])

$$(\ell_i, x^\alpha)=0, \quad \alpha=0,1,2,...,m-1, \quad i=1,2,3,4. \tag{8}$$

The following theorem is valid.

Theorem 1. The optimal coefficients for the quadrature formulas of the form (6) in the Sobolev space $L_2^{(m)}(-1,1)$ are defined as follows

$$\mathring{C}_{k-1,i}[0]=h^{-1}\left[F_{k-1,i}[1]-F_{k-1,i}[0]+\frac{h}{2}\left(\frac{g_{k-1,i}}{(k-1)!}-\nu_{k-2,i}\right)\right],$$

$$\mathring{C}_{k-1,i}[\beta]=h^{-1}\Big[F_{k-1,i}[\beta-1]-2F_{k-1,i}[\beta]+F_{k-1,i}[\beta+1]\Big], \text{for} \quad \beta=1,...,N-1$$

$$\mathring{C}_{k-1,i}[N]=h^{-1}\left[F_{k-1,i}[N-1]-F_{k-1,i}[N]+\frac{h}{2}\left(\frac{g_{k-1,i}}{(k-1)!}-\nu_{k-2,i}\right)\right],$$

$k=0,1,2,...,m-1$, where

$$\nu_{k-2,i}=\sum_{\alpha=0}^{k-2}\sum_{\gamma=0}^{N}C_{\alpha,i}[\gamma]\frac{(h\gamma)^{k-\alpha-1}}{(k-\alpha-1)!},$$

$F_{k-1,i}$ and $g_{k-1,i}$ are known.

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Optimal quadrature formulas for calculating Fourier coefficients in $W_2^{(m,0)}(0,1)$ space

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DOI 10.5281/zenodo.18217438

Keywords: Hilbert space, error functional, optimal quadrature formula, Fourier coefficients.

Statement of the problem

We consider the following quadrature formula

$$\int_0^1 e^{2\pi i\omega x}\varphi(x)dx \cong \sum_{\beta=0}^{N} C_\beta \varphi(x_\beta), \tag{1}$$

where C_β are coefficients, x_β are nodal points, $i^2=-1$ and $\omega \in \mathbb{Z}\backslash\{0\}$. The functions $\varphi(x)$ belong to the Hilbert space $W_2^{(m,0)}(0,1)$, which is defined as

$$W_2^{(m,0)}(0,1) = \{\varphi(x) : [0,1] \to \mathbb{R}, \varphi^{(m-1)}(x) \text{ is absolutely continuous}, \varphi^{(m)}(x) \in L_2(0,1)\}$$

is the Hilbert space of complex-valued functions, m is odd and in this space the inner product of two functions φ and ψ is defined by the equality

$$<\varphi,\psi>=\int_0^1 \left(\varphi^{(m)}(x)+\varphi(x)\right)\left(\overline{\psi^{(m)}}(x)+\overline{\psi}(x)\right)dx, \tag{2}$$

where $\overline{\psi}$ is the conjugate function of the function ψ, and the norm of the function φ is accordingly determined by the formula

$$\|\varphi\| =<\varphi,\varphi>^{1/2}.$$

and

$$<\varphi,\varphi>^{1/2}=\int_0^1 \left(\varphi^{(m)}(x)+\varphi(x)\right)\left(\overline{\varphi^{(m)}}(x)+\overline{\varphi}(x)\right)dx<\infty.$$

Note that the coefficients C_β depend on ω, N and m, i.e. $C_\beta = C_\beta(\omega, N, m)$.

The difference between the integral and the quadrature sum in the considered quadrature formula (1)

$$\int_0^1 e^{2\pi i\omega x}\varphi(x)dx - \sum_{\beta=0}^{N} C_\beta\varphi(x_\beta) = \int_{-\infty}^{\infty} (\varepsilon_{[0,1]}(x)e^{2\pi i\omega x} - \sum_{\beta=0}^{N} C_\beta\delta(x-x_\beta))\varphi(x)dx =$$

$$= \int_{-\infty}^{\infty} \ell(x)\varphi(x)dx = (\ell,\varphi) \tag{3}$$

is called the *error* of the quadrature formula.

For the derived expression (3), the following *error functional* corresponding to the conjugate space $W_2^{(m,0)^*}(0,1)$ is obtained

$$\ell(x) = \varepsilon_{[0,1]}(x)e^{2\pi i\omega x} - \sum_{\beta=0}^{N} C_\beta\delta(x-x_\beta), \tag{4}$$

where $\varepsilon_{[0,1]}(x)$ is the characteristic function of the interval $[0,1]$, and $\delta(x)$ is Dirac's delta-function.

According to the definition of the norm of a linear continuous functional, the following is known:

$$\|\ell\|_{W_2^{(m,0)^*}} = \sup_{\|\varphi\|\neq 0} \frac{|(\ell,\varphi)|}{\|\varphi\|}.$$

From this equality, the Cauchy-Schwarz inequality follows

$$|(\ell,\varphi)| \le \|\ell\|_{W_2^{(m,0)^*}} \|\varphi\|_{W_2^{(m,0)}}.$$

To construct an optimal quadrature formula of the form (1) in the space $W_2^{(m,0)}(0,1)$, we need to find the coefficients C_β that satisfy the following equality

$$\|\mathring{\ell}\|_{W_2^{(m,0)^*}} = \inf_{C_\beta} \|\ell\|_{W_2^{(m,0)^*}(0,1)} \tag{5}$$

Thus, in this work we will sequentially solve the following problems.

Problem 1. Find the form of the norm of the error functional (4) of the quadrature formula (1) in the $W_2^{(m,0)}(0,1)$ space.

For this, we construct a quadrature formula at nodal points x_β where $x_\beta = h\beta$ ($h = \frac{1}{N}, N \in \mathbb{N}$) are equidistantly distributed points.

The coefficients C_β that minimize the norm of the error functional $\ell(x)$, that is are called *optimal coefficients* and are denoted by $\mathring{C}_\beta$. The quadrature formula constructed using these optimal coefficients is called an *optimal quadrature formula*. Therefore, to construct an optimal quadrature formula in the $W_2^{(m,0)}(0,1)$ space, it is necessary to solve the following problem.

Problem 2. Find the optimal coefficients $\mathring{C}_\beta$ that achieve the value in (5).

Finding the extremal function and expressing the norm of the error functional

To solve problem 1, that is, to find the form of the norm of the error functional (4), we use the concept of an extremal function.

The function $\psi_\ell(x)$ is called an *extremal function* corresponding to the error functional $\ell(x)$ if the following equality holds [1,2,3]

$$(\ell, \psi_\ell) = \|\ell\|_{W_2^{(m,0)*}} \cdot \|\psi_\ell\|_{W_2^{(m,0)}}. \tag{6}$$

To obtain an explicit expression for the norm of the error functional (4) in the $W_2^{(m,0)*}(0,1)$ space we use the concept of the extremal function ψ_ℓ. Since $W_2^{(m,0)}(0,1)$ is a Hilbert space, the extremal function ψ_ℓ in this space can be found using Riesz's theorem. Then for the functional ℓ and for any $\varphi \in W_2^{(m,0)}(0,1)$ there exists a unique function $\psi_\ell \in W_2^{(m,0)}(0,1)$ for which the following equation holds

$$(\ell, \varphi) =< \psi_\ell, \varphi >_{W_2^{(m,0)}} \tag{7}$$

where

$$< \psi_\ell, \varphi >_{W_2^{(m,0)}} = \int_0^1 \left(\overline{\psi_\ell^{(m)}}(x) + \overline{\psi_\ell}(x)\right)\left(\varphi^{(m)}(x) + \varphi(x)\right) dx, \tag{8}$$

is an inner product defined in the $W_2^{(m,0)}(0,1)$ space. From (7), taking into account (8), for the extremal function ψ_ℓ we obtain the following boundary value problem

$$\psi_\ell^{(2m)}(x) - \psi_\ell(x) = (-1)^m \overline{\ell}(x) \tag{9}$$

with the boundary conditions

$$\left(\psi_\ell^{(m+s)}(x) + \psi_\ell^{(s)}(x)\right)_{x=0}^{x=1} = 0, \quad s = 0, 1, ..., m-1, \tag{10}$$

where $\overline{\ell}$ is the adjoint functional to the functional ℓ.

Theorem. The solution of equation (9) with boundary conditions (10) is an extremal function ψ_ℓ of the error functional ℓ of quadrature formula (1) and has the form

$$\psi_\ell(x) = (-1)^m \overline{\ell}(x) * G_m(x) + d_0 e^{-x} + \sum_{k=1}^{\frac{m-1}{2}} e^{-x\cos\frac{2\pi k}{m}} \left[d_{1k}\cos\left(x\sin\frac{2\pi k}{m}\right) + d_{2k}\sin\left(x\sin\frac{2\pi k}{m}\right)\right], \tag{11}$$

where

$$G_m(x) = \frac{\operatorname{sgn} x}{2m}\left[\sinh(x) + \sum_{k=1}^{m-1} e^{x\cos\frac{\pi k}{m}}\cos\left(x\sin\frac{\pi k}{m} + \frac{\pi k}{m}\right)\right] \tag{12}$$

is a solution to the equation

$$G_m^{(2m)} - G_m(x) = \delta(x), \tag{13}$$

d_0, d_{1k} and d_{2k} are any complex numbers, $*$ is convolution operation.

The theorem can be proved in the same way as Theorem 2.1 in [4].

Since the error functional (4) is defined on the space $W_2^{(m,0)}(0,1)$, it satisfies the following conditions:

$$\left(\ell, e^{-x}\right) = 0, \left(\ell, e^{-x\cos\frac{2\pi k}{m}} \cos\left(x\sin\frac{2\pi k}{m}\right)\right) = 0,$$

$$\left(\ell, e^{-x\cos\frac{2\pi k}{m}} \sin\left(x\sin\frac{2\pi k}{m}\right)\right) = 0, k = \overline{1, \frac{m-1}{2}}. \tag{14}$$

From this it is clear that for quadrature formulas of the form (1) to exist, the condition $N+1 \geq m$ must be satisfied. Equalities (14) mean that our quadrature formula is exact for the functions $e^{-x\cos\frac{2\pi k}{m}} \cos\left(x\sin\frac{2\pi k}{m}\right)$, $e^{-x\cos\frac{2\pi k}{m}} \sin\left(x\sin\frac{2\pi k}{m}\right)$ $\left(k = \overline{1, \frac{m-1}{2}}\right)$ and e^{-x}.

Now, using Theorem we obtain a representation of the squared norm of the error functional (4). Taking into account the definition of convolution and equality (4), we calculate the convolution $\overline{\ell}(x) * G_m(x)$, i.e.

$$\overline{\ell}(x) * G_m(x) = \int\limits_{-\infty}^{\infty} \overline{\ell}(y) G_m(x-y)dy = \int\limits_{0}^{1} e^{-2\pi i\omega y} G_m(x-y)dy - \sum_{\beta 0}^{N} \overline{C}_\beta G_m(x-h\beta),$$

where $\overline{\ell}$ and $\overline{C}_\beta$ are conjugate to ℓ and C_β respectively.

Then, keeping in mind (7), (8) and the theorem, we obtain

$$\|\ell\|^2 = (\ell, \psi_\ell) = <\psi_\ell, \psi_\ell>_{W_2^{(m,0)}} = \int\limits_{-\infty}^{\infty} \ell(x)\psi_\ell(x)dx = (-1)^m \int\limits_{-\infty}^{\infty} \ell(x) \cdot \left(\overline{\ell}(x) * G_m(x)\right) dx,$$

i.e.

$$\|\ell\|^2 = (-1)^m \int\limits_{-\infty}^{\infty} \left(e^{2\pi i\omega x}\varepsilon_{[0,1]}(x) - \sum_{\beta=0}^{N} C_\beta(x-h\beta)\right) \cdot$$

$$\cdot \left(\int\limits_{0}^{1} e^{-2\pi i\omega y} G_m(x-y)dy - \sum_{\beta=0}^{N} \overline{C}_\gamma G_m(x-h\gamma)\right) dx.$$

From here we get

$$\|\ell\|^2 = (-1)^m \left[\sum_{\beta=0}^{N}\sum_{\gamma=0}^{N} C_\beta\overline{C}_\gamma G_m(h\beta - h\gamma) - \sum_{\beta=0}^{N}\int\limits_{0}^{1} \left(\overline{C}_\beta e^{2\pi i\omega x} + C_\beta e^{-2\pi i\omega x}\right) G_m(x-h\beta)dx\right.$$

$$\left. + \int\limits_{0}^{1}\int\limits_{0}^{1} e^{2\pi i\omega x} e^{-2\pi i\omega y} G_m(x-y)dxdy\right]. \tag{15}$$

Let us now show that the right side of (15) is real. Indeed, let $C_\beta = C_\beta^R + iC_\beta^I$, where $i^2 = -1$, C_β^R and C_β^I are real. Using Euler's formula $e^{2\pi i\omega x} = \cos(2\pi\omega x) + i\sin(2\pi\omega x)$, we

obtain the following equalities

$$\sum_{\beta=0}^{N}\sum_{\gamma=0}^{N} C_\beta \overline{C}_\gamma G_m(h\beta - h\gamma) = \sum_{\beta=0}^{N}\sum_{\gamma=0}^{N} \left(C_\beta^R C_\gamma^R + C_\beta^I C_\gamma^I\right) G_m(h\beta - h\gamma),$$

$$\overline{C}_\beta e^{2\pi i\omega x} + C_\beta e^{-2\pi i\omega x} = 2C_\beta^R \cos(2\pi\omega x) + 2C_\beta^I \sin(2\pi\omega x),$$

$$\int_0^1\int_0^1 e^{2\pi i\omega x} e^{-2\pi i\omega y} G_m(x-y)dxdy = \int_0^1\int_0^1 \cos\left[2\pi\omega(x-y)\right] G_m(x-y)dxdy.$$

Taking into account the last three equalities, from (15) for the norm of the error functional we have:

$$\|\ell\|^2 = (-1)^m \left[\sum_{\beta=0}^{N}\sum_{\gamma=0}^{N} \left(C_\beta^R C_\gamma^R + C_\beta^I C_\gamma^I\right) G_m(h\beta - h\gamma) - 2\sum_{\beta=0}^{N} C_\beta^R \int_0^1 \cos(2\pi\omega x)\, G_m(x - h\beta)dx\right.$$

$$\left. -2\sum_{\beta=0}^{N} C_\beta^I \int_0^1 \sin(2\pi\omega x)\, G_m(x - h\beta)dx + \int_0^1\int_0^1 \cos\left[2\pi\omega(x-y)\right] G_m(x-y)dxdy\right] \quad (16)$$

and from (14) we have the following equalities

$$\sum_{\beta=0}^{N} C_\beta^R e^{-h\beta} = \int_0^1 e^{-x}\cos(2\pi\omega x)\, dx, \quad (17)$$

$$\sum_{\beta=0}^{N} C_\beta^R e^{-h\beta\cos\frac{2\pi k}{m}} \cos\left(h\beta\sin\frac{2\pi k}{m}\right) = \int_0^1 e^{-x\cos\frac{2\pi k}{m}} \cos\left(x\sin\frac{2\pi k}{m}\right)\cos(2\pi\omega x)\, dx, \quad (18)$$

$$\sum_{\beta=0}^{N} C_\beta^R e^{-h\beta\cos\frac{2\pi k}{m}} \sin\left(h\beta\sin\frac{2\pi k}{m}\right) = \int_0^1 e^{-x\cos\frac{2\pi k}{m}} \sin\left(x\sin\frac{2\pi k}{m}\right)\cos(2\pi\omega x)\, dx, \quad (19)$$

$$\sum_{\beta=0}^{N} C_\beta^I e^{-h\beta} = \int_0^1 e^{-x}\sin(2\pi\omega x)\, dx, \quad (20)$$

$$\sum_{\beta=0}^{N} C_\beta^I e^{-h\beta\cos\frac{2\pi k}{m}} \cos\left(h\beta\sin\frac{2\pi k}{m}\right) = \int_0^1 e^{-x\cos\frac{2\pi k}{m}} \cos\left(x\sin\frac{2\pi k}{m}\right)\sin(2\pi\omega x)\, dx, \quad (21)$$

$$\sum_{\beta=0}^{N} C_\beta^I e^{-h\beta\cos\frac{2\pi k}{m}} \sin\left(h\beta\sin\frac{2\pi k}{m}\right) = \int_0^1 e^{-x\cos\frac{2\pi k}{m}} \sin\left(x\sin\frac{2\pi k}{m}\right)\sin(2\pi\omega x)\, dx. \quad (22)$$

Thus, problem 1 is solved. Next, we will solve problem 2.

System for optimal coefficients of quadrature formulas

The square of the norm (16) of the error functional (4) is a multivariable square function of the coefficients C_β^R and C_β^I. To find the minimum of the norm (16) of the error functional (4) with respect to the coefficients C_β^R and C_β^I under conditions (17)вЂ“(22), we apply Lagrange's method for finding a conditional extremum.

Let's consider the function

$$\Psi\left(C_0^R,...,C_N^R,C_0^I,...,C_N^I,d_0^R,d_0^I,d_{11}^R,...,d_{1\frac{m-1}{2}}^R,d_{11}^I,...,d_{1\frac{m-1}{2}}^I,d_{21}^R,...,d_{2\frac{m-1}{2}}^R,d_{21}^I,...,d_{2\frac{m-1}{2}}^I\right)=\|\ell\|^2$$

$$-2(-1)^m d_0^R\left(\int_0^1 e^{-x}\cos(2\pi\omega x)\,dx-\sum_{\beta=0}^{N}C_\beta^R e^{-h\beta}\right)-$$

$$-2(-1)^m d_0^I\left(\int_0^1 e^{-x}\sin(2\pi\omega x)\,dx-\sum_{\beta=0}^{N}C_\beta^I e^{-h\beta}\right)$$

$$-2(-1)^m d_{1k}^R\left(\int_0^1 e^{-x\cos\frac{2\pi k}{m}}\cos\left(x\sin\frac{2\pi k}{m}\right)\cos(2\pi\omega x)\,dx-\sum_{\beta=0}^{N}C_\beta^R e^{-h\beta\cos\frac{2\pi k}{m}}\cos\left(h\beta\sin\frac{2\pi k}{m}\right)\right)$$

$$-2(-1)^m d_{1k}^I\left(\int_0^1 e^{-x\cos\frac{2\pi k}{m}}\cos\left(x\sin\frac{2\pi k}{m}\right)\sin(2\pi\omega x)\,dx-\sum_{\beta=0}^{N}C_\beta^I e^{-h\beta\cos\frac{2\pi k}{m}}\cos\left(h\beta\sin\frac{2\pi k}{m}\right)\right)$$

$$-2(-1)^m d_{2k}^R\left(\int_0^1 e^{-x\cos\frac{2\pi k}{m}}\sin\left(x\sin\frac{2\pi k}{m}\right)\cos(2\pi\omega x)\,dx-\sum_{\beta=0}^{N}C_\beta^R e^{-h\beta\cos\frac{2\pi k}{m}}\sin\left(h\beta\sin\frac{2\pi k}{m}\right)\right)$$

$$-2(-1)^m d_{2k}^I\left(\int_0^1 e^{-x\cos\frac{2\pi k}{m}}\sin\left(x\sin\frac{2\pi k}{m}\right)\sin(2\pi\omega x)\,dx-\sum_{\beta=0}^{N}C_\beta^I e^{-h\beta\cos\frac{2\pi k}{m}}\sin\left(h\beta\sin\frac{2\pi k}{m}\right)\right).$$

By equating the partial derivatives of the function Ψ with respect to $C_\beta^R, C_\beta^I\left(\beta=\overline{0,N}\right), d_0^R, d_0^I,$
$d_{1k}^R, d_{1k}^I, d_{2k}^R$ and $d_{2k}^I\left(k=\overline{1,\frac{m-1}{2}}\right)$ to zero, we obtain the following system of linear equations for
$\beta=0,1,...,N,$

$$\sum_{\gamma=0}^{N}C_\gamma^R G_m(h\beta-h\gamma)+d_0^R e^{-h\beta}+d_{1k}^R e^{-h\beta\cos\frac{2\pi k}{m}}\cos\left(h\beta\sin\frac{2\pi k}{m}\right)$$

$$+d_{2k}^R e^{-h\beta\cos\frac{2\pi k}{m}}\sin\left(h\beta\sin\frac{2\pi k}{m}\right)=\int_0^1\cos(2\pi\omega x)\,G_m(x-h\beta)dx, \qquad (23)$$

$$\sum_{\gamma=0}^{N}C_\gamma^R e^{-h\gamma}=\int_0^1 e^{-x}\cos(2\pi\omega x)\,dx, \qquad (24)$$

$$\sum_{\gamma=0}^{N} C_{\gamma}^{R} e^{-h\gamma\cos\frac{2\pi k}{m}} \cos\left(h\gamma\sin\frac{2\pi k}{m}\right) = \int_{0}^{1} e^{-x\cos\frac{2\pi k}{m}} \cos\left(x\sin\frac{2\pi k}{m}\right)\cos\left(2\pi\omega x\right)dx, \quad (25)$$

$$\sum_{\gamma=0}^{N} C_{\gamma}^{R} e^{-h\gamma\cos\frac{2\pi k}{m}} \sin\left(h\gamma\sin\frac{2\pi k}{m}\right) = \int_{0}^{1} e^{-x\cos\frac{2\pi k}{m}} \sin\left(x\sin\frac{2\pi k}{m}\right)\cos\left(2\pi\omega x\right)dx, \quad (26)$$

$$\sum_{\gamma=0}^{N} C_{\gamma}^{I} G_m(h\beta - h\gamma) + d_0^I e^{-h\beta} + d_{1k}^I e^{-h\beta\cos\frac{2\pi k}{m}} \cos\left(h\beta\sin\frac{2\pi k}{m}\right)$$

$$+d_{2k}^I e^{-h\beta\cos\frac{2\pi k}{m}} \sin\left(h\beta\sin\frac{2\pi k}{m}\right) = \int_{0}^{1} \sin\left(2\pi\omega x\right) G_m(x - h\beta)dx, \quad (27)$$

$$\sum_{\gamma=0}^{N} C_{\gamma}^{I} e^{-h\gamma} = \int_{0}^{1} e^{-x}\sin\left(2\pi\omega x\right)dx, \quad (28)$$

$$\sum_{\gamma=0}^{N} C_{\gamma}^{I} e^{-h\gamma\cos\frac{2\pi k}{m}} \cos\left(h\gamma\sin\frac{2\pi k}{m}\right) = \int_{0}^{1} e^{-x\cos\frac{2\pi k}{m}} \cos\left(x\sin\frac{2\pi k}{m}\right)\sin\left(2\pi\omega x\right)dx, \quad (29)$$

$$\sum_{\gamma=0}^{N} C_{\gamma}^{I} e^{-h\gamma\cos\frac{2\pi k}{m}} \sin\left(h\gamma\sin\frac{2\pi k}{m}\right) = \int_{0}^{1} e^{-x\cos\frac{2\pi k}{m}} \sin\left(x\sin\frac{2\pi k}{m}\right)\sin\left(2\pi\omega x\right)dx. \quad (30)$$

Now, multiplying both sides of equalities (27) - (30) by i and adding to both sides (23) - (26) respectively, using the notations $C_\beta = C_\beta^R + C_\beta^I, d_0 = d_0^R + id_0^I, d_{1k} = d_{1k}^R + id_{1k}^I$ and $d_{2k} = d_{2k}^R + id_{2k}^I \left(k = \overline{1, \frac{m-1}{2}}\right)$ for the coefficients of optimal quadrature formulas of the form (1), we obtain the following system $N+m+1$ of linear equations and $\beta = 0, 1, ..., N$,

$$\sum_{\gamma=0}^{N} C_{\gamma} G_m(h\beta - h\gamma) + d_0 e^{-h\beta} + d_{1k} e^{-h\beta\cos\frac{2\pi k}{m}} \cos\left(h\beta\sin\frac{2\pi k}{m}\right)$$

$$+d_{2k} e^{-h\beta\cos\frac{2\pi k}{m}} \sin\left(h\beta\sin\frac{2\pi k}{m}\right) = f_m(h\beta), \quad \beta = 0, 1, ..., N, \quad (31)$$

$$\sum_{\gamma=0}^{N} C_{\gamma} e^{-h\gamma} = \int_{0}^{1} e^{2\pi i\omega x} e^{-x} dx, \quad (32)$$

$$\sum_{\gamma=0}^{N} C_{\gamma} e^{-h\gamma\cos\frac{2\pi k}{m}} \cos\left(h\gamma\sin\frac{2\pi k}{m}\right) = \int_{0}^{1} e^{2\pi i\omega x} e^{-x\cos\frac{2\pi k}{m}} \cos\left(x\sin\frac{2\pi k}{m}\right)dx, \quad (33)$$

$$\sum_{\gamma=0}^{N} C_{\gamma} e^{-h\gamma\cos\frac{2\pi k}{m}} \sin\left(h\gamma\sin\frac{2\pi k}{m}\right) = \int_{0}^{1} e^{2\pi i\omega x} e^{-x\cos\frac{2\pi k}{m}} \sin\left(x\sin\frac{2\pi k}{m}\right)dx, \quad (34)$$

where $G_m(x)$ is defined by equality (13),

$$f_m(h\beta) = \int_0^1 e^{2\pi i\omega x} G_m(x - h\beta)dx. \tag{35}$$

It should be noted that system (31) - (34) has a unique solution and this solution yields the minimum of the squared norm (16) of the error functional (4) under conditions (32) - (34). The uniqueness of the solution to this system is obtained in [5].

From (16) and from the work [5] it follows that the square of the norm of the error functional ℓ, which is a quadratic function of the coefficients C_β, has a unique minimum at some specific value $C_\beta = \mathring{C}_\beta$. The quadrature formula and the coefficients $\mathring{C}_\beta\,(\beta = 0, 1, ..., N)$, corresponding to this minimum are called the optimal quadrature formula and $\mathring{C}_\beta$ are called the optimal coefficients.

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Optimal quadrature formula for the approximate calculation of a singular integral

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DOI 10.5281/zenodo.18217450

Keywords: Hilbert space, singular integral, error functional, optimal quadrature formula, singular integral equations, Hilbert kernel.

Statement of the problem

We consider the following quadrature formula

$$\int_0^{2\pi} \varphi(x) \cot \frac{x-t}{2} dx \cong \sum_{\beta=0}^{N} C_\beta \varphi(x_\beta), \tag{1}$$

where C_β are coefficients, x_β are nodal points, and $x_\beta \in [0, 2\pi]$. The functions $\varphi(x)$ belong to the Hilbert space $L_2^{(3)}(0, 2\pi)$, which is defined as

$$L_2^{(3)}(0,2\pi) = \{\varphi(x) : [0,2\pi] \to \mathbb{R}, \varphi''(x) \text{ is absolutely continuous}, \varphi'''(x) \in L_2(0,2\pi)\}.$$

In this $L_2^{(3)}(0, 2\pi)$ space, the inner product of functions φ and ψ is defined as follows

$$< \varphi, \psi >= \int_0^{2\pi} \varphi'''(x)\psi'''(x)dx, \tag{2}$$

and the norm of the function φ in this space is calculated using the inner product (**??**) as follows

$$\|\varphi\| = \sqrt{\int_0^{2\pi} \Big(\varphi'''(x)\Big)^2 dx}. \tag{3}$$

The difference between the integral and the quadrature sum in the considered quadrature formula (1)

$$\int_0^{2\pi} \varphi(x) \cot \frac{x-t}{2} dx - \sum_{\beta=0}^{N} C_\beta \varphi(x_\beta) = \int_{-\infty}^{\infty} (\varepsilon_{[0,2\pi]}(x) \cot \frac{x-t}{2} - \sum_{\beta=0}^{N} C_\beta \delta(x - x_\beta))\varphi(x)dx =$$

$$= \int_{-\infty}^{\infty} \ell(x)\varphi(x)dx = (\ell, \varphi) \tag{4}$$

is called the *error* of the quadrature formula.

For the derived expression (4), the following *error functional* corresponding to the conjugate space $L_2^{(3)^*}(0,2\pi)$ is obtained

$$\ell(x) = \varepsilon_{[0,2\pi]}(x)\cot\frac{x-t}{2} - \sum_{\beta=0}^{N} C_\beta \delta(x-x_\beta), \tag{5}$$

where $\varepsilon_{[0,2\pi]}(x)$ is the characteristic function of the interval $[0,2\pi]$, and $\delta(x)$ is Dirac's delta-function.

According to the definition of the norm of a linear continuous functional, the following is known:

$$\|\ell\|_{L_2^{(3)^*}} = \sup_{\|\varphi\|\neq 0} \frac{|(\ell,\varphi)|}{\|\varphi\|}.$$

From this equality, the Cauchy-Schwarz inequality follows

$$|(\ell,\varphi)| \leq \|\ell\|_{L_2^{(3)^*}} \|\varphi\|_{L_2^{(3)}}. \tag{6}$$

According to the Cauchy-Schwarz inequality, the following conditions hold for the norm (3) of the function $\varphi(x)$

$$(\ell, x^\alpha) = 0, \qquad \alpha = 0,1,2. \tag{7}$$

According to the conditions (7), the quadrature formula (1) is exact for the function $d_2x^2 + d_1x + d_0$. These conditions ensure that the error functional is bounded.

Thus, the upper bound of the error (4) of the quadrature formula (1) is estimated using the norm of the error functional $\ell(x)$. From this, the following problem arises.

Problem 1. Find the form of the norm of the error functional $\ell(x)$ belonging to the conjugate space $L_2^{(3)^*}(0,2\pi)$.

In this work, we solve the Sard problem. For this, we construct a quadrature formula at nodal points x_β where $x_\beta = h\beta$ ($h = \frac{2\pi}{N}, N \in \mathbb{N}$) are equidistantly distributed points.

The coefficients C_β that minimize the norm of the error functional $\ell(x)$, that is

$$\inf_{C_\beta} \|\ell\|_{L_2^{(3)^*}(0,2\pi)} \tag{8}$$

are called *optimal coefficients* and are denoted by $\mathring{C}_\beta$. The quadrature formula constructed using these optimal coefficients is called an *optimal quadrature formula*. Therefore, to construct an optimal quadrature formula in the sense of Sard in the space $L_2^{(3)}(0,2\pi)$, it is necessary to solve the following problem.

Problem 2. Find the optimal coefficients $\mathring{C}_\beta$ that achieve the value in (8).

Finding the extremal function and expressing the norm of the error functional

To solve problem (1), that is, to find the form of the norm of the error functional (5), we use the concept of an extremal function.

The function $\psi_\ell(x)$ is called an *extremal function* corresponding to the error functional $\ell(x)$ if the following equality holds [1]

$$(\ell, \psi_\ell) = \|\ell\|_{L_2^{(3)*}} \cdot \|\psi_\ell\|_{L_2^{(3)}}. \tag{9}$$

The concept of an extremal function was first introduced by S.L. Sobolev and is defined as follows

$$\psi_\ell = -\ell(x) * G_3(x) + P_2(x),$$

where $G_3(x) = \frac{|x|^5}{2 \cdot 5!}$, and $P_2(x)$ is a second-degree polynomial.

We note that $*$ is the convolution and the convolution of two functions

$$\varphi(x) * \psi(x) = \int\limits_{-\infty}^{\infty} \varphi(x-y)\psi(y)dy = \int\limits_{-\infty}^{\infty} \varphi(y)\psi(x-y)dy$$

is defined by the formula.

Since the space $L_2^3(0, 2\pi)$ we are considering is a Hilbert space, we present the Riesz theorem on the general form of a linear continuous functional.

(Riesz theorem) Let H be a Hilbert space. Then for each linear continuous functional $\ell \in H^*$, there exists a unique function ψ_ℓ in the space H itself, called the Riesz element, which satisfies the condition $(\ell, \varphi) =< \psi_\ell, \varphi >$ corresponding to this functional, and the following equality holds for this function

$$\|\ell\|_{H^*} = \|\psi_\ell\|_H$$

and conversely.

For any $\psi_\ell \in H$, there exists a unique functional $\ell \in H^*$ that satisfies the conditions $(\ell, \varphi) =< \psi_\ell, \varphi >$ and $\|\ell\|_{H^*} = \|\psi_\ell\|_H$.

According to the theorem, the extremal function $\psi_\ell(x)$ is a Riesz element and

$$(\ell, \psi_\ell) = \|\ell\|^2 = \|\psi_\ell\|^2.$$

Based on the theorem, we calculate the square of the norm of the error functional

$$\|\ell\|^2 = -\int\limits_0^{2\pi}\int\limits_0^{2\pi} \frac{|x-y|^5}{2 \cdot 5!} \cot\frac{x-t}{2} \cot\frac{y-t}{2} dxdy +$$

$$+2\sum_{\beta=0}^{N} C_\beta \int\limits_0^{2\pi} \frac{|x-h\beta|^5}{2 \cdot 5!} \cot\frac{x-t}{2} dx - \sum_{\beta=0}^{N}\sum_{\gamma=0}^{N} C_\beta C_\gamma \frac{|h\beta - h\gamma|^5}{2 \cdot 5!} \tag{10}$$

Thus, the square of the norm of the error functional $\ell(x)$ has been determined, and problem 1 has been solved.

Finding the conditional minimum of the error functional

Now we will tackle the second problem. The square of the norm of the error functional obtained in (10) is a multi-variable function with respect to the coefficients C_β. According to the rule for finding the conditional extremum of a multi-variable function, we will determine its local minimum in conjunction with the conditions (7). To achieve this, we construct the Lagrange function:

$$\Phi(C_\beta, d_2, d_1, d_0) = \|\ell\|^2 + 2\sum_{\alpha=0}^{2} d_\alpha(\ell, x^\alpha), \qquad \beta = 0, 1, ..., N.$$

According to (7), the following holds true here

$$(\ell, x^\alpha) = \int_0^{2\pi} x^\alpha \cot\frac{x-t}{2}dx - \sum_{\beta=0}^{N} C_\beta(h\beta)^\alpha = 0, \quad \alpha = 0, 1, 2.$$

By setting the partial derivatives of the resulting function Φ with respect to the coefficients C_β and the unknowns d_2, d_1, d_0 to zero, we obtain the following system of linear equations

$$\sum_{\gamma=0}^{N} C_\gamma \frac{|h\beta - h\gamma|^5}{2\cdot 5!} + d_2(h\beta)^2 + d_1(h\beta) + d_0 = f_3(h\beta), \quad \beta = 0, 1, ..., N, \tag{11}$$

$$\sum_{\gamma=0}^{N} C_\gamma(h\gamma)^2 = g_2, \tag{12}$$

$$\sum_{\gamma=0}^{N} C_\gamma(h\gamma) = g_1, \tag{13}$$

$$\sum_{\gamma=0}^{N} C_\gamma = g_0, \tag{14}$$

where

$$f_3(h\beta) = \int_0^{2\pi} \frac{|x-h\beta|^5}{2\cdot 5!}\cot\frac{x-t}{2}dx =$$

$$= \frac{1}{5!}\left[\sum_{k=0}^{\infty}(-1)^k \frac{B_{2k}}{(2k+5)(2k)!}\Big((2\pi-t)^{2k+5} - 2(h\beta-t)^{2k+5} - t^{2k+5}\Big) + \right.$$

$$+5(t-h\beta)\sum_{k=0}^{\infty}(-1)^k\frac{B_{2k}}{(2k+4)(2k)!}\Big((2\pi-t)^{2k+4}-2(h\beta-t)^{2k+4}+t^{2k+4}\Big)+$$

$$+10(t-h\beta)^2\sum_{k=0}^{\infty}(-1)^k\frac{B_{2k}}{(2k+3)(2k)!}\Big((2\pi-t)^{2k+3}-2(h\beta-t)^{2k+3}-t^{2k+3}\Big)+$$

$$+10(t-h\beta)^3\sum_{k=0}^{\infty}(-1)^k\frac{B_{2k}}{(2k+2)(2k)!}\Big((2\pi-t)^{2k+2}-2(h\beta-t)^{2k+2}+t^{2k+2}\Big)+$$

$$+5(t-h\beta)^4\sum_{k=0}^{\infty}(-1)^k\frac{B_{2k}}{(2k+1)(2k)!}\Big((2\pi-t)^{2k+1}-2(h\beta-t)^{2k+1}-t^{2k+1}\Big)+$$

$$+2(t-h\beta)^5\ln\left|\frac{\sin\frac{t}{2}}{\sin\frac{h\beta-t}{2}}\right|\Big],$$

$$g_2=2\sum_{k=0}^{\infty}(-1)^k\frac{B_{2k}}{(2k+2)(2k)!}\Big((2\pi-t)^{2k+2}-t^{2k+2}\Big)+$$

$$+4t\sum_{k=0}^{\infty}(-1)^k\frac{B_{2k}}{(2k+1)(2k)!}\Big((2\pi-t)^{2k+1}+t^{2k+1}\Big),$$

$$g_1=2\sum_{k=0}^{\infty}(-1)^k\frac{B_{2k}}{(2k+1)(2k)!}\Big((2\pi-t)^{2k+1}+t^{2k+1}\Big),$$

$$g_0=\int_0^{2\pi}\cot\frac{x-t}{2}dx=0,$$

here, B_{2k} are the Bernoulli numbers, and the following formula was used [3]

$$\int x^p\cot x dx=\sum_{k=0}^{\infty}(-1)^k\frac{2^{2k}B_{2k}}{(2k+p)(2k)!}x^{2k+p},\qquad[p\geq 1,|x|<\pi].$$

The system (11)-(14) consists of $(N+4)$ unknowns and $(N+4)$ linear equations with respect to $C_\gamma,\gamma=0,1,...,N,d_0,d_1,d_2$. To solve this system, we will employ the functional method proposed by Sobolev. For this method, it is crucial to use a discrete analogue of the differential operator d^6/dx^6.

The discrete analog of the differential operator d^6/dx^6 satisfying the equality $D_3(h\beta)*G_3(h\beta)=\delta(h\beta)$ has the following form [2]

$$D_3(h\beta)=\frac{5!}{h^6}\begin{cases}\sum\limits_{t=1}^{2}K_t\cdot q_t^{|\beta|-1}, & |\beta|\geq 2,\\ 1+\sum\limits_{t=1}^{2}K_t, & |\beta|=1,\\ -2^5+\sum\limits_{t=1}^{2}\frac{K_t}{q_t}, & \beta=0,\end{cases}\tag{15}$$

where $\delta(h\beta)$ is a discrete delta function, $K_1 = \dfrac{(1-q_1)^7}{E_5(q_1)}$, $K_2 = \dfrac{(1-q_2)^7}{E_5(q_2)}$, $E_5(x)$ is a fifth-degree Euler-Frobenius polynomial, and q_1 and q_2 are roots of the Euler-Frobenius polynomial $E_4(q)$ satisfying the inequality $|q_t| < 1, \quad (t = 1, 2)$.

The optimal coefficients of the quadrature formula (1), with nodes equidistantly spaced in the space $L_2^{(3)}(0, 2\pi)$, have the following form

$$C_0 = \frac{5!}{h^5}\Bigg[\frac{g_1h^4}{48} + \frac{g_2h^3}{24} + a_2^-h^2 - a_1^-h - 31\cdot f_3(0) + f_3(1) + \sum_{n=1}^{2}\frac{K_nF_n}{q_n} +$$

$$+\sum_{n=1}^{2}\frac{K_n}{q_n}\left(\frac{D_{n4}g_1h^4}{48} + \frac{D_{n3}g_2h^3}{24} + D_{n2}a_2^-h^2 - D_{n1}a_1^-h + D_{n0}f_3(0)\right) +$$

$$+\sum_{n=1}^{2}K_nq_n^{N-1}\left(-\frac{D_{n4}g_1h^4}{48} + D_{n3}h^3\left(\frac{g_2}{24} - \frac{g_1hN}{12}\right) +\right.$$

$$\left.+D_{n2}h^2\left(\frac{g_2hN}{8} - \frac{g_1h^2N^2}{8} + a_2^+\right) + + D_{n1}h\left(\frac{g_2h^2N^2}{8} - \frac{g_1h^3N^3}{12} + 2a_2^+hN + a_1^+\right) + D_{n0}f_3(N)\right)\Bigg];$$

$$C_\beta = \frac{5!}{h^5}\Bigg[\sum_{n=1}^{2}K_nq_n^{\beta-1}\left(\frac{D_{n4}g_1h^4}{48} + \frac{D_{n3}g_2h^3}{24} + D_{n2}a_2^-h^2 - D_{n1}a_1^-h + D_{n0}f_3(0)\right) +$$

$$+\sum_{\gamma=0}^{N}\sum_{n=1}^{2}K_nq_n^{|\beta-\gamma|-1}f_3(\gamma) + f_3(\beta-1) - 32f_3(\beta) + f_3(\beta+1) +$$

$$+\sum_{n=1}^{2}K_nq_n^{N-\beta-1}\left(-\frac{D_{n4}g_1h^4}{48} +\right.$$

$$+D_{n3}h^3\left(\frac{g_2}{24} - \frac{g_1hN}{12}\right) + D_{n2}h^2\left(\frac{g_2hN}{8} - \frac{g_1h^2N^2}{8} + a_2^+\right) +$$

$$\left.+D_{n1}h\left(\frac{g_2h^2N^2}{8} - \frac{g_1h^3N^3}{12} + 2a_2^+hN + a_1^+\right) + D_{n0}f_3(N)\right)\Bigg], \quad \beta = 1, 2, ..., N-1;$$

$$C_N = \frac{5!}{h^5}\Bigg[-\frac{g_1h^4}{48} + \frac{g_2h^3}{24} - \frac{g_1h^4N}{12} + \frac{g_2h^3N}{8} - \frac{g_1h^4N^2}{8} + \frac{g_2h^3N^2}{8} -$$

$$-\frac{g_1h^4N^3}{12} + 2a_2^+h^2N + a_2^+h^2 + a_1h - 31\cdot f_3(N) + f_3(N-1) +$$

$$+\sum_{n=1}^{2}K_nq_n^{N-1}\left(\frac{D_{n4}g_1h^4}{48} + \frac{D_{n3}g_2h^3}{24} + D_{n2}a_2^-h^2 - D_{n1}a_1^-h + D_{n0}f_3(0)\right) +$$

$$+\sum_{n=1}^{2}\sum_{\gamma=0}^{N}K_nq_n^{N-\gamma-1}f_3(\gamma) + \sum_{n=1}^{2}\frac{K_n}{q_n}\left(-\frac{D_{n4}g_1h^4}{48} + D_{n3}h^3\left(\frac{g_2}{24} - \frac{g_1hN}{12}\right) +\right.$$

$$+D_{n2}h^2\left(\frac{g_2hN}{8}-\frac{g_1h^2N^2}{8}+a_2^+\right)+$$

$$+D_{n1}h\left(\frac{g_2h^2N^2}{8}-\frac{g_1h^3N^3}{12}+2a_2^+hN+a_1^+\right)+D_{n0}f_3(N)\Bigg)\Bigg];$$

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Functional Central Limit Theorem for Random Fields with Values in Some Infinite Dimensional Spaces

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DOI 10.5281/zenodo.18217460

Let $\{X_i,\ \ i\in\mathbb{Z}\}$ be a sequence of random variables taking values in a separable Hilbert space H endowed with the inner product $(\cdot,\cdot)$ and the corresponding norm $\|\cdot\|=\sqrt{(\cdot,\cdot)}$. Assume that $\{X_i,\ \ i\in\mathbb{Z}\}$ can be represented as

$$X_i=f(\varepsilon_{i-s}),\qquad s\in\mathbb{Z},i\in\mathbb{Z} \tag{1}$$

where $\{\varepsilon_j,\ \ j\in\mathbb{Z}\}$ is a sequence of independent and identically distributed random variables taking values in H and f is a measurable function(for real-valued case see [1], [2]).

Let $\{\varepsilon_j^{|},\ \ j\in\mathbb{Z}\}$ be an independent copy of $\{\varepsilon_j,\ \ j\in\mathbb{Z}\}$ and

$$X_i^*=f(\varepsilon_{i-s}^*,\ \ s\in\mathbb{Z}),$$

where

$$\varepsilon_j^*=\begin{cases}\varepsilon_j, & j\neq 0,\\ \varepsilon_j^{|}, & j=0.\end{cases}$$

Introduce the following notation:

$\{e_j, j \geq 1\}$ - an orthonormal basis of H,

$$X_i = \sum_{j=1}^{\infty} X_i^{(j)} e_j,$$

$$S_n = \frac{1}{\sqrt{n}} \sum_{i \in \Gamma_n} X_i, \qquad i = 1, 2, ..., n,$$

$$S_n(t) = \frac{1}{\sqrt{n}} \sum_{i=1}^{[nt]} X_i, \qquad t \in [0, 1],$$

$$t_{ij} = \lim_{n \to \infty} E S_n^{(i)} S_n^{(j)}, \qquad i, j = 1, 2, ...,$$

$$\delta_{ij} = \sqrt{E(X_i^{(j)} - X_i^{*(j)})^2}.$$

$$\triangle_p(j) = \sum_{t \in \mathbb{Z}} \delta_j(t, p).$$

Our aim is to prove functional central limit theorem for $\{X_i, \quad i \in \mathbb{Z}\}$. Central limit theorems were proved in [3],[4].

Theorem 1. Let $\{X_i, \quad i \in \mathbb{Z}\}$ be a centered, sequence of random variables taking values in a separable Hilbert space H and satisfying the following conditions

$$\sum_{i \in \mathbb{Z}} \delta_{ij} < \infty, \qquad j = 1, 2,$$

$$\sum_{i=1}^{\infty} t_{ii} < \infty, \quad t_{jj} > 0 \qquad j = 1, 2, ...,$$

Then the following weak convergence holds

$$S_n(t) \Rightarrow W(t) \quad \text{as} \quad n \to \infty$$

where $W(t), \quad t \in [0, 1]$ is a Brownian motion with values in H and $W(1)$, is a Gaussian random variable with values in H with mean zero and covariance matrix $T = (t_{ij})$. Analogous results are obtained for the sequences with values in ℓ_p $(1 \leq p \leq 2)$ (a space of the sequences $x = (x^{(1)}, x^{(2)}, ...)$ such that $\sum\limits_{i=1}^{\infty} |x^{(i)}|^p < \infty$ with a norm $\|x\| = \left(\sum\limits_{i=1}^{\infty} |x^{(i)}|^p \right)^{\frac{1}{p}}$ and c_0 (a space of all sequences $x = (x^{(1)}, x^{(2)}, ...)$ such that $\lim\limits_{n \to \infty} x^{(n)} = 0$ with a norm $\|x\| = \sup\limits_i |x^{(i)}|$.

In $\ell_p \quad (1 \leq p \leq 2)$ and c_0 spaces as a basis $\{e_j, \quad j \geq 1\}$ we take a standard basis i.e. $e_j = (0, 0, ..., 1, 0, ...)$ where $i-$th component is 1 and others are 0 . In the next theorems we use above notation.

Theorem 2. Let $\{X_i, \quad i \in \mathbb{Z}\}$ be a centered, sequence of random variables taking values in ℓ_p $(1 \leq p \leq 2)$ and satisfying the following conditions

$$\sum_{i \in \mathbb{Z}} \delta_{ij} < \infty, \quad j = 1, 2,$$

$$\sum_{i=1}^{\infty} t_{ii}^{\frac{p}{2}} < \infty, \quad t_{jj} = 1, 2, ...,$$

Then the following weak convergence holds

$$S_n(t) \Rightarrow W(t) \quad \text{as} \quad n \to \infty$$

where $W(t), \ \ t \in [0,1]$ is a Brownian motion with values in ℓ_p $(1 \leq p \leq 2)$ and $W(1)$ is a Gaussian random variable with values in ℓ_p $(1 \leq p \leq 2)$ with mean zero and covariance matrix $T = (t_{ij})$.

Theorem 3. Let $\{X_i, \ \ i \in \mathbb{Z}\}$ be a centered, sequence of random variables taking values in c_0 and satisfying the following conditions

$$\sum_{i \in \mathbb{Z}} \delta_{ij} < \infty, \quad j = 1, 2,$$

$$\sum_{i=1}^{\infty} t_{ii} < \infty,$$

Then the following weak convergence holds

$$S_n(t) \Rightarrow W(t) \quad \text{as} \quad n \to \infty$$

where $W(t), \ \ t \in [0,1]$ is a Brownian motion with values in c_0 and $W(1)$ is a Gaussian random variable with values in c_0 with mean zero and covariance matrix $T = (t_{ij})$.

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Leadership emergence or disappearance in agent based networks

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A new dynamic mathematical model describes leadership emergence or disappearance in agent based networks is proposed. Through a generalised Verhulst–Lotka–Volterra model, a triad of agents operates in a market where each agent detains a quota. The triad is composed of a leader, who leads communication, and two followers. Communications flows both ways from leader to followers and vice versa. Competition, collaboration and cheating are allowed. Stability solutions are investigated analytically through a fixed point analysis. Various solutions exist depending on the type of behavioural interactions. Results show that communication counts: survival of the leader is a condition for stability. All configurations with the leader out of the market are unstable. Conversely, the two followers position is highly difficult. For followers, cheating the leader, especially if the leader is collaborating, can be a disaster. By the way, collaboration with the leader may not always ensure market survival. Cheating is a cause of instability. This is a joint work with my team. The published paper is attached.

δ-derivations on the twisted Heisenberg-Virasoro algebra

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DOI 10.5281/zenodo.18217466

Keywords: Lie algebra, twisted Heisenberg-Virasoro algebra, derivation, δ-derivation.

The architectural analysis of Lie algebras-encompassing their structural stability, cohomological invariants, and representation-theoretic properties-is fundamentally dependent on the study of linear operators known as derivations. In recent decades, the transition from classical derivations to their generalized counterparts has facilitated a deeper understanding of algebraic symmetries. Among these, the concept of the δ-derivation stands out as a critical advancement. Originally formalized by Filippov [1], this operator provides a robust, multi-faceted theoretical matrix that consolidates several disparate classes of linear mappings into a single parameterized framework.

By varying the scalar δ, one can interpolate between distinct algebraic structures. This flexibility allows researchers to treat ordinary derivations, elements of the centroid, and antiderivations as specific manifestations of a more universal identity. Such a unified perspective is particularly advantageous when investigating infinite-dimensional structures like Virasoro-type algebras, where traditional derivation-inner properties might be influenced by external parameters.

Definition. Let F be the underlying field of a Lie algebra $(L, [\cdot, \cdot])$. For any fixed element $\delta \in F$, a linear transformation φ mapping L into itself is formally categorized as a **δ-derivation** if, for every pair of elements $x, y \in L$, the following generalized structural relation is satisfied:

$$\varphi([x, y]) = \delta\left([\varphi(x), y] + [x, \varphi(y)]\right). \tag{1}$$

This definition gives rise to several distinguished sub-classes based on the value of δ:

- **Case $\delta = 1$:** The mapping φ recovers the classical definition of a derivation, adhering to the standard Leibniz rule which governs the infinitesimal automorphisms of the algebra.
- **Case $\delta = \frac{1}{2}$:** The operator relates to the *centroid* of the algebra, $\mathrm{Cent}(L)$. Such mappings are essential for determining the scalar extensions and the simplicity of the underlying algebraic structure.
- **Case $\delta = -1$:** The mapping is recognized as an *antiderivation*, a structure that appears frequently in the study of Jordan algebras and specific types of commutative cohomology.
- **Case $\delta = 0$:** For non-perfect Lie algebras, this identifies mappings that vanish on the derived subalgebra, offering insights into the abelian quotients of the system.

The presence or absence of non-trivial δ-derivations for values other than these specific integers often serves as a "rigidity test"for the Lie algebra, indicating a high degree of structural constraint, as is observed in the case of the twisted Heisenberg-Virasoro algebra.

The literature on Lie theory consistently highlights that the presence of non-trivial δ-derivations is a statistically rare and structurally constrained phenomenon. Extensive research across various algebraic families indicates that these operators do not exist arbitrarily; rather, their manifestation is governed by the underlying symmetry of the algebra. A cornerstone of this research is the work of Filippov [1], who rigorously established that for prime Lie algebras, non-zero δ-derivations are restricted to a highly specific, discrete set of exceptional values for the parameter δ.

This discovery suggested that the bracket structure of prime algebras imposes a form of "arithmetic rigidity"on the types of linear deformations they can support. These foundational insights were subsequently generalized to more complex mathematical environments, including the study of infinite-dimensional Lie algebras and the specialized realm of Lie superalgebras. Such expansions have been instrumental in unveiling the intricate interdependencies between the theory of δ-derivations and other fundamental algebraic constructs. Specifically, the analysis of these operators has provided new pathways into the study of:

1. **Centroid Topologies:** Understanding how δ-derivations (specifically at $\delta = \frac{1}{2}$) interact with the space of linear maps that commute with the adjoint action.

2. **Cohomological Invariants:** Utilizing δ-derivations to probe the first and second cohomology groups, which clarify the relationship between internal structure and external extensions.

3. **Algebraic Rigidity:** Demonstrating that the absence of non-trivial δ-derivations for values outside the set $\{0, 1, \frac{1}{2}\}$ often serves as a hallmark of structural stability in high-order systems like the twisted Heisenberg-Virasoro algebra.

These theoretical advancements demonstrate that δ-derivations are not merely a curiosity of linear algebra, but are instead deep probes into the categorical nature of Lie-type structures. In the following sections, we apply these global constraints to the specific case of the twisted Heisenberg-Virasoro algebra to determine its unique parameter spectrum.

A related structure is the centroid $\mathrm{Cent}(L)$, consisting of all linear maps $\chi : L \to L$ satisfying

$$\chi([x, y]) = [\chi(x), y] = [x, \chi(y)],$$

for all $x, y \in L$. Elements of the centroid commute with all inner adjoint maps ad_x, and they play a significant role in understanding automorphisms and derivations of the algebra. It is straightforward to check that every centroid element satisfies condition (1) with $\delta = \frac{1}{2}$, and hence centroid maps provide natural examples of $\frac{1}{2}$-derivations.

Special values of δ lead to structurally important subclasses of δ-derivations. For instance, if $\delta = 0$ and L is not perfect (that is, $[L, L] \neq L$), then any nonzero linear map annihilating the derived algebra $[L, L]$ automatically satisfies (1), and therefore yields a nontrivial 0-derivation. In contrast, when L is perfect, and especially when L is simple, such maps cannot exist, which implies that no nonzero 0-derivations occur in those cases.

When $\delta = -1$, the operator φ is referred to as an antiderivation. This type of map has been studied independently in classical literature, including Jacobson's foundational text [2], and appears naturally in the investigation of various noncommutative Jordan algebras [3]. Furthermore, an intimate connection between antiderivations and commutative 2-cocycles on Lie algebras was established in [4], revealing deeper cohomological implications of these operators.

Filippov pursued an extensive analysis of δ-derivations in a series of works [1-2,5-7]. One of his key results, proved in [7], states that prime Lie algebras admit no nonzero δ-derivations except in the exceptional cases

$$\delta = \{-1; 0; 1; \tfrac{1}{2}\}.$$

He further employed antiderivations to examine Lie algebras satisfying certain polynomial identities of degree five [7], and proved that any prime Lie algebra possessing a nonzero antiderivation necessarily satisfies the standard degree-5 identity, a strong structural restriction.

In addition, Filippov observed in [5] that the set of $\frac{1}{2}$-derivations of a prime Lie algebra forms a commutative ring, and he raised the natural question of whether such a ring must always be free of zero divisors. This question was later addressed by Zusmanovich [8], who constructed explicit examples showing that the Zassenhaus algebra possesses $\frac{1}{2}$-derivations that are zero divisors, thus providing a definitive negative answer.

The theory was subsequently extended beyond ordinary Lie algebras. Kaygorodov introduced analogous notions of δ-derivations for nonassociative superalgebras in [9, 10], significantly broadening the applicability of the concept. Among his results is the theorem that finite-dimensional classical Lie superalgebras over a field of characteristic zero do not admit any nontrivial δ-derivations, thereby generalizing Filippov's conclusions to the superalgebraic setting.

In this paper, we focus on the twisted Heisenberg-Virasoro algebra at level zero, which arises as the universal central extension of the Lie algebra of differential operators of order at most one on the circle. This algebra plays a prominent role in representation theory and mathematical physics, as it combines Virasoro and Heisenberg structures through a nontrivial cocycle interaction. The derivation algebra of the twisted Heisenberg-Virasoro algebra is known and admits a decomposition into inner derivations and several explicitly described outer derivations.

Building upon these structural results, we provide a complete classification of δ-derivations of the twisted Heisenberg-Virasoro algebra. Our approach is purely algebraic and is based on a detailed analysis of the defining identities of δ-derivations applied to the standard basis elements of the algebra. By deriving and solving the resulting systems of linear equations for the coefficients, we obtain strong constraints on the possible forms of such operators.

Our main result shows that nontrivial δ-derivations of the twisted Heisenberg-Virasoro algebra exist only for the values $\delta = 0$, $\delta = 1$, and $\delta = \frac{1}{2}$. In all other cases, every δ-derivation is necessarily trivial. This result complements earlier work on ordinary derivations and $\frac{1}{2}$-derivations and provides a complete picture of the δ-derivation structure for this important class of infinite-dimensional Lie algebras.

We now turn to the twisted Heisenberg-Virasoro algebra at level zero. This Lie algebra was introduced as the universal central extension of the Lie algebra of differential operators on the circle of order at most one. Denote this algebra by $\mathcal{L}$. It admits a basis of the form

$$\{L_n, I_n, C_L, C_I, C_{LI} | n \in Z\}$$

with the following commutation relations

$$\begin{aligned}
[L_n, L_m] &= (n-m)L_{n+m} + \delta_{n,-m}\frac{n^3-n}{12}C_L, \\
[L_n, I_m] &= -mI_{n+m} + \delta_{n,-m}(n^2+n)C_{LI}, \\
[I_n, I_m] &= m\,\delta_{n,-m}C_I, \\
[\mathcal{L}, C_L] &= [\mathcal{L}, C_{LI}] = [\mathcal{L}, C_I] = 0,
\end{aligned}$$

where $m, n \in Z$.

This Lie algebra has an infinite-dimensional Heisenberg subalgebra and a Virasoro subalgebra. These subalgebras, however, do not form a semidirect product, but instead, the natural action of the Virasoro subalgebra on the Heisenberg subalgebra is twisted with a 2-cocycle. The structure and representation theory of the twisted Heisenberg-Virasoro algebra were studied by V. Kac, Y. Billing, C. Jiang, R. Shen and D.Liu etc (see [11-14]).

The following lemma comes from [14], which determines the derivations of the twisted Heisenberg-Virasoro algebra.

Lemma. [14] $\mathrm{Der}\mathcal{L} = \mathrm{ad}\mathcal{L} \oplus CD_1 \oplus CD_2 \oplus CD_3$, where

$$\begin{aligned}
D_1: &\quad D_1|_{L\oplus CC_L} = 0, \qquad D_1|_{I\oplus CC_I} = \mathrm{id}, \qquad D_1(C_{LI}) = 2\,C_I; \\
D_2: &\quad D_2(L_n) = n\,I_n + \delta_{n,0}\,C_{LI}, \qquad D_2(I_n) = -\delta_{n,0}\,C_I, \\
&\quad D_2(C_L) = 24\,C_{LI}, \qquad D_2(C_{LI}) = -C_I, \qquad D_2(C_I) = 0; \\
D_3: &\quad D_3(L_n) = (n+1)\,I_n, \qquad D_3(C_L) = 24\,C_{LI}, \qquad D_3(C_{LI}) = -C_I, \\
&\quad D_3|_{I\oplus CC_I} = 0.
\end{aligned}$$

The $\frac{1}{2}$-derivations of the twisted Heisenberg-Virasoro algebra were calculated in [15].

Theorem. [15] Every $\frac{1}{2}$-derivation on the twisted Heisenberg-Virasoro algebra is trivial.

In the present paper, we provide a complete description of the δ-derivations of the twisted Heisenberg-Virasoro algebra. In particular, we prove that all nontrivial δ-derivations occur only for the values $\delta = 0$, $\delta = 1$, and $\delta = \frac{1}{2}$, and that for any other value of δ the algebra admits no nonzero δ-derivations.

The core result of this study is the following classification theorem.

Theorem. A linear operator φ on the twisted Heisenberg–Virasoro algebra $\mathcal{L}$ is a nontrivial δ-derivation if and only if $\delta \in \{0, 1, \frac{1}{2}\}$. For any $\delta \notin \{0, 1, \frac{1}{2}\}$, the only solution to the δ-derivation identity is the zero map.

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Exact Analytical Solution of a Weighted Integral Geometry Problem

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DOI 10.5281/zenodo.18217469

Introduction. Reconstruction of an unknown function from its integral characteristics is one of the fundamental problems of computational mathematics, mathematical modeling, and inverse problem theory. Such problems arise naturally in a wide range of applied fields, including computer tomography, medical imaging, nondestructive testing, geophysics, and remote sensing. Within the framework of integral geometry, these problems are formulated as inverse problems for integral transforms defined on special families of curves or surfaces.

Of particular interest are weighted integral transforms associated with families of curves possessing prescribed geometric properties, such as parabolic curves. For such transforms, uniqueness theorems have been established, analytic inversion formulas have been derived, and stability estimates in Sobolev spaces have been obtained. The construction of inversion formulas is typically based on harmonic analysis techniques, in particular on the application of direct and inverse Fourier transforms.

Within the framework of weighted integral geometry on linear domains, the uniqueness of solutions for integral transforms defined over families of parabolic curves has been rigorously established. Considerable attention has been devoted to the derivation of inversion formulas, analytic solution representations, and stability estimates in Sobolev spaces. These inversion formulas fundamentally rely on the application of both direct and inverse Fourier transforms. As demonstrated in [1], the theory of integral geometry is closely related to representation theory of Lie groups, inverse boundary value problems for differential equations, and various practical applications, including tomographic imaging.

For different families of broken lines and surfaces with specific geometric properties, uniqueness theorems, stability estimates, and analytical inversion formulas for weighted integral transforms have been obtained. Problems involving weighted families of parabolic curves have likewise been thoroughly investigated [2]. The moderately ill-posed character of such problems, the uniqueness of solutions within classes of smooth compactly supported functions, and the derivation of stable inversion procedures based on Fourier methods are examined in detail in [3, 4].

Statement of the problem. Consider the following problem

$$\int_{(x,y)} g(x,\xi)u(\xi,\eta)ds = f(x,y) \tag{1}$$

here

$$g(x,\xi,y,\eta) = a_0 + a_1(y-\eta) + a_2(y-\eta)^2,\ (x,y) = (x,y) : |x-\xi| = y-\eta$$

The considered integral problem belongs to the class of inverse problems for Fredholm integral equations of the first kind and is ill-posed in the sense of Hadamard. This means that for arbitrary input data $f(x,y)$ a solution may not exist, and even if it exists, neither uniqueness nor stability is guaranteed. In particular, small perturbations in the right-hand side of the equation may lead to significant changes in the reconstructed function.

Equation (1) can be equivalently rewritten in the following form:

$$\frac{1}{\sqrt{2}}\int_0^y \big(a_0 + a_1(y-\eta) + a_2(y-\eta)^2\big)u\big(x-(y-\eta),\eta\big)\,d\eta$$
$$+\frac{1}{\sqrt{2}}\int_0^y \big(a_0 + a_1(y-\eta) + a_2(y-\eta)^2\big)u\big(x+(y-\eta),\eta\big)\,d\eta = f(x,y) \qquad (2)$$

We apply the Fourier transform with respect to the variable to both sides of equation (2), which yields:

$$\int_0^y \hat{u}(\lambda,\eta)\big(a_0 + a_1(y-\eta) + a_2(y-\eta)^2\big)\cos\big(\lambda(y-\eta)\big)\,d\eta =$$
$$= \int_0^\infty \int_0^\infty e^{-\mu\eta}e^{i\mu\tau}\hat{u}(\lambda,\eta)\cos(\lambda\tau)\big(a_0 + a_1\tau + a_2\tau^2\big)\,d\tau\,d\eta =$$
$$= \int_0^\infty e^{i\mu\eta}\hat{u}(\lambda,\eta)\,d\eta \int_0^\infty e^{-i\mu\tau}\big(a_0 + a_1\tau + a_2\tau^2\big)\cos(\lambda\tau)\,d\tau.$$

or

$$\hat{\tilde{f}}(\lambda,\mu) = \hat{\tilde{u}}(\lambda,\mu)\,I(\lambda,\mu)$$

where $I(\lambda,\mu) = \int_0^{+\infty} e^{-i\mu\tau}(a_0 + a_1\tau + a_2\tau^2)cos(\lambda\tau)d\tau$

As a result, we obtain

$$\tilde{\hat{u}}(\lambda,\mu) = \frac{\sqrt{2}\tilde{\hat{f}}(\lambda,\mu)}{(\frac{ia_0\mu}{\lambda^2\tau^2-\mu^2} - a_1\frac{\mu^2+\lambda^2\tau^2}{(\lambda^2\tau^2-\mu^2)^2} + a_2\frac{2i\mu(-\mu^2-3\lambda^2\tau^2)}{(\lambda^2\tau^2-\mu^2)^3})}$$

Taking the inverse Fourier transform with respect to μ and λ , we arrive at the following expression:

$$u(x,y) = \int_{-\infty}^{+\infty}\int_0^{+\infty} \frac{\sqrt{2}\tilde{\hat{f}}(\lambda,\mu)(\lambda^2\tau^2-\mu^2)^3 e^{-i\mu y - i\lambda x}}{ia_0\mu(\lambda^2\tau^2-\mu^2) - a_1(\lambda^2\tau^2-\mu^2)^2 - 2ia_2\mu(\mu^2+3\lambda^2\tau^2)}d\mu d\lambda$$

The numerical evaluation of the obtained integral representation is associated with substantial difficulties arising from both the analytical structure of the inversion kernel and the properties of the Fourier transforms involved. These difficulties are intrinsic to the inverse nature of the problem and reflect its moderately ill-posed character.

A major source of instability is the presence of singular spectral factors of the form $(\lambda^2\tau^2-\mu^2)^{-k}$, which appear in the denominator of the inversion formula. These factors generate poles in the spectral domain along the critical manifolds $\mu = \pm\lambda\tau$, where the denominator vanishes. In the neighbourhood of these points, the integrand exhibits

rapid growth and strong oscillatory behaviour, which makes the numerical integration highly sensitive to perturbations in the data. As a result, even small measurement errors or discretization inaccuracies in the right-hand side function $f(x, y)$ may lead to large deviations in the reconstructed solution.

Another fundamental difficulty is related to the amplification of high-frequency components in the Fourier domain. Since the inversion formula involves division by small spectral values, the high-frequency noise contained in the measured data is significantly magnified. This effect is characteristic of inverse problems for integral equations of the first kind and represents a manifestation of their ill-posedness in the sense of Hadamard. In practice, this leads to spurious oscillations in the reconstructed function and to the loss of physical interpretability of the numerical solution.

For these reasons, direct numerical evaluation of the inversion integral is impractical without the use of regularization techniques. In order to obtain a stable and physically meaningful reconstruction, it is necessary to introduce spectral filtering or Tikhonov-type regularization, which suppresses the influence of high-frequency noise and prevents the blow-up of the solution near the singular spectral sets. Such approaches make it possible to balance stability and accuracy and provide a reliable numerical implementation of the analytical inversion formula.

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Solving P° Multi-Criteria Optimization Problem of Agricultural Production Using the Weighted Sum Method

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DOI 10.5281/zenodo.18217476

Introduction. Today, the efficient use of resources and ensuring long-term system stability are among the most critical issues in agriculture. Specifically, the growing demand for food, coupled with limited water, land, and energy resources, requires approaches that simultaneously consider economic efficiency, environmental sustainability, and social factors. Therefore, agricultural planning often needs to optimize several goals at once: increasing economic profit, reducing production costs, limiting water consumption, and using other resources wisely [1–3].

In this research, the multi-objective optimization model for agricultural production proposed in [4] was reviewed and improved as follows.

Multi-Objective Optimization Model.

Objective functions:

$$f_1(x,z) = \sum_{i\in I}\sum_{j\in J} q_{ij}x_{ij} + \sum_{i\in I}\sum_{k\in K} \widehat{q}_{ik}z_{ik} \to \min, \tag{1}$$

$$f_2(x,z) = \sum_{i\in I}\sum_{j\in J} P_{ij}a_{ij}x_{ij} + \sum_{i\in I}\sum_{k\in K} \widehat{P}_{ik}z_{ik} \to \max, \tag{2}$$

$$f_3(x,z) = \sum_{i\in I}\sum_{j\in J} \omega_{ij}x_{ij} + \sum_{i\in I}\sum_{k\in K} \widehat{\omega}_{ik}z_{ik} \to \min\,. \tag{3}$$

Constraints:

$$\sum_{i\in I} a_{ij}x_{ij} - \sum_{i\in I} y_{ij} \geq A_j, \qquad \forall j \in J, \tag{4}$$

$$\sum_{i\in I} z_{ik} \geq B_k, \qquad \forall k \in K, \tag{5}$$

$$\sum_{j\in J} b_{jm}y_{ij} \geq \sum_{k\in K} \widehat{b}_{ikm}z_{ik}, \qquad \forall i \in I,\ \forall m \in M, \tag{6}$$

$$\sum_{j\in J} t_{ij}x_{ij} + \sum_{k\in K} \widehat{t}_{ik}z_{ik} \leq T_i, \qquad \forall i \in I, \tag{7}$$

$$\sum_{j\in J} x_{ij} \leq S_i, \qquad \forall i \in I, \tag{8}$$

$$0 \leq y_{ij} \leq a_{ij}x_{ij}, \qquad \forall i \in I,\ \forall j \in J, \tag{9}$$

$$0 \leq x_{ij} \leq \delta_{ij}X_{ij}^{\max}, \qquad \forall i \in I,\ \forall j \in J, \tag{10}$$

$$0 \leq z_{ik} \leq \gamma_{ik}Z_{ik}^{\max}, \qquad \forall i \in I,\ \forall k \in K. \tag{11}$$

We define the set of feasible solutions satisfying constraints (4)–(11) as D. The detailed problem statement, including the definitions of indices, parameters, and decision variables, is fully described in [4].

Solution Algorithm. Solving the three-objective problem (1)–(3) directly and simultaneously complicates the practical computing process. Therefore, it is appropriate to normalize the objectives and then transform them into a single-objective linear programming problem using the weighted sum method [5].

The algorithm consists of the following steps:

Step 1. Finding extreme values for each objective. To ensure correct normalization, the minimum and maximum possible values for each objective within the feasible region D are determined beforehand by solving individual optimization problems:

$$F_r^{\min} = \min_{(x,z)\in D} f_r(x,z), \qquad F_r^{\max} = \max_{(x,z)\in D} f_r(x,z), \qquad r \in \{1,2,3\}.$$

Step 2. Normalization. Since the objectives have different units of measurement, we scale them to the interval $[0,1]$. In this step, all objectives are converted into a minimization form:

$$F_1^{\text{norm}}(x,z) = \frac{f_1(x,z) - F_1^{\min}}{F_1^{\max} - F_1^{\min}},$$

$$F_2^{\text{norm}}(x,z) = \frac{F_2^{\max} - f_2(x,z)}{F_2^{\max} - F_2^{\min}},$$

$$F_3^{\text{norm}}(x,z) = \frac{f_3(x,z) - F_3^{\min}}{F_3^{\max} - F_3^{\min}},$$

where $F_r^{\max} \neq F_r^{\min}$, $r \in \{1,2,3\}$.

As a result, the values of $F_1^{\text{norm}}, F_2^{\text{norm}}, F_3^{\text{norm}}$ range between 0 and 1, making it possible to compare and combine them directly.

Step 3. Constructing a single objective function using the weighted sum method. In this step, weights representing the priorities of the decision-maker are selected:

$$0 \le \alpha_1 \le 1, \quad 0 \le \alpha_2 \le 1, \quad 0 \le \alpha_3 \le 1, \quad \alpha_1 + \alpha_2 + \alpha_3 = 1.$$

The single objective function is then formulated as:

$$F(x,z) = \alpha_1 F_1^{\text{norm}}(x,z) + \alpha_2 F_2^{\text{norm}}(x,z) + \alpha_3 F_3^{\text{norm}}(x,z) \to \min. \tag{12}$$

By substituting the original objective functions (1)–(3) into (12) and simplifying, we obtain the following compact objective function:

$$\begin{aligned} F(x,z) = &\sum_{i\in I}\sum_{j\in J^i} \left(\alpha_1 k_1 q_{ij} - \alpha_2 k_2 P_{ij} a_{ij} + \alpha_3 k_3 \omega_{ij}\right) x_{ij} \\ &+ \sum_{i\in I}\sum_{k\in K^i} \left(\alpha_1 k_1 \widehat{q}_{ik} - \alpha_2 k_2 \widehat{P}_{ik} + \alpha_3 k_3 \widehat{\omega}_{ik}\right) z_{ik}, \end{aligned} \tag{13}$$

where

$$k_1 = \frac{1}{F_1^{\max} - F_1^{\min}}, \qquad k_2 = \frac{1}{F_2^{\max} - F_2^{\min}}, \qquad k_3 = \frac{1}{F_3^{\max} - F_3^{\min}}.$$

As a result, we obtain a single-objective linear programming problem consisting of objective function (13) subject to constraints (4)–(11).

Step 4. Pareto Analysis. Various combinations of the weights $\alpha_1, \alpha_2, \alpha_3$ are selected. For each combination, the optimization problem (13) is solved subject to constraints (4)–(11), and the corresponding values of f_1, f_2, f_3 are collected. Then, dominated solutions are removed, and the remaining ones are considered as Pareto-efficient solutions.

Conclusion. In this study, a three-objective linear optimization model was presented for the joint planning of crop production and livestock farming, incorporating cost, profit, and water consumption criteria. The model includes constraints related to product and livestock demand, feed balance, as well as resource and land limitations. The multi-criteria problem was transformed into a single-objective linear programming problem using normalization and the weighted sum method. The proposed approach is particularly suitable for practical planning in regions with limited water resources.

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Stability Estimation of the Problem of Integral Geometry on a Family of Parabolas

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DOI 10.5281/zenodo.18217492

Introduction. The papers [1,2,3] investigate weakly ill-posed integral geometry problems involving families of parabolas. Theorems regarding the existence, uniqueness, and stability of solutions were established, and conversion formulas were derived [4,5].

Radon's seminal work [6] forms the foundation of the theory of integral geometry problems [7], which underpins various applications in imaging [8], seismology [9] and numerous other fields.

This paper [10] addresses integral geometry problems involving a family of parabolas and derives an approximate regularized sequential conversion formula. The current work builds on this, focusing on estimating the error in the space using the derived conversion formula. To validate the results, several test examples with numerical solutions are provided.

Statement of the problem 1: Reconstruct the function of two variables $u(x, y)$ within the strip $L_H = (x, y) : x \in R^1, 0 \leq \eta \leq y \leq H$ if its integrals over the curves in the family $Y(x, y)$ are known

$$\int_{Y(x,y)} u(\xi, \eta) d\xi = f(x, y) \tag{1}$$

where the expression describes any curve from the family

$Y(x, y) = (\xi, \eta) : x - \xi = \sqrt{y - \eta},\ 0 \leq \eta \leq y$

Statement of the problem 2: Within the region $\Omega_H = (x, y) : x \in R^1, y \leq \eta \leq H$ reconstruct the function of two variables $u(x, y)$ if the integrals of this function along the curves in the family $\Lambda(x, y)$ are known

$$\int_{\Lambda(x,y)} u(\xi, \eta) d\xi = f(x, y) \tag{2}$$

where the expression gives any curve from the family

$\Lambda(x, y) = (\xi, \eta) : x - \xi = \sqrt{\eta - y},\ y \leq \eta \leq H$

For these two problems, we have the following lemmas. The proofs of these lemmas can be found in [10].

Lemma 1. Consider a function $f(x, y)$ known within the strip L_H . Then the solution of equation (1) in the class of C^2 finite functions, which have support in the strip L_H , is unique and can be represented as

$$\hat{u}(\lambda, y) = \frac{\partial}{\pi \partial y} \int_0^y \frac{ch(\lambda \sqrt{y - \eta})}{\sqrt{y - \eta}} \hat{f}(\lambda, \eta) d\eta \tag{3}$$

where $\hat{u}(\lambda, y)$ and $\hat{f}(\lambda, y)$ represent the Fourier transform of the first variable applied to the functions $u(x, y)$ and $f(x, y)$, respectively. Lemma 2. Let $f(x, y)$ be a function twice continuously differentiable concerning its arguments and finite within the domain Ω . The unique solution of the equation (2), which is also twice continuously differentiable and has a finite value, can be expressed as:

$$\hat{u}(\lambda, y) = -\frac{\partial}{\pi \partial y} \int_y^H \frac{ch(\lambda \sqrt{\eta - y})}{\sqrt{\eta - y}} \hat{f}(\lambda, \eta) d\eta \tag{4}$$

where $\hat{u}(\lambda, y)$ and $\hat{f}(\lambda, y)$ represent the Fourier transform of the first variable applied to the functions $u(x, y)$ and $f(x, y)$, respectively.

Theorem 1. Let the function $f(x,y)$ be defined in the domain L_H . We consider the regularizing operator for equation (3), which is given by:

$R(u,\alpha) = \mathcal{F}^{-1}[\hat{u}e^{-\alpha_1^2\lambda^2}]$

Then, on the rectangle L_H with support $\overline{L_H}$, Problem 1 in the class of twice continuously differentiable, finite functions gives the following solution:

$u_{\alpha_1}(x,y) = \frac{1}{2\alpha_1\sqrt{\pi^3}}\frac{\partial}{\partial y}\int_0^y\int_{-\infty}^{+\infty} e^{\frac{y-\eta-(x-\xi)^2}{4\alpha_1^2}} cos\frac{(x-\xi)\sqrt{y-\eta}}{2\alpha_1^2}\frac{f(\xi,\eta)}{\sqrt{y-\eta}}d\xi d\eta$

and satisfies the following inequality:

$\|u_{\alpha_1} - u\|_{L_2(\overline{L_H})} \leq C_1\alpha_1^2\|f\|_{L_2(\overline{L_H})}$

where $\alpha_1 > 0$ is a regularization parameter, $\mathcal{F}^{-1}$ is the inverse Fourier transform, and C_1 is a positive constant.

Theorem 2. Let the function $f(x,y)$ be defined within the domain Ω . We examine the regularization operator for equation (4).

$R(u,\alpha) = \mathcal{F}^{-1}[\hat{u}e^{-\alpha_1^2\lambda^2}]$

Thus, the solution to Problem 2, in the class of twice continuously differentiable, finite functions with support in the rectangle Ω , is represented as follows:

$u_{\alpha_2}(x,y) = \frac{1}{2\alpha_2\sqrt{\pi^3}}\frac{\partial}{\partial y}\int_y^H\int_{-\infty}^{+\infty} e^{\frac{\eta-y-(x-\xi)^2}{4\alpha_2^2}} cos\frac{(x-\xi)\sqrt{\eta-y}}{2\alpha_2^2}\frac{f(\xi,\eta)}{\sqrt{\eta-y}}d\xi d\eta$

and the inequality holds true

$\|u_{\alpha_2} - u\|_{L_2(\hat{\Omega})} \leq C_2\alpha_2^2\|f\|_{L_2(\hat{\Omega})}$

where α_2 is a positive regularization parameter, $\mathcal{F}^{-1}$ is the inverse Fourier transform, and C_2 is a positive constant.

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Mathematical Modeling of Water Flows in Water Management Facilities Using Complex Systems with Distributed Parameters

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DOI 10.5281/zenodo.18217503

Introduction. Efficient management of water resources is one of the most critical challenges in modern agriculture and environmental engineering. Large-scale irrigation systems consist of main canals, distribution channels, hydraulic structures, and control devices characterized by complex dynamic behavior. Water flow and water level variations in such systems depend on time and spatial coordinates, which makes them typical examples of systems with distributed parameters [1].

Automatic water level regulation systems are widely used to ensure stable water delivery to consumers under varying operating conditions. These systems involve feedback control mechanisms, boundary effects, external disturbances, and nonlinear interactions between different elements of the hydraulic network.

Water management facilities, including irrigation canals, reservoirs, and water distribution channels, are inherently characterized by spatially distributed hydraulic variables. Unlike lumped systems, where state variables are assumed to be uniform throughout the object, water management systems exhibit significant variations of water level, flow rate, and hydraulic energy along their spatial extent. These variations arise due to channel geometry, roughness, slope, and the presence of hydraulic structures such as gates, regulators, and pumping stations [2].

External influences on water management systems are applied at multiple spatial locations and at different times. Water withdrawals for irrigation, inflows from tributaries,

precipitation, seepage losses, and control actions at regulating structures introduce localized disturbances that affect the system dynamics. As a result, the response of the system to any external action cannot be considered instantaneous or uniform across the entire domain.

Dynamic processes in open channels propagate along the canal with finite velocity, leading to the formation of wave phenomena and time delays between input actions and observed responses. Changes in discharge or gate positions at upstream locations influence downstream water levels after a certain delay, which depends on hydraulic and geometric properties of the channel. These propagation effects play a critical role in the stability and performance of automatic water level regulation systems [3].

Furthermore, the interaction between adjacent canal sections introduces coupling effects that must be considered in the modeling process. The behavior of each section depends not only on local conditions but also on upstream and downstream boundary influences. This interdependence makes the overall system highly interconnected and sensitive to disturbances.

Due to these characteristics, water management facilities cannot be adequately described using simplified lumped-parameter models. Instead, distributed parameter models are required to capture the spatial and temporal evolution of hydraulic variables. Such models provide a more realistic representation of physical processes and allow accurate analysis of transient behavior, control effectiveness, and system stability.

The use of distributed parameter representations is particularly important in the design and analysis of automatic regulation systems, where precise prediction of system responses is necessary for developing reliable control strategies. By accounting for spatial distribution and dynamic propagation effects, these models form a solid foundation for numerical simulation, optimization, and decision support in modern water resource management.

Modeling Approach. The structural modeling approach is based on the decomposition of a complex water management system into a set of interconnected subsystems, each representing a typical physical or functional element of the irrigation network. These subsystems include canal sections, hydraulic structures, regulating devices, sensors, and control units. By considering each element as an individual block with defined inputs and outputs, the overall system can be analyzed in a systematic and hierarchical manner [4].

In this approach, the first step involves identifying the boundaries of each subsystem and determining the nature of interactions between adjacent elements. Canal sections are connected through boundary conditions that reflect the continuity of water flow and water level, while control elements introduce feedback and external influences. This representation allows the complex spatial structure of the water management facility to be translated into a clear and organized structural diagram.

Structural diagrams play a central role in the modeling process. They provide a visual and analytical description of information flows, control actions, and physical interactions within the system. Each connection in the diagram represents a specific type of interaction, such as serial transmission of hydraulic effects, parallel distribution of flows, or feedback loops introduced by automatic regulators. This explicit representation of interactions helps to identify key dynamic relationships and potential sources of instability.

A major advantage of the structural modeling approach is the ability to apply

systematic transformation rules to simplify complex schemes. Through sequential transformations, interconnected blocks can be combined into equivalent representations that preserve the essential dynamic properties of the original system. Serial connections are reduced by combining their effects, parallel structures are represented by aggregated interactions, and feedback loops are transformed into equivalent blocks with modified characteristics.

These transformations significantly reduce the complexity of the mathematical model while maintaining accuracy. As a result, the final structural representation consists of a minimal number of blocks, each characterized by well-defined inputвЂ“output relationships. This simplified structure is particularly well suited for numerical modeling and discrete-time simulation.

Another important feature of the structural approach is its modularity. Since each subsystem is modeled independently, changes in system configuration, such as the addition of new control devices or modification of canal geometry, can be incorporated without redesigning the entire model. This flexibility is essential for practical applications, where water management systems are often subject to operational changes and modernization.

The structural modeling approach also facilitates the development of numerical algorithms and software implementations. By aligning the mathematical model with the structural diagram, it becomes possible to design modular software components that correspond directly to physical subsystems. This correspondence enhances transparency, simplifies debugging, and improves the reliability of simulation results.

Overall, the structural modeling approach provides a powerful framework for analyzing complex water management systems with distributed parameters. It bridges the gap between detailed physical modeling and practical numerical implementation, enabling efficient simulation, analysis, and optimization of automatic water regulation systems.

Based on the developed algorithms, modular software tools were implemented to simulate automatic water level regulation systems. Simulation results demonstrate accurate representation of dynamic behavior and validate the effectiveness of the proposed modeling approach.

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Navigating the Globalisation of the Digital Economy: The Construction and Challenges of the Legal Framework for Cross-Border Personal Information Transfer in Uzbekistan

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DOI 10.5281/zenodo.18217519

Abstract

Uzbekistan's Personal Data Act, which came into force in 2019, represents a significant legislative advancement in the nation's data protection landscape and aims to align with international standards. The law establishes fundamental principles for personal data processing, grants data subjects essential rights, and defines obligations for data controllers, thereby forming an initial legal framework for the development of the digital economy. However, its provisions remain relatively general, and specific implementing regulations and enforcement practicesвЂ”particularly regarding cross-border personal data flowsвЂ”require further refinement. Challenges such as unclear alternative mechanisms, limited practical guidance, and potentially complex approval procedures create uncertainties for enterprises engaged in international operations. This study proposes recommendations for improving Uzbekistan's legal framework and institutional arrangements for cross-border personal data transfers, drawing on judicial practice and legislative developments.

Keywords: Personal information; Cross-border flow; Digital sovereignty; Flexible regulation; Regional mutual recognition.

INTRODUCTION

In the contemporary era of rapid digital economic globalisation, the cross-border flow of personal information has become a highly contested global issue. Legal questions related to data flow rules, regulatory authorities, and the scope of personal information directly

affect individual rights, corporate development, public interests, economic growth, and national security.

ANALYSIS AND RESULTS

1. Legal framework and cross-border rules: flexible practices within rigid structures

Uzbekistan's personal information protection system is centred on the 2019 Personal Data Act (Law No. ZRU-547), which constitutes the country's first comprehensive regulation of personal data processing. Article 2 defines personal data broadly and applies to most processing activities, while exempting non-commercial household use, state secrets, and certain law enforcement activities.

The 2021 amendment (Law No. ZRU-666) introduced strict data localisation requirements. Article 27-1 mandates that enterprises processing personal data of Uzbek citizens utilise domestic technical infrastructure and register relevant databases in the State Register of Personal Databases. This provision has had significant implications for multinational digital platforms.

Sector-specific legislation further differentiates regulatory practices. Financial institutions, healthcare providers, and telecommunications operators face additional restrictions on cross-border data transfers. Compared with the EU's GDPR, Uzbekistan adopts a more restrictive negative-list approach and imposes substantially lower administrative penalties, limiting deterrence.

Institutionally, regulatory oversight was historically exercised by the State Personalisation Centre, later replaced in 2024 by the Personnel Research Agency under the Ministry of Justice. Uzbekistan also participates in regional and international data governance initiatives, including cooperation within the Shanghai Cooperation Organisation and observer participation in the Eurasian Economic Union.

Despite these efforts, international assessments highlight challenges related to operational feasibility, transparency, and compliance costs, particularly for foreign enterprises.

2. Practical constraints and systemic reform

Technical capacity constraints, regulatory ambiguity, and limited transparency continue to hinder effective governance. Data localisation policies have increased operational costs and reduced foreign investment in the ICT sector. To address these issues, authorities are exploring a tiered data classification system and enhanced regional cooperation, including the proposed Central Asian Digital Single Market initiative.

Technological self-reliance is also being promoted through domestic encryption development and regional satellite cooperation projects, aimed at strengthening digital sovereignty while enhancing competitiveness.

Conclusion

Firstly, detailed rules under the Personal Data Protection Act should be revised to clarify consent requirements, incorporate modern data governance principles, and provide practical compliance guidance for cross-border transfers.

Secondly, regulatory capacity should be strengthened through the adoption of

regulatory technology tools, electronic filing systems, and risk-based supervision mechanisms.

Thirdly, regional mutual recognition mechanisms should be advanced through the development of Central Asian data governance standards.

Finally, deeper international cooperation is required to balance digital sovereignty with economic openness and to secure a stronger voice for developing economies in global digital governance.

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Cauchy Problem for a Nonstationary Third-Order Composite Type Equation on a Plane

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DOI 10.5281/zenodo.18217526

Abstract

This work provides a method for constructing a solution to the Cauchy problem for a non-stationary third-order composite equation in space (x, y, t). The authors determine the largest uniqueness class depending on the asymptotic properties of the fundamental solutions.

Keywords: the Cauchy problem, the Airy function, variation, functions with bounded variation, existence.

1. Introduction

The aim of this work is to study some properties of solutions of the equation

$$u_{xxx} + u_{yyy} - u_t = 0, \tag{1}$$

in the domain

$$D = \{(x, y, t) : -\infty < x < \infty,\ -\infty < y < \infty,\ 0 < t \le T\},$$

with the initial condition

$$u(x, y, 0) = u_0(x, y), \qquad -\infty < x < \infty,\ -\infty < y < \infty. \tag{2}$$

Reference [2] established a foundational framework for the equation

$$u_{xxx} - u_y = 0, \tag{MC}$$

including the construction of a fundamental solution, the development of potential theory, and a methodology for addressing boundary value and Cauchy problems.

Building upon this, work [3] extended the analysis by deriving a solution of the Cauchy problem within a broader class and investigating its properties. Following this approach, [4] applied the same method to solve the Cauchy problem for a higher-order odd equation.

$$\frac{\partial^{2n+1} u}{\partial x^{2n+1}} + (-1)^n \frac{\partial u}{\partial y} = 0.$$

The solutions of equation

$$u_{xxx} + u_{yyy} - u_t = 0 \tag{1}$$

and the linear Zakharov–Kuznetsov equation (equations, as referenced in [6–7])

$$u_{xxx} + u_{yyy} + u_t = 0 \tag{3}$$

exhibit similar asymptotic behavior at infinity.

As a multidimensional generalization of the Korteweg–de Vries equation, the Zakharov–Kuznetsov equation (3) describes ion–acoustic wave propagation in plasma [7].

A. N. Tikhonov [15] introduced correctness classes for the Cauchy problem in growing function spaces using the heat equation. Later, generalized functions [9–11] aided in studying initial–boundary value problems for even–order equations. The theory of linear even–order equations, especially parabolic ones, is now well developed [5,12–13].

A fundamental solution for equation (1) within the space $\mathbb{R}^{n+1}$ was constructed in [1].

Statement of the problem

It is required to determine, in the domain

$$D = \{(x, y, t) : -\infty < x < \infty,\ -\infty < y < \infty,\ 0 < t \le T\},$$

a function $u(x, y, t)$ that has a regular solution of the following equation:

$$u_{xxx} + u_{yyy} - u_t = 0. \tag{1}$$

The function $u(x,y,t)$ is required to satisfy the following condition in the domain D:

$$u(x,y,0)=u_0(x,y), \qquad -\infty<x<\infty,\ -\infty<y<\infty. \tag{2}$$

Here $u_0(x,y)$ is a given function.

The following function is a fundamental solution of equation (1) [1].

$$U(x,y;\xi,\eta)=(t-\tau)^{-\frac{2}{3}}\,f\left(\frac{x-\xi}{(t-\tau)^{\frac{1}{3}}}\right)\,f\left(\frac{y-\eta}{(t-\tau)^{\frac{1}{3}}}\right), \qquad x\neq\xi,\ y\neq\eta,\ t>\tau. \tag{4}$$

where

$$f(t)=\int_0^{+\infty}\cos\bigl(\lambda^3-\lambda t\bigr)\,d\lambda, \qquad -\infty<t<+\infty, \tag{14}$$

called Airy functions, satisfy the following equation [2]:

$$z''(\xi)+\frac{1}{3}\,\xi\,z(\xi)=0. \tag{5}$$

The following relations hold:

$$\int_0^{+\infty}f(t)\,dt=\frac{2\pi}{3}, \qquad \int_{-\infty}^{0}f(t)\,dt=\frac{\pi}{3}. \tag{6}$$

Moreover, for $t\neq 0$ the following equality is valid:

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}U(x,y,t)\,dx\,dy=\int_{-\infty}^{+\infty}f(\xi)\,d\xi\cdot\int_{-\infty}^{+\infty}f(\tau)\,d\tau=\pi^2. \tag{7}$$

Theorem 1. Let $u_0(x,y)$ be a continuous function of bounded variation on a bounded domain

$$D=\{(x,y):a\le x\le b,\ a\le y\le b\}\subset\mathbb{R}^2,$$

and let u_0 have bounded variation in D. Then the function

$$u(x,y,t)=\frac{1}{\pi^2}\iint_{\mathbb{R}^2}U(x-\xi,\ y-\eta,\ t)\,u_0(\xi,\eta)\,d\xi\,d\eta \tag{8}$$

for $t>0$ satisfies equation (1), and for any $(x_0,y_0)\in D$ the following limit relation holds:

$$\lim_{t\to 0^+}u(x,y,t)=\frac{2}{3}\,u_0(x_0-0,y_0-0)+\frac{1}{3}\,u_0(x_0+0,y_0+0). \tag{9}$$

The validity of the first part of the theorem follows immediately from the properties of the fundamental solution of equation (1) and the assumptions of the theorem. The proof of the second part is carried out separately with respect to each spatial variable. Since this proof is similar to that presented in [3] and does not involve any essential difficulties, it is omitted.

Theorem 2. Let the function $u_0(x,y)$, defined on any bounded domain

$$D_{(a_j,b_j)}=\{(x,y):a_j\le x\le b_j,\ a_j\le y\le b_j\}, \qquad j=1,2,$$

where $D_{(a_j,b_j)} \subset \mathbb{R}^2$, be continuous and have bounded variation. Assume also that the variation of the function

$$|z|^{\frac{3}{4}+\delta_1}\,\psi(z)$$

is bounded for any $z < a_0$, where $a_0 = \text{const}$.

Moreover, let the following asymptotic estimates hold:

$$u_0(x,y) \sim O\Big(\exp\big\{\text{const}\,(|x|^{\frac{3}{2}} + |y|^{\frac{3}{2}})\big\}\Big), \qquad x \to \infty,\ y \to \infty, \tag{15}$$

$$u_0(x,y) \sim O\Big(\psi(y)\,\exp\big\{\text{const}\,|x|^{\frac{3}{2}}\big\}\Big), \qquad x \to \infty,\ y < a, \tag{16}$$

$$u_0(x,y) \sim O\Big(\psi(x)\,\exp\big\{\text{const}\,|y|^{\frac{3}{2}}\big\}\Big), \qquad x < a,\ y \to \infty, \tag{17}$$

$$u_0(x,y) \sim O\big(\psi(x)\psi(y)\big), \qquad x < a,\ y < a, \tag{18}$$

where δ_1, δ_2 are sufficiently small positive numbers.

Therefore, for $t > 0$, the formula

$$u(x,y,t) = \frac{1}{\pi^2}\iint_{\mathbb{R}^2} U(x-\xi,\ y-\eta,\ t)\,u_0(\xi,\eta)\,d\xi\,d\eta \tag{10}$$

provides a solution of equation (1) and satisfies the condition

$$\lim_{(x,y,t)\to(x_0,y_0,0^+)} u(x,y,t) = u_0(x_0,y_0). \tag{11}$$

Proof. We prove the first part of the theorem. To this end, we formally differentiate expression (10) with respect to the variables x and y, which yields

$$\begin{gathered} u_{xx}(x,y,t) + u_{yy}(x,y,t) = \\ = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\Big[U_{xx}(x-\xi,\ y-\eta,\ t) + U_{yy}(x-\xi,\ y-\eta,\ t)\Big]u_0(\xi,\eta)\,d\xi\,d\eta = \\ = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\Big[U_{xx}(x,y,t) + U_{yy}(x,y,t)\Big]u_0(x-\xi,\ y-\eta)\,d\xi\,d\eta. \end{gathered}$$

Using relation (5) in the calculation of the derivatives, we find

$$\begin{aligned} \pi^2 t\big(u_{xx}+u_{yy}\big) &= -\frac{1}{3}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(\xi)f(\eta)\,u_0\Big(x-\xi t^{\frac{1}{3}},\ y-\eta t^{\frac{1}{3}}\Big)\,d\xi\,d\eta \\ &\quad -\frac{1}{3}\int_{-\infty}^{+\infty} f(\eta)\,d\eta\int_{-\infty}^{+\infty}\xi f'(\xi)\,u_0\Big(x-\xi t^{\frac{1}{3}},\ y-\eta t^{\frac{1}{3}}\Big)\,d\xi \\ &\quad -\frac{1}{3}\int_{-\infty}^{+\infty} f(\xi)\,d\xi\int_{-\infty}^{+\infty}\eta f'(\eta)\,u_0\Big(x-\xi t^{\frac{1}{3}},\ y-\eta t^{\frac{1}{3}}\Big)\,d\eta \\ &= -\frac{1}{3}\big\{u_1(x,y) + u_2(x,y) + u_3(x,y)\big\}. \end{aligned} \tag{12}$$

We will show that if the conditions of Theorem 2 are satisfied, then the formally differentiated integral in (12) converges uniformly.

The following relations are valid for the Airy functions [3]:

$$\begin{aligned} f(z) &\sim z^{-\frac{1}{4}} \exp\left(-\frac{2}{3}|z|^{\frac{3}{2}}\right)\left(\sqrt{\pi}+O\left(|z|^{-\frac{3}{2}}\right)\right), \\ f'(z) &\sim z^{\frac{1}{4}} \exp\left(-\frac{2}{3}|z|^{\frac{3}{2}}\right)\left(\sqrt{\pi}+O\left(|z|^{-\frac{3}{2}}\right)\right), \end{aligned} \tag{13}$$

For sufficiently large negative z, the following asymptotic relations hold:

$$\begin{aligned} f(z) &\sim z^{-\frac{1}{4}} \cos\left(\frac{2}{3}|z|^{\frac{3}{2}}-\frac{\pi}{4}\right)\left(\sqrt{\pi}+O\left(|z|^{-\frac{3}{2}}\right)\right), \\ f'(z) &\sim z^{\frac{1}{4}} \sin\left(\frac{2}{3}|z|^{\frac{3}{2}}-\frac{\pi}{4}\right)\left(\sqrt{\pi}+O\left(|z|^{-\frac{3}{2}}\right)\right), \end{aligned} \tag{14}$$

for sufficiently large positive z.

First, we examine the second term on the right-hand side of (12). The third term is treated similarly.

From (12), for sufficiently large positive numbers r and s, we obtain

$$u_2(x,y,t)=\int_{-\infty}^{+\infty} f(\eta)\,d\eta \int_{-\infty}^{+\infty} \xi f'(\xi)\,u_0\left(x-\xi t^{\frac{1}{3}},\, y-\eta t^{\frac{1}{3}}\right)\,d\xi =$$

$$= \left(\int_{-\infty}^{-r}+\int_{-r}^{r}+\int_{r}^{+\infty}\right) f(\eta)\,d\eta \left(\int_{-\infty}^{-s}+\int_{-s}^{s}+\int_{s}^{+\infty}\right) \xi f'(\xi)\,u_0\left(x-\xi t^{\frac{1}{3}},\, y-\eta t^{\frac{1}{3}}\right)\,d\xi =$$

$$= \left(\int_{-\infty}^{-r}+\int_{-r}^{r}+\int_{r}^{+\infty}\right) f(\eta)\,d\eta \left(\int_{-\infty}^{-s}+\int_{-s}^{s}+\int_{s}^{+\infty}\right) \left(J_1, J_2, J_3\right) d\xi.$$

Let $\xi \in (-\infty, -s]$ and $\eta \in (-\infty, -r]$. Then, by virtue of condition (13), we obtain

$$\begin{aligned} J_1(x,y,t) &= \int_{-\infty}^{-r} f(\eta)\,d\eta \int_{-\infty}^{-s} \xi f'\left(x-\xi t^{\frac{1}{3}}\right) u_0\left(x-\xi t^{\frac{1}{3}},\, y-\eta t^{\frac{1}{3}}\right)\,d\xi \\ &\sim O\left(\int_{r}^{+\infty} \eta^{-\frac{1}{4}} \exp\left(-\eta^{\frac{3}{2}}\left[\frac{2}{3}-Cy^{-\delta_2}\left(\frac{y}{\eta t^{\frac{1}{3}}}\right)^{\frac{3}{2}-\delta_2}\right]\right) d\eta\right) \\ &\quad \times O\left(\int_{s}^{+\infty} \xi^{-\frac{1}{4}} \exp\left(-\xi^{\frac{3}{2}}\left[\frac{2}{3}-Cy^{-\delta_1}\left(\frac{y}{\xi t^{\frac{1}{3}}}\right)^{\frac{3}{2}-\delta_1}\right]\right) d\xi\right). \end{aligned}$$

This integral converges uniformly to zero as $s \to \infty$ and $r \to \infty$.

Similarly, it can be shown that in the cases $\xi \in (-\infty, -s]$, $\eta \in [r, +\infty)$ and $\xi \in [s, +\infty)$, $\eta \in (-\infty, -r]$ the corresponding integrals also converge uniformly.

Let $\xi \in [s, +\infty)$ and $\eta \in [r, +\infty)$. The convergence of the integrals J_1 and J_2 follows from [16].

Following a similar approach for the remaining integral, we obtain uniform convergence of the integrals for $t > t_0 > 0$ and $a \leq x \leq b$. Consequently, differentiation under the integral sign is justified. Using functions with compact support, it is proved that representation (10) satisfies relation (11).

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On a Method for Constructing Composite Optimal Quadrature Formulas in the Hilbert space of Periodic Functions

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In recent years, several results based on the application of functional-theoretical concepts have been obtained in particular problems of computational mathematics. One can point to several problems in which functional-theoretical approaches yield general results: the construction of quadrature and cubature formulas, interpolation of functions of one and many variables, and the approximate solution of integral and differential equations (see [1]).

Of the listed questions, the question of approximate integral calculation has been developed in more detail than the others (see [2]). Let us consider a composite quadrature formula of the form:

$$\int_0^1 \varphi(x)\,dx \approx \sum_{\beta=0}^{N}\sum_{\gamma=0}^{m-1} C_{\beta\gamma}\varphi^{\gamma}(x_\beta) \tag{1}$$

here φ is from the Hilbert space $W_2^{(m,m-1)}$, $C_{\beta\gamma}$ are the coefficients of the quadrature formula, and $x_\beta \in [0,1]$. Furthermore, we will consider the error of the quadrature formula as a linear functional of the form:

$$l(x) = \varepsilon_{[0,1]}(x) - \sum_{\beta=0}^{N}\sum_{\gamma=0}^{m-1} C_{\beta\gamma}\delta^{(\gamma)}(x - x_\beta) \tag{2}$$

in the space $W_2^{(m,m-1)}$ is classes of functions possessing generalized derivatives of order m with an integrable square and the norm:

$$\|\varphi\|_{W_2^{(m,m-1)}} = \Big(\int_0^1 (\varphi^{(m)}(x) + \varphi^{(m-1)}(x))^2 dx\Big)^{1/2} \tag{3}$$

where

$$\varepsilon_{[0,1]}(x)$$

denotes the characteristic function of the interval $[0,1]$.
The space $W_2^{\sim(m,m-1)}$ is the class of periodic functions from $W_2^{(m,m-1)}$, i.e.,

$\varphi^{(\lambda)}(0) = \varphi^{(\lambda)}(1)$ for $\lambda = 0, 1, ..., m-1$. Furthermore, $\int_0^1 \varphi(x)\ dx = 0$. Elements of the space $W_2^{\sim(m,m-1)}$ are functions that differ from each other by a constant term. The norm of a function in the space of periodic functions $W_2^{\sim(m,m-1)}$ is determined by the formula (1.3).

Theorem 1. Let $\varphi \in L_2^{(m)}$, then the following function:

$$\varphi^{\sim} = \varphi(x) - \int_0^1 \varphi(t)dt - \sum_{k=1}^{m} \frac{B_k(x)}{k!}(\varphi^{(k-1)}(1) - \varphi^{(k-1)}(0))$$

is a periodic function in the space, i.e., an element of the space $W_2^{\sim(m,m-1)}$, where $\varphi^{\sim(\lambda)}(0) = \varphi^{\sim(\lambda)}(1)$ for $\lambda = 0, 1, ..., m-1$
$\int_0^1 \varphi^{\sim}(x)dx = 0$

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Numerical methods for solving the Neumann problem for the wave equation

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DOI 10.5281/zenodo.18217534

Keywords: Lyapunov function, hyperbolic system, difference scheme, mixed problem, numerical solution

Abstract This paper investigates the numerical solution of the Neumann boundary value problem for the one-dimensional wave equation. In addition, the study focuses on transforming the classical second-order wave equation into a system of first-order symmetric hyperbolic equations and applying appropriate boundary and initial conditions. Using the method of Riemann invariants, is reduced to a canonical form suitable for numerical treatment. A difference scheme is constructed to approximate the solution, and its stability is analyzed through the Lyapunov functional method. Conditions ensuring exponential stability of the numerical scheme are derived. Numerical experiments demonstrate the accuracy and stability of the proposed approach, confirming the theoretical results.

Introduction

Neumann problems are applied to the wave propagation equation and to the management of processes associated with wave propagation, especially the control of gas or oil pressure in pipelines. The existence of solutions to differential problems of this type has been investigated in [1]-[3], and their numerical solutions were analyzed in [4]-[6]. In this article, we attempt to apply Lyapunov stability theory to the numerical solution of Neumann problems for one-dimensional hyperbolic systems. In particular, we study the problem of constructing and analyzing a difference scheme for the numerical computation of stable solutions to the Neumann problem for a one-dimensional linear symmetric t-hyperbolic system. The main idea of this research is to investigate the Lyapunov stability of solutions to mixed problems for one-dimensional hyperbolic systems by constructing a Lyapunov function and obtaining a priori estimates in functional spaces

Formulation of the problem.

$$u_{tt} = a^2 u_{xx}, \quad 0 \le x \le l, \quad t > 0 \tag{1}$$

$$u(0,x) = \varphi(x), \quad u_t(0,x) = \psi(x) \tag{2}$$

$$u_x(t,0) = \gamma_1 u_t(t,0), \quad u_x(t,l) = -\gamma_2 u_t(t,l) \tag{3}$$

here,

$$a_1, \gamma_1, \gamma_1 > 0$$

are real numbers.

The main objective of this study is to investigate the numerical solution of the Neumann problem associated with the wave propagation equation. The problem is reformulated as a system of two first-order partial differential equations.

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \frac{\partial}{\partial t} \begin{pmatrix} w_1(t,x) \\ w_2(t,x) \end{pmatrix} + \begin{pmatrix} 0 & -a^2 \\ -1 & 0 \end{pmatrix} \frac{\partial}{\partial x} \begin{pmatrix} w_1(t,x) \\ w_2(t,x) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{4}$$

$$at, t = 0 \;\; w_1(0,x) = \varphi(x), \quad w_2(0,x) = \psi_x(x) \tag{5}$$

initial conditions at $x = 0$ and $x = l$,

$$w_2(t,0) = \gamma_1 w_1(t,0), \quad w_2(t,l) = -\gamma_2 w_1(t,l) \tag{6}$$

We have obtained a mixed problem posed for the symmetric t-hyperbolic system of equations (4)-(6) that satisfies the boundary conditions. Now, in this problem, we make the following substitution to bring it into canonical form:

$$\begin{pmatrix} w_1(t,x) \\ w_2(t,x) \end{pmatrix} = \begin{pmatrix} -a & a \\ 1 & 1 \end{pmatrix} \begin{pmatrix} u_1(t,x) \\ u_2(t,x) \end{pmatrix} \tag{7}$$

and

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \frac{\partial}{\partial t} \begin{pmatrix} u_1(t,x) \\ u_2(t,x) \end{pmatrix} + \mathbf{K} \frac{\partial}{\partial x} \begin{pmatrix} u_1(t,x) \\ u_2(t,x) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{8}$$

here

$$\mathbf{K} = \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix}.$$

The initial conditions with respect to Riemann invariants are:

$$u_1(0,x) = \Phi(x) \triangleq \triangle = \triangleq \frac{1}{2}\varphi_x(x) - \frac{1}{2a}\psi(x), u_2(0,x) = \Psi(x) \triangleq \frac{1}{2a}\psi(x) - \frac{1}{2}\varphi_x(x) \quad (9)$$

and

$$u_1(t,0) = -\frac{1-\gamma_1 a}{1+\gamma_1 a}\, u_2(t,0), \quad u_2(t,l) = \frac{1-\gamma_2 a}{1+\gamma_2 a}\, u_1(t,l). \quad (10)$$

boundary conditions are written in this form. Let the initial data

$$\varphi(x), \psi(x) \in W_2^1((0,l), R^n)$$

satisfy the compatibility condition:

$$\begin{cases} a\varphi_x(0) = \psi(0), & x = 0,\ t = 0, \\ a\gamma_2\varphi_x(l) = -\psi(l), & x = l,\ t = 0, \end{cases} \quad (11)$$

here $W_2^1((0,l), R^n)$ is the Sobolev space. Lyapunov function

$$L(t) = \int_0^l (\mu(x) \cdot U(t,x), U(t,x))\, dx \quad (12)$$

we get in sight. Here

$$\mu(x) = \begin{pmatrix} e^{-vx}\mu_1 & 0 \\ 0 & e^{vx}\mu_2 \end{pmatrix}, \quad U(t,x) = \begin{pmatrix} u_1(t,x) \\ u_2(t,x) \end{pmatrix}.$$

become a matrix, $v,\ \mu_1,\ \mu_2$- positive numbers. In this work, the exponential stagnation of the mixed problem (8)-(10) are studied and a difference scheme is built as numerical solution. Also, the convergence scheme is proved and the results of a numerical experiment are presented.

Theorem(Exponential stability). System (8) with boundary conditions (10) is exponentially stable in the L^2, if there exists such $\nu > 0$ and $\mu_i > 0,\ i = 1,2$ that the following matrices

$$\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \begin{pmatrix} e^{-\nu l}\mu_1 & 0 \\ 0 & \mu_2 \end{pmatrix} - \begin{pmatrix} 0 & r \\ s & 0 \end{pmatrix} \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \begin{pmatrix} \mu_1 & 0 \\ 0 & e^{\nu l}\mu_2 \end{pmatrix} \begin{pmatrix} 0 & s \\ r & 0 \end{pmatrix} \quad (13)$$

are positive definite.

Proof. We calculate the derivative of the Lyapunov function:

$$L'(t) = \int_0^l \partial_t(\mu(x)U, U)\, dx = \int_0^l (\mu(x)\partial_t U, U)\, dx + \int_0^l (\mu(x)U, \partial_t U)\, dx$$

$$= \int_0^l (\mu(x)(-\mathbf{K}\partial_x U), U)\, dx + \int_0^l (\mu(x)U, (-\mathbf{K}\partial_x U))\, dx$$

$$= -\int_0^l \left[(\mu(x)\mathbf{K}\partial_x U, U) + (\mu(x)U, \mathbf{K}\partial_x U)\right] dx.$$

since $\mu'(x)K = -\nu|K|\mu(x)$ and $(\mu(x)K\partial_x U, U) = \partial_x(\mu(x)KU, U) - (\mu'(x)KU, U)$, we have the following identity:

$$L'(t) = -\int_0^l \partial_x(\mu(x)\mathbf{K}U, U)\, dx - \int_0^l (\nu|\mathbf{K}|\mu(x)U, U)\, dx = -(\mathbf{K}\mu(x)U, U)\Big|_0^l - \int_0^l (\nu|\mathbf{K}|\mu(x)U, U)\, dx,$$

where

$$|\mathbf{K}| = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}.$$

Transform separately the each term of the penultimate obtained identity:

$$\begin{aligned}
-(K\mu(x)U, U)\Big|_0^l &= -\Big[(K\mu(l)U(t,l), U(t,l)) - (K\mu(0)U(t,0), U(t,0))\Big] \\
&= -\left(\begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix} \begin{pmatrix} e^{-vl}\mu_1 & 0 \\ 0 & e^{vl}\mu_2 \end{pmatrix} \begin{bmatrix} u_1(t,l) \\ u_2(t,l) \end{bmatrix}, \begin{bmatrix} u_1(t,l) \\ u_2(t,l) \end{bmatrix} \right) \\
&\quad + \left(\begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix} \begin{pmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{pmatrix} \begin{bmatrix} u_1(t,0) \\ u_2(t,0) \end{bmatrix}, \begin{bmatrix} u_1(t,0) \\ u_2(t,0) \end{bmatrix} \right) \\
&= -\left(\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \begin{pmatrix} e^{-vl}\mu_1 & 0 \\ 0 & \mu_2 \end{pmatrix} \begin{bmatrix} u_1(t,l) \\ u_2(t,0) \end{bmatrix}, \begin{bmatrix} u_1(t,l) \\ u_2(t,0) \end{bmatrix} \right) \\
&\quad + \left(\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \begin{pmatrix} \mu_1 & 0 \\ 0 & e^{vl}\mu_2 \end{pmatrix} \begin{bmatrix} u_1(t,0) \\ u_2(t,l) \end{bmatrix}, \begin{bmatrix} u_1(t,0) \\ u_2(t,l) \end{bmatrix} \right) \\
&= -\left(\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \begin{pmatrix} e^{-vl}\mu_1 & 0 \\ 0 & \mu_2 \end{pmatrix} \begin{bmatrix} u_1(t,l) \\ u_2(t,0) \end{bmatrix}, \begin{bmatrix} u_1(t,l) \\ u_2(t,0) \end{bmatrix} \right) \\
&\quad + \left(\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \begin{pmatrix} \mu_1 & 0 \\ 0 & e^{vl}\mu_2 \end{pmatrix} \begin{pmatrix} 0 & s \\ r & 0 \end{pmatrix} \begin{bmatrix} u_1(t,l) \\ u_2(t,0) \end{bmatrix}, \begin{pmatrix} 0 & s \\ r & 0 \end{pmatrix} \begin{bmatrix} u_1(t,l) \\ u_2(t,0) \end{bmatrix} \right) \\
&= -\left(\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \begin{pmatrix} e^{-vl}\mu_1 & 0 \\ 0 & \mu_2 \end{pmatrix} \begin{bmatrix} u_1(t,l) \\ u_2(t,0) \end{bmatrix}, \begin{bmatrix} u_1(t,l) \\ u_2(t,0) \end{bmatrix} \right) \\
&\quad + \left(\begin{pmatrix} as & 0 \\ 0 & ar \end{pmatrix} \begin{pmatrix} \mu_1 s & 0 \\ 0 & e^{vl}\mu_2 r \end{pmatrix} \begin{bmatrix} u_1(t,l) \\ u_2(t,0) \end{bmatrix}, \begin{bmatrix} u_1(t,l) \\ u_2(t,l) \end{bmatrix} \right) < 0
\end{aligned}$$

Taking these transformations into account, we obtain

$$L'(t) = -\left(\mathbf{K}\mu(x)U, U\right)\Big|_0^l - \int_0^l \left(\left[\nu\left|\mathbf{K}\right|\mu(x)\right]U, U\right) dx < 0.$$

If we choose large enough, then the inequality holds

$$\left(\left[\nu\left|\mathbf{K}\right|\mu(x)\right]U, U\right) > a(\mu(x)U, U)$$

Then we have $L'(t) < -\alpha L(t)$. Hence,

$$L(t) \le e^{-\alpha t} L(0), \quad t > 0.$$

However, since there is such a constant $\gamma > 0$, such that

$$\frac{1}{\gamma} \|U(t,\cdot)\|^2_{L^2((0,l),R^n)} \le L(t) \le \gamma \|U(t,\cdot)\|^2_{L^2((0,l),R^n)}$$

we obtain

$$\|U(t,\cdot)\|_{L^2((0,l),R^n)} \le \gamma e^{-\nu t/2} \left\| \begin{pmatrix} \varphi \\ \psi \end{pmatrix} \right\|_{L^2((0,l),R^n)}, \quad t \in [0,+\infty).$$

The differefce scheme

We take the equidistant grid:

$$t_n = n\Delta t, \quad n = \overline{0,N}, \ N\Delta t = T, \ x_{j-\frac{1}{2}} = j\Delta x,$$

$$x_j = \left(j + \frac{1}{2}\right)\Delta x, \ M\Delta x = l, \ j = \overline{0,M-1},$$

$$U_j^n = \frac{1}{\Delta x} \int\limits_{x_{j-\frac{1}{2}}}^{x_{j+\frac{1}{2}}} U(t^n,x)dx, \ \ j = \overline{0,M-1}$$

We consider the following mixed problem for the one-dimensional hyperbolic system (8)-(10) and integrate (8) over the interval $\left[x_{j-\frac{1}{2}}, x_{j+\frac{1}{2}}\right]$

$$\partial_t \int\limits_{x_{j-\frac{1}{2}}}^{x_{j+\frac{1}{2}}} U(t,x)dx = \mathbf{K}\left(U\left(t, x_{j-\frac{1}{2}}\right) - U\left(t, x_{j+\frac{1}{2}}\right)\right) j = \overline{0,M-1}. \tag{14}$$

We integrate the resulting equality over t on the interval $[t_n, t_{n+1}]$:

$$\int\limits_{x_{j-\frac{1}{2}}}^{x_{j+\frac{1}{2}}} U(t_{n+1},x)dx - \int\limits_{x_{j-\frac{1}{2}}}^{x_{j+\frac{1}{2}}} U(t_n,x)dx = \mathbf{K}\left(\int\limits_{t_n}^{t_{n+1}} U(t, x_{j-\frac{1}{2}})dt - \int\limits_{t_n}^{t_{n+1}} U(t, x_{j+\frac{1}{2}})dt\right) \tag{15}$$

$$j = \overline{1..M-1}$$

or

$$U_j^{n+1} = U_j^n - \frac{\Delta t}{\Delta x}\mathbf{K}\left(U^n_{j+\frac{1}{2}} - U^n_{j-\frac{1}{2}}\right) \tag{16}$$

$$j = \overline{0,M-1}, \ \ n = \overline{0,N-1}$$

Let us assume that the KFL condition is satisfied $a\frac{\Delta t}{\Delta x} \le 1$

$$\begin{bmatrix} (u_1)_j^{n+1} \\ (u_2)_j^{n+1} \end{bmatrix} = \begin{bmatrix} (u_1)_j^n \\ (u_2)_j^n \end{bmatrix} - a\frac{\Delta t}{\Delta x}\begin{bmatrix} (u_1)_j^n - (u_1)_{j-1}^n \\ (u_2)_j^n - (u_2)_{j+1}^n \end{bmatrix} \tag{17}$$

$$j = \overline{0, M-1}, \;\; n = \overline{0, N-1}.$$

The initial conditions (9) are approximated exactly

$$(u_1)_j^0 = \frac{1}{\Delta x} \int\limits_{x_{j-\frac{1}{2}}}^{x_{j+\frac{1}{2}}} \Phi(x)dx, \;\; (u_2)_j^0 = \frac{1}{\Delta x} \int\limits_{x_{j-\frac{1}{2}}}^{x_{j+\frac{1}{2}}} \Psi(x)dx, \tag{18}$$

$$j = \overline{0..M-1}$$

The boundary conditions are approximated as follows

$$\begin{pmatrix} (u_1)_{-1}^{n+1} \\ (u_2)_M^{n+1} \end{pmatrix} = \begin{pmatrix} 0 & s \\ r & 0 \end{pmatrix} \begin{pmatrix} (u_1)_{M-1}^{n+1} \\ (u_2)_0^{n+1} \end{pmatrix}. \tag{19}$$

Definition. The difference scheme (17) with the difference boundary condition (19) is exponentially stable if there exist constants $\eta > 0$ and $C > 0$ such that, for any initial condition,

$$U_j^0 \in L^2((x_{j-\frac{1}{2}}, x_{j+\frac{1}{2}}), R^n)$$

the solution of the difference boundary value problem (17)-(19) satisfies the inequality

$$\Delta x \sum_{j=0}^{M-1} \left(U_j^n, U_j^n\right) \le e^{-\eta t_n} \Delta x \sum_{j=0}^{M-1} \left(U_j^0, U_j^0\right),$$

$$n = \overline{1, N}.$$

As a discrete Lyapunov function we propose

$$L^n = \Delta x \sum_{j=0}^{M-1} (U_j^n, \mu_j U_j^n), \;\; \mu_j = \mu(x_j), \tag{20}$$

$$j = \overline{1..M-1}$$

Theorem. Let $T > 0$ and be a discrete Lyapunov function defined by formula (20). If the CFL condition is satisfied $a\frac{\Delta t}{\Delta x} \le 1$ and there exist real numbers $\nu > 0$ and $\mu_i > 0, \;\; i = 1, 2$ such that $0 < a(1 - e^{-\nu \Delta x}) < 1$ and

$$\begin{bmatrix} \mu_1 a e^{-\nu x_M} & 0 \\ 0 & \mu_2 a e^{\nu x_{-1}} \end{bmatrix} - \begin{pmatrix} 0 & r \\ s & 0 \end{pmatrix} \begin{bmatrix} \mu_1 a e^{-\nu x_0} & 0 \\ 0 & \mu_2 a e^{\nu x_{M-1}} \end{bmatrix} \begin{pmatrix} 0 & s \\ r & 0 \end{pmatrix}$$

positive definite matrix, then the numerical solution

$$U_j^n = \begin{pmatrix} (u_1)_j^n \\ (u_2)_j^n \end{pmatrix}$$

of the difference boundary value problem (17)-(19) is exponentially stable for the norm L^2 the norm.

Proof. Let us introduce the following notation$[W_j^n = \begin{bmatrix} (u)_{j-1}^n \\ (v)_{j+1}^n \end{bmatrix}, \; \mathbf{D} = \frac{\Delta t}{\Delta x} \; |\mathbf{K}|$. Taking these notations into account, we calculate the difference ratio for the discrete Lyapunov function: 1).

$$\frac{L^{n+1} - L^n}{\Delta t} = \frac{\Delta x}{\Delta t} \sum_{j=0}^{M-1} \left[(U_j^{n+1}, \mu_j U_j^{n+1}) - (U_j^n, \mu_j U_j^n)\right]$$

$$= \frac{\Delta x}{\Delta t} \sum_{j=0}^{M-1} \left[\left(\left\{ U_j^n - \mathbf{D} \left[U_j^n - W_j^n \right] \right\}, \mu_j \left\{ U_j^n - \mathbf{D} \left[U_j^n - W_j^n \right] \right\} \right) \right.$$

$$\left. - \frac{\Delta x}{\Delta t} \sum_{j=0}^{M-1} \left(U_j^n, \mu_j U_j^n \right) \right] = \frac{\Delta x}{\Delta t} \sum_{j=0}^{M-1} \left[\left(U_j^n, \mu_j U_j^n \right) - \left(U_j^n, \mu_j U_j^n \right) \right] - \frac{\Delta x}{\Delta t} \sum_{j=0}^{M-1} \left[2 \left(\mu_j U_j^n, \mathbf{D} \left[U_j^n - W_j^n \right] \right) \right];$$

2).

$$\frac{\Delta x}{\Delta t} \sum_{j=0}^{M-1} \left(\mu_j \mathbf{D} \left[U_j^n - W_j^n \right], \mathbf{D} \left[U_j^n - W_j^n \right] \right)$$

$$= \frac{\Delta x}{\Delta t} \sum_{j=0}^{M-1} \left(\mu_j \mathbf{D} \left[U_j^n - W_j^n \right], \mathbf{D} \left[U_j^n - W_j^n \right] \right) - \sum_{j=0}^{M-1} 2 \left(\mu_j U_j^n, \mathbf{D} \left[U_j^n - W_j^n \right] \right) =$$

$$= \frac{\Delta x}{\Delta t} \sum_{j=0}^{M-1} \left[\left(\mu_j \mathbf{D} U_j^n, \mathbf{D} U_j^n \right) \right.$$

$$\left. - \frac{\Delta x}{\Delta t} \sum_{j=0}^{M-1} \left(\mu_j \mathbf{D} W_j^n, \mathbf{D} W_j^n \right) - 2 \left(\mu_j \mathbf{D} U_j^n, \mathbf{D} W_j^n \right) - 2 \left(\mu_j U_j^n, \mathbf{D} U_j^n \right) + 2 \left(\mu_j U_j^n, \mathbf{D} W_j^n \right) \right.$$

$$= \frac{\Delta x}{\Delta t} \sum_{j=0}^{M-1} \left[\left(\mu_j \mathbf{D} U_j^n, \mathbf{D} U_j^n \right) \right] + 2 \left(\mu_j (\mathbf{E} - \mathbf{D}) U_j^n, \mathbf{D} W_j^n \right) +$$

$$+ \frac{\Delta x}{\Delta t} \sum_{j=0}^{M-1} \left(\mu_j \mathbf{D} W_j^n, \mathbf{D} W_j^n \right) - 2 \left(\mu_j U_j^n, \mathbf{D} U_j^n \right)$$

By virtue of the CFL condition, the matrix $(\mathbf{E} - \mathbf{D}) \geq 0$ and $(\mathbf{E} - \mathbf{D}) \mu_j \mathbf{D}$- a positive definite diagonal matrix. Taking into account the boundary conditions

$$\begin{pmatrix} (u_1)_{-1}^{n+1} \\ (u_2)_J^{n+1} \end{pmatrix} = \begin{pmatrix} 0 & s \\ r & 0 \end{pmatrix} \begin{pmatrix} (u_1)_{M-1}^{n+1} \\ (u_2)_0^{n+1} \end{pmatrix}$$

we will receive

$$\frac{\Delta x}{\Delta t} \sum_{j=0}^{M-1} \left(\mu_j W_j^\kappa, \mathbf{D} W_j^\kappa \right) = e^{-\nu \Delta x} \sum_{j=0}^{M-1} \left(U_j^n, \mu_j \left| \mathbf{K} \right| U_j^n \right) +$$

$$+ \left(\begin{pmatrix} r \, (u_1)_{M-1}^{n+1} \\ s \, (u_2)_0^{n+1} \end{pmatrix}, \begin{bmatrix} \mu^+ e^{-\nu x_0} a \cdot s & 0 \\ 0 & \mu^- e^{\nu x_{M-1}} a \cdot r \end{bmatrix} \begin{pmatrix} (u_1)_{M-1}^{n+1} \\ (u_2)_0^{n+1} \end{pmatrix} \right)$$

$$- \left(\begin{bmatrix} (u_1)_{M-1}^n \\ (u_2)_0^n \end{bmatrix}, \begin{bmatrix} \mu^+ e^{-\nu x_M} a & 0 \\ 0 & \mu^- e^{\nu x_{-1}} a \end{bmatrix} \begin{bmatrix} (u_1)_{M-1}^n \\ (u_2)_0^n \end{bmatrix} \right)$$

Let us assume that the dissipativity condition is satisfied

$$\begin{bmatrix} \mu^+ a e^{-\nu x_M} & 0 \\ 0 & \mu^- a e^{\nu x_{-1}} \end{bmatrix} - \begin{bmatrix} \mu^+ e^{-\nu x_0} a \cdot s^2 & 0 \\ 0 & \mu^- e^{\nu x_{M-1}} a \cdot r^2 \end{bmatrix} > 0$$

Then we have

$$\frac{\Delta x}{\Delta t} \sum_{j=0}^{M-1} \left(\mu_j W_j^n, \mathbf{D} W_j^n \right) \le e^{-\nu \Delta x} \sum_{j=0}^{M-1} \left(U_j^n, \mu_j \left| \mathbf{K} \right| U_j^n \right)$$

and therefore

$$\frac{L^{n+1} - L^n}{\Delta t} \le e^{-\nu \Delta t} \sum_{j=0}^{M-1} (U_j^n, \mu_j \mid K U_j^n) - \sum_{j=0}^{M-1} (U_j^n, \mu_j \mid K U_j^n) =$$

$$= (e^{-\nu \Delta t} - 1) \sum_{j=0}^{M-1} (U_j^n, \mu_j \mid K U_j^n) \le 0$$

Here $\eta = a(1 - e^{-\nu \Delta x})$. Let $0 < \eta < 1$. Then

$$\frac{L^{n+1} - L^n}{\Delta t} \le -\eta L^n.$$

Recursively applying this inequality we obtain

$$L^{n+1} < (1 - \Delta t \eta)^{n+1} L^0 \le e^{-\eta \Delta t (n+1)} L^0 = e^{-\eta t_{n+1}} L^0,$$

$$n = 0, \ldots, N-1.$$

Let us denote

$$_1 = \min_{\substack{1 \le i \le n \\ 0 \le j \le M-1}} \left\{ \sigma_{ij} : |\mu_j - \sigma_{ij} \mathbf{E}| = 0 \right\},$$

$$_2 = \max_{\substack{1 \le i \le n \\ 0 \le j \le M-1}} \left\{ \sigma_{ij} : |\mu_j - \sigma_{ij} \mathbf{E}| = 0 \right\}.$$

Then

$$_1 \mathbf{E} \le \mu_j \le _2 \mathbf{E},$$

$$j = 0, \ldots, M-1.$$

Series Solution and Exact Result

$$\Delta x \sum_{j=0}^{M-1} \left(U_j^n, U_j^n \right) \le L^n \le C_2 e^{-\eta t_n} \Delta x \sum_{j=0}^{M-1} \left(U_j^0, U_j^0 \right),$$

$$n = 0, \ldots, N.$$

$$\Delta x \sum_{j=0}^{M-1} \left(U_j^n, U_j^n \right) \le e^{-\eta t_n} \Delta x \sum_{j=0}^{M-1} \left(U_j^0, U_j^0 \right),$$

$$n = 0, \ldots, N; \ = _2 / C_1.$$

Therefore, the numerical solution U_j^n of the mixed problem is exponentially stable in L_2 the norm.

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A Symmetric 1–Volterra Quadratic Stochastic Operator for Portfolio–Share Dynamics

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DOI 10.5281/zenodo.18217547

Abstract

We study a discrete-time nonlinear stochastic operator on the 2-simplex that models the evolution of portfolio shares among three competing financial choices under pairwise social interaction. Focusing on a symmetric (neutral) 1вЂ‘Volterra quadratic stochastic operator, we show that the conservative share is invariant over time and the remaining two shares satisfy a one-dimensional affine recursion with an explicit solution. For any initial state with a nonzero conservative segment, the dynamics converge geometrically to a continuum of fixed points in which the residual share is split equally between deposits and alternative assets. Numerical iterations illustrate both monotone and oscillatory convergence depending on the invariant conservative share. [1–5]

Keywords: quadratic stochastic operator; 1вЂ‘Volterra operator; discretevЂ‘time dynamics; portfolio shares; fixed points; system modeling. [1–5]

1. Introduction

Discrete-time models are natural for many socio-economic processes where decisions are updated periodically (e.g., monthly rebalancing, quarterly reporting). When agents influence each other through pairwise communication (wordвЂ˜ofвЂ˜mouth) or imitation, the evolution of population shares can be described by nonlinear maps on a simplex. Quadratic stochastic operators (QSOs) are a classical class of such maps originating from pairwise interaction mechanisms; Volterra and в„“вЂ˜Volterra subclasses have been studied in detail in the dynamical systems literature. [1,2]

In this paper we work directly with a symmetric (neutral) operator, corresponding to 50–50 crossвЂ˜influence in pairwise meetings. This symmetry yields a transparent reduced dynamics with closed-form convergence results, making the model suitable for system analysis and computer modeling in economics and international relations. [1]

2. Notation and the Symmetric Operator (Hybrid Notation)

(Economic-layer notation.) Let s_t^{FX}, s_t^{DEP}, and s_t^{ALT} denote the population shares of: foreign-currency holding (conservative choice), domestic deposits, and alternative assets (e.g., gold/crypto) at discrete time t. The state vector $s_t = \left(s_t^{FX},\ s_t^{DEP},\ s_t^{ALT}\right)$ lies in the simplex Δ^2, i.e., all shares are nonnegative and sum to one.

(Mathematical-layer shorthand.) To keep formulas compact, we set $f_t := s_t^{FX}$, ($d_t := s_t^{DEP}$, and $r_t := s_t^{ALT}$. Hence $f_t + d_t + r_t = 1$ for all t.

$$f_t + d_t + r_t = 1, \quad f_t \geq 0,\ d_t \geq 0,\ r_t \geq 0, \quad t = 0, 1, 2, \ldots$$

$$\begin{cases} f_{t+1} = f_t \\ d_{t+1} = (1 - 2 f_t)\, d_t + f_t (1 - f_t) \quad \textit{(Eq. 1)} \\ r_{t+1} = 1 - f_{t+1} - d_{t+1} \end{cases}$$

3. Reduced One-Dimensional Dynamics and Explicit Solution

From (Eq. 1) we obtain invariance of the conservative share: $f_t \equiv f_0$. Substituting $f_t = f_0$ yields an affine recursion for d_t:

$d_{t+1} = \rho\, d_t + \sigma, \quad \rho := 1 - 2 f_0, \quad \sigma := f_0 (1 - f_0)$ *(Eq. 2)*

If $f_0 \in (0, 1)$, then $|\rho| < 1$, so (Eq. 2) converges geometrically. For $\rho \neq 1$, the closed-form solution is:

$d_t = \rho^t d_0 + \sigma \frac{1 - \rho^t}{1 - \rho}, \quad t \geq 0$ *(Eq. 3)*

The unique fixed point is $d^* = \frac{\sigma}{1-\rho} = \frac{1-f_0}{2}$, hence $r^* = \frac{1-f_0}{2}$. Therefore:

$(d_t,\ r_t) \to \left(\frac{1 - f_0}{2}, \frac{1 - f_0}{2}\right) \quad as \quad t \to \infty$ *(Eq. 4)*

4. Fixed-Point Set (Symmetric Case)

For the symmetric operator, the fixed-point set contains a continuum consisting of the entire edge $f = 0$ and the line $d = r$, together with the simplex vertices (as described in the source notes). In hybrid notation:

$$\{(0,\ \alpha,\ 1 - \alpha) :\ 0 < \alpha < 1\} \cup \left\{(1 - 2\alpha,\ \alpha,\ \alpha) :\ 0 < \alpha < \frac{1}{2}\right\} \cup \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$$

5. Numerical Illustration

We iterate (Eq. 2) for several values of f_0 and initial d_0. Figure 1 shows monotone convergence for $f_0 = 0.30$ ($\rho = 0.40$) and alternating convergence for $f_0 = 0.70$ ($\rho = -0.40$), both approaching $d^* = r^* = \frac{1-f_0}{2}$.

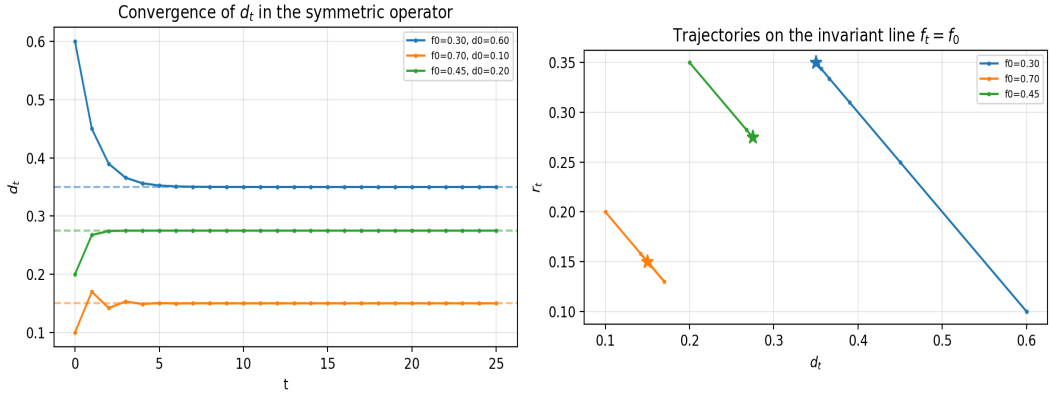

Figure 1. Iterations of the symmetric map: time series of d_t and trajectories in the (d_t, r_t)-plane for different invariant f_0.

6. Conclusion

By focusing directly on the symmetric operator, we obtained a compact hybrid-notation model with explicit reduced dynamics. The conservative share is invariant, and the remaining two shares converge to equal splitting of the residual share. This special case provides an interpretable benchmark for computer experiments and for future extensions where symmetry is relaxed or where other nonlinear stochastic operators are coupled to the share dynamics. [1–5]

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O'zbekiston aholisi umr davomiyligi funksiyasini parametrik statistik modellar asosida baholash

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DOI 10.5281/zenodo.18217579

Annotatsiya: Mazkur maqolada O'zbekiston Respublikasi aholisi umr davomiyligi funksiyasini parametrik statistik modellar asosida baholash masalalari ko'rib chiqiladi. Tadqiqotda umr davomiyligi musbat qiymatlarni qabul qiluvchi tasodifiy miqdor sifatida qaralib, rasmiy demografik ma'lumotlar asosida empirik umr davomiyligi funksiyasi quriladi. O'limlilik jarayonini tavsiflash maqsadida Gompertz hamda Weibull parametrik modellaridan foydalaniladi. Modellarning noma'lum parametrlarini baholashda haqiqatga maksimal o'xshashlik usuli qo'llaniladi. Olingan natijalar tanlangan parametrik modellar empirik umr davomiyligi funksiyasiga yetarli darajada yaxshi mos kelishini hamda o'limlilik jarayonining asosiy xususiyatlarini aniq aks ettirishini ko'rsatadi. Tadqiqot natijalari demografik tahlil, sug'urta va pensiya tizimlarini rejalashtirishda ilmiy-amaliy ahamiyatga ega.

Kalit so'zlar: umr davomiyligi, parametrik modellar, Gompertz modeli, Weibull modeli, demografik prognozlash

Аннотация: В данной статье рассматриваются вопросы оценки функции продолжительности жизни населения Республики Узбекистан на основе параметрических статистических моделей. В исследовании продолжительность жизни трактуется как случайная величина, принимающая положительные значения, и на основе официальных демографических данных строится эмпирическая функция выживания. Для описания процесса смертности используются параметрические модели Гомпертца и Вейбулла. Оценивание неизвестных параметров моделей осуществляется с применением метода максимального правдоподобия. Полученные результаты показывают, что выбранные параметрические модели обеспечивают достаточно хорошее приближение эмпирической функции продолжительности жизни и адекватно отражают основные особенности процесса смертности. Результаты исследования имеют научно-практическое значение для демографического анализа, а также для планирования систем страхования и пенсионного обеспечения.

Ключевые слова: продолжительность жизни, параметрические модели, модель Гомпертца, модель Вейбулла, демографическое прогнозирование.

Abstract: This article addresses the problem of estimating the life expectancy function of the population of the Republic of Uzbekistan using parametric statistical models. In the study, life expectancy is treated as a positive-valued random variable, and an empirical survival function is constructed based on official demographic data. To describe the mortality process, the Gompertz and Weibull parametric models are employed. The unknown parameters of the models are estimated using the maximum likelihood method.

The results demonstrate that the selected parametric models provide a sufficiently good approximation of the empirical life expectancy function and accurately capture the main characteristics of the mortality process. The findings of the study are of scientific and practical relevance for demographic analysis, as well as for planning insurance and pension systems.

Keywords: life expectancy, parametric models, Gompertz model, Weibull model, demographic forecasting.

Umr davomiyligi funksiyasi (survival function) $S(x)$ ehtimollar nazariyasi, statistik tahlil va aktuar matematikasining asosiy komponentlaridan biri hisoblanadi. Bu funksiya musbat qiymatlarni qabul qiluvchi tasodifiy miqdor T ni, umr davomiyligi yoki beto'xtov ishlash vaqti - x qiymatidan katta bo'lish ehtimolini bildiradi

$$S(x) = P(T > x) = 1 - F(x)$$

umr davomiyligi funksiyasini zichlik funksiyasi orqali ham ifodalash mumkin

$$S(x) = \int_0^\infty f(t)dt$$

Umr davomiyligi funksiyasi orqali hazard (xavf) funksiyasi

$$\mu(x) = \frac{f(x)}{S(x)} = -\frac{d}{dx} lnS(x),$$

kumulativ xavf funksiyasi

$$H(x) = -lnS(x) = \int_0^x \mu(t)dt,$$

x yoshdagi insonning kutilayotgan qolgan umrining o'rtacha qiymati

$$e_x = \int_0^\infty S(x,t)dt$$

kiritiladi. Bu funksiyalar birgalikda umr davomiyligining statistik strukturasini aniqlashda muhim ahamiyatga ega. Umr davomiyligi funksiyasi sug'urta sohasida keng qo'llaniladi, masalan, hayotni sug'urtalashda $S(x)$ yordamida sug'urta mukofotlari va kompaniya zaxiralari hisoblanadi; pensiya tizimlarida kutilayotgan yashab qolish ehtimolligi asosida pensiya to'lovlari rejasi shakllanadi; umr davomiyligi ortishi bilan bog'liq risklarni prognoz qilishda qo'llaniladi; aholi hayot sifati, umr davomiyligi va farovonlik darajasini o'lchashda asosiy ko'rsatkichlardan biri hisoblanadi, $S(x)$ asosida davlat byudjeti, sog'liqni saqlash va pensiya tizimi uchun zarur prognozlar amalga oshiriladi, e_x (o'rtacha umr davomiyligi) ko'rsatkich pensiya fondlari, hayot sug'urtasi, uzoq muddatli qarzlar va kreditlar uchun hal qiluvchi ahamiyatga.

Umr davomiyligi funksiyasi $S(x)$ g'oyasi inson umrining statistik, iqtisodiy va demografik modellashtirilishida uzoq tarixga ega. Quyida asosiy bosqichlarni ko'rib o'tamiz. John Graunt (1662) Londondagi o'lim statistikasini yig'ib, birinchi hayot jadvallarini yaratgan. U yoshga nisbatan yashab qolish ehtimolligini statistik tarzda ifodalab bergan. Graunt ishlari aholi sog'lig'i, migratsiya va tug'ilishlar haqida birinchi

ilmiy kuzatishlar bo'lgan. Edmond Halley (1693) Breslau (hozirgi Vrotslav, Polsha) aholisiga oid o'lim ma'lumotlariga asoslangan to'liq hayot jadvalini yaratgan. U har bir yosh uchun $S(x)$ ni quyidagicha baholagan

$$S(x) = \frac{Tirik\ qolganlar\ soni}{Yosh\ guruhi\ boshlanishidagi\ soni}$$

Bu jadval sug'urta kompaniyalarining ilk matematik vositasiga aylangan. Abraham de Muavr (1725) hayotni chiziqli kamayuvchi ehtimollik modeli sifatida tavsiflagan:

$$S(x) = 1 - \frac{x}{\omega},\ 0 \le x \le \omega$$

bu yerda ω – maksimal umr (masalan, $\omega = 86$ yil). Modelning soddaligi uni sug'urta va aktuar hisob-kitoblarida qo'llashni osonlashtirgan. Benjamin Gompertz (1825) biologik qarish jarayonini eksponensial o'lim stavkasi bilan ifodalalashni taklif qilgan:

$$\mu(x) = ae^{bx}, \Rightarrow\ S(x) = exp(-\frac{a}{b}(e^{bx} - 1))$$

Bu model inson umrida yosh ortishi bilan xavfning tezlashishini aks ettiradi. William Makeham (1860) Gompertz modelini kengaytirib, doimiy xavf (background mortality) ni qo'shdi

$$\mu(x) = A + ae^{bx}, \Rightarrow S(x) = exp(-Ax - \frac{a}{b}(e^{bx} - 1))$$

O'zbekiston Respublikasi uchun umr davomiyligi taqsimot funksiyasini baholash uchun yuqorida keltirilgan modellardan hamda 2008 yilda olingan ma'lumotlardan foydalanamiz.

x (yosh)	l_x (aholi soni)	d_x (Vafot etganlar soni)	x yosh	l_x (aholi soni)	d_x (Vafot etganlar soni)
0	100000	1307	50	91336	558
1	98693	263	55	88047	861
5	98194	39	60	82447	1443
10	98034	29	65	73599	1550
15	97879	33	70	62879	2776
20	97638	66	75	47716	2973
25	97230	109	80	30933	2961
30	96608	149	85	17012	1827
35	95801	178	90	6968	851
40	94755	256	95	2782	297
45	93392	339	100	1485	601

1-jadval. O'zbekiston Respublikasi aholisi uchun umr davomiyligi jadvali (2008 yil)

Avval berilgan jadval asosida empirik umr davomiyligi funksiyasini qurib olamiz hamda mazkur empirik umr davomiyligi funksiyasi asosida (ya'ni empirik umr davomiyligi

funksiyasiga eng yaqin bo'lgan) ikki hil modelni tanlaymiz. Bular Gomperts modeli va Weibull modellaridan foydalanamiz. Gomperts modeli umumiy holda

$$\mu(x) = A + Be^{Cx},\ S(x) = exp(-Ax - \frac{B}{C}(e^{Cx} - 1))$$

formula bilan aniqlanadi. A parametri yoshga bog'liq bo'lmagan tashqi xavflar (masalan, zo'ravonlik yoki keskin kasalliklar)ni, B va C parametrlar biologik qarishning eksponensial tezligini bildiradi.

Weibull modelining umumiy ko'rinishi

$$\mu(x) = \frac{\beta}{\alpha}(\frac{x}{\alpha})^{\beta-1},\ S(x) = exp(-(\frac{x}{\alpha})^{\beta})$$

Weibull modeli ishonchlilik nazariyasi, sanoat statistikasi va umr davomiyligi tahlilida keng qo'llaniladigan parametrik model hisoblanadi. Bu ikki modelni ko'rinishidan tanlanma bo'yicha noma'lum parametrlarni baholashda aniq analitik yechim olish imkoniyati mavjud emaslini ko'rish mumkin. Shuning uchun noma'lum parametrlarni sonli usullar yordamida baholaymiz.

Weibull modelida parametrlarni baholash. Weibull modelining umr davomiyligi funksiyasi quyidagicha aniqlanadi

$$S(x) = exp(-(\frac{x}{\alpha})^{\beta})$$

bu yerda $\alpha > 0$ - masshtab parametri, $\beta > 0$ - shakl parametri. Avval noma'lum parametrlarni haqiqatga maksimal o'xshashlik usuli yordamida baholaymiz. Agar bizda n ta to'liq kuzatilgan tanlanmalar mavjud bo'lsa: $x_1,\ x_2, ..., x_n$, u holda haqiqatga o'xshashlik funksiyasi

$$L(\alpha, \beta) = \prod_{i=1}^{n} f(x_i) = \prod_{i=1}^{n} [\frac{\beta}{\alpha}(\frac{x_i}{\alpha})^{\beta-1} e^{-(\frac{x_i}{\alpha})^{\beta}}]$$

va uning logarifmi

$$l(\alpha, \beta) = nln\beta - nln\alpha + (\beta - 1)\sum lnx_i - \beta\sum(\frac{x_i}{\alpha})^{\beta}$$

bo'ladi. Bu $l(\alpha, \beta)$ funksiyasini noma'lum parametrlar bo'yicha hosilalarini nolga teng qilib yechamiz

$$\frac{\partial l}{\partial \alpha} = 0,\ \frac{\partial l}{\partial \beta} = 0$$

Bu tenglamalarni analitik yechimini topib bo'lmaydi. Shuning uchun sonli usullardan foydalanib (buning uchun maxsus dasturiy vositalardan foydalanamiz) α va β ni topamiz. Berilgan tanlanma (1-jadval) bo'yicha α va β ning qiymatlari: $\alpha \approx 78.18$ va $\beta \approx 6.23$ bo'ladi.

Gompertz modelida parametrlarni baholash. Gompertz modelining umr davomiyligi funksiyasi quyidagicha aniqlanadi

$$S(x) = exp(-\frac{a}{b}(e^{bx} - 1))$$

bu yerda $a > 0$ bosh xavf intensivligi, $b > 0$ qarish tezligi. Bu modelda ham haqiqatga maksimal o'xshashlik usuli yordamida baholaymiz. Haqiqatga o'xshashlik funksiyasi logarifmining ko'rinishi

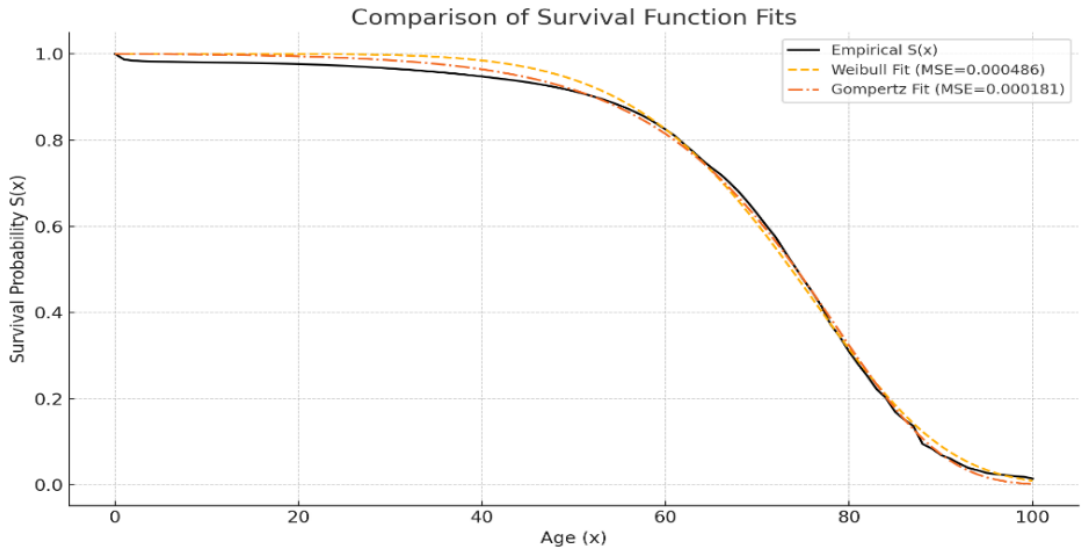
$$l(a, b) = \sum_{i=1}^{n}[lna + bx_i - \frac{a}{b}(e^{bx_i} - 1)]$$

Bu modelda ham tenglamalarni analitik yechimini topib bo'lmaydi. Shuning uchun sonli usullardan foydalanib noma'lum parametrlarni baholaymiz: $a \approx 0.0001074$ va $b \approx 0.08494$.

Noma'lum parametrlarni baholashda kichik kvadratlar usulidan ham foydalanish mumkin. Bunda modeldagi $S(x, \theta)$ funksiya bilan $\hat{S}(x) = \frac{l_x}{l_0}$ orasidagi farqni minimallashtirish orqali parametrlar aniqlanadi:

$$min_\theta \sum_{i=1}^{n}(S(x_i; \theta) - \hat{S}(x_i))^2$$

Quyida berilgan o'limlik jadvali asosida qurilgan empirik umr davomiyligi funksiyasi va o'rganilgan modellar bo'yicha baholangan umr davomiyligi funksiyalarini solishtirma grafigini chizamiz:



1-rasm. O'zbekiston aholisi umr davomiyligi bo'yicha empirik va parametrik (Weibull, Gompertz) funksiyalarining qiyosiy tahlili

Xulosa

Umr davomiyligi funksiyasi ehtimollar nazariyasi, statistik tahlil va aktuar hisoblarning muhim tushunchalaridan biri bo'lib, sug'urta, pensiya va sog'liqni saqlash tizimlarida ijtimoiy-iqtisodiy prognozlar tuzishda keng qo'llaniladi. Mazkur tadqiqotda O'zbekiston Respublikasi aholisi bo'yicha 2008-yil statistik ma'lumotlari asosida empirik umr davomiyligi funksiyasi Gompertz va Weibull parametrik modellari bilan qiyosiy tahlil qilindi. Natijalar tanlangan modellar empirik funksiyaga yaxshi mos kelishini ko'rsatib, ularning real demografik jarayonlarni tavsiflashdagi samaradorligini tasdiqlaydi. Gompertz modeli biologik qarish jarayonini hisobga olib, o'lim xavfining yosh ortishi bilan tezlashishini ifodalaydi, Weibull modeli esa xavfning vaqtga bog'liq o'zgarishini moslashuvchan tarzda modellashtirish imkonini beradi.

Parametrlarni baholashda analitik yechimlarning mavjud emasligi sababli, haqiqatga maksimal o'xshashlik (MLE) va kichik kvadratlar (OLS) kabi usullardan foydalangan holda sonli optimizatsiya yondashuvlari qo'llanildi. Tadqiqotda parametrlar maxsus dasturiy vositalar yordamida hisoblab chiqildi va har bir model asosida qurilgan nazariy umr davomiyligi funksiyalari empirik funksiyalar bilan solishtirildi.

Natijalar umr davomiyligini modellashtirish orqali aholi salomatligi, pensiya tizimining barqarorligi va sug'urta risklarini aniqlashda aniq va asosli qarorlar qabul qilish imkonini berishini ko'rsatdi. Ayniqsa, hozirgi davrda O'zbekistonda demografik o'zgarishlar, umr ko'rishning ortishi va iqtisodiy islohotlar fonida bunday modellar strategik rejalashtirish uchun nihoyatda muhim hisoblanadi.

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Cheksiz izotropik fazoda qalin sferik qobiqdan nostatsionar to‘lqinlar

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DOI 10.5281/zenodo.18217590

Kirish. Tutash muhitlarda nostatsionar to‘lqin jarayonlarini matematik modellashtirish va o‘rganish to‘lqinlar dinamikasining murakkab, shu bilan birga dolzarb muammolaridandir. To‘lqinlar dinamikasi muammolarining dolzarbligi muhandislikning turli sohalarida dinamik kuchlar ta’siri ostida ishlaydigan yangi konstruksiyalarini yaratilishi, shuningdek, geofizika, seysmologiya, konchilik sanoati, fuqarolik va sanoat qurilishidan kelib chiqadigan amaliy muammolar bilan aniqlanadi.

Hozirgi vaqtda uzluksiz muhitlarda to‘lqinlarning tarqalishi va difraksiyasini o‘rganishga bag‘ishlangan juda ko‘p sondagi ilmiy tadqiqot ishlari mavjud. Masalan, [1] da suyuqlik va qattiq jismlardagi zarba to‘lqini jarayonlarini modellashtirishning ucho‘lchovli uslubiyoti, shuningdek, suyuqlik va konstruksiyalarning o‘zaro ta’sirini modellashtirish masalalari o‘rganilgan. [2] da esa sirtlarga normal tipdagi zarbalar paytida aylanma yupqa qobiqlarida paydo bo‘ladigan elliptik chegaraviy qatlami uchun chegara masalaning yechish usuli ishlab chiqilgan. Maqolada elliptik chegaraviy qatlami tenglamalarini yechish usuli eksponensial formadagi Laplas yechimining tasvirlarining asimptotik shakldan foydalanishga asoslangan hamda sharsimon qobiq holi uchun olingan analitik yechimlar asosida normal kuchlanishning sonli natijalari olingan.

Elastik yarim fazoda sferik bo‘shliqdan nostatsionar ko‘ndalang siljish to‘lqinlarining tarqalishi [3] tadqiqotda o‘rganilgan. Ko‘chish vektori va kuchlanish tenzorining komponentlari uchun formulalar olingan. [4] maqolada tekisliklar bilan chegaralangan ideal suyuqliklar qatlamida sferik bo‘shliqdan nostatsionar to‘lqin tarqalishining o‘qsimetrik masalasi qaralgan. Laplas tasvir fazosida masala cheksiz matritsali tenglamani yechishga keltirilgan. Sonli natijalar sferik bo‘shliq yaqinida bosim o‘zgarishi grafiklar shaklida tasvirlangan.

[5] da yassi seysmik to‘lqinning ko‘ndalang tushishi natijasida yuzaga kelgan yer osti quvurining statsionar tebranishlari o‘rganilgan. Qiya to‘lqinni quvur bo‘ylab yuqori tezlikda tarqaladigan bir nechta bo‘ylama va ko‘ndalang to‘lqinlar shaklida ifodalanishi mumkinligi ko‘rsatilgan. Maqola [6] qalin devorli sferik qobiqdan elastik fazoda kososimmetrik ko‘ndalang siljish to‘lqinlarining tarqalishi masalasiga bag‘ishlangan. O‘lchovsiz vaqtga nisbatan integral Laplas almashtirishi qo‘llanilgan. Fazoda siljish vektori va kuchlanish tenzorining komponentlari uchun formulalar olingan. Originalga o‘tish qoldiq nazariyasi yordamida amalga oshirilgan. Sonli tajribalar o‘tkazilgan. [7] maqolada izotropik sferik qatlamda kososimmetrik ko‘ndalang to‘lqinlarning tarqalishi masalasi ko‘rib chiqilgan. O‘lchovsiz vaqtga nisbatan integral Laplas almashtirishi

va o‘zgaruvchilarni to‘liq bo‘lmagan ajratish usuli qo‘llanilgan. Ko‘chish vektori va kuchlanish tensorining komponentlari uchun formulalar olinadi. Sonli tajribalar natijalari grafiklar shaklida berilgan.

Ushbu maqola chegaralanmagan izotropik fazoda qalin sferik qobiqdan nostatsionar bo‘ylama to‘lqin tarqalishi haqidagi masalani o‘rganishga bag‘ishlangan. Ishning maqsadi va vazifasi – masalaning yechish algoritmini ishlab chiqish va sonli eksperimentlar asosida qobiq hamda atrof muhitda nostatsionar to‘lqin jarayonlarini tahlil etishdan iboratdir.

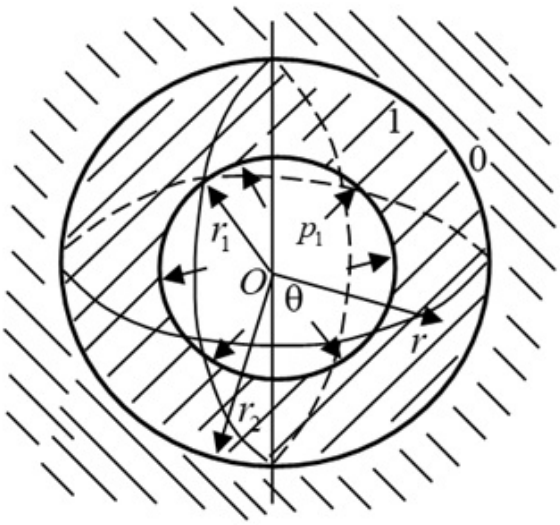
Masalaning qo’yilishi. Aytaylik, cheksiz bir jinsli izotropik fazoda deformatsiyalanuvchi sferik qobiq joylashgan bo‘lib, u ichki va tashqi tomondan radiuslari mos ravishda r_1 va r_2 $(r_1 < r_2)$ bo‘lgan sferik sirtlar bilan chegaralangan (rasm 1). Muhit qobiq va harakatini o‘rganish uchun boshlang‘ich nuqtasi O qobiq markazida bo‘lgan (r, θ, ϑ) sferik koordinatalar sistemasi kiritilgan.

Vaqtning $\tau = 0$ paytida sferik qobiq ichki sirtiga tekis taqsimlagan normal $p_1(\tau, \theta)$ sirt kuchi ta’sir etadi. Natijada qobiq va atrof muhitda faqat nostatsionar bo‘ylama to‘lqin tarqaladi.

Masalaning o‘qsimmetrik ekanligini e’tiborga olib, muhit va qobiq harakati ko‘chish ϕ_k potensiallariga nisbatan to‘lqin tenglamalari orqali tasvirlanadi ($k = 0, 1$; indeks “0” muhitni, “1” indeks esa sferik qobiqni bildiradi, nuqtalar bilan vaqtga bo‘yicha differetsiallash belgilangan):

$$\gamma_k^2 \ddot{\phi}_k = \Delta \phi_k \tag{1}$$

bu yerda Δ - sferik koordinatalar sistemasida Laplas operatori.



Rasm1.

Boshlang‘ich shartlar – bir jinsli:

$$\phi_k|_{\tau=0} = \dot{\phi}_k|_{\tau=0} = 0. \tag{2}$$

Cheksizlikda esa to‘lqin so‘nadi:

$$\lim_{r \to \infty} \phi_0 = 0 \tag{3}$$

Sferik qobiq ichki sirtida chegaraviy shart

$$\sigma_{rr}^{(1)}|_{r=r_2} = p_1(\tau,\theta) \tag{4}$$

ko'rinishga ega.

Sferik qobiq bilan atrof muhit o'rtasidagi kontakt shartlar

$$\sigma_{rr}^{(1)}|_{r=r_2} = \sigma_{rr}^{(0)}|_{r=r_2},\ w_0|_{r=r_2} = w_1|_{r=r_2} \tag{5}$$

tengliklar bilan ifodalanadi. Bu yerda w_k va $\sigma_{rr}^{(k)}$ lar ($k = 0, 1$) mos ravishda muhit va qobiqning kuchlanish-deformatsiya holati komponentalari.

Ko'chish ϕ_k potensiallari $w_k, \sigma_{rr}^{(k)}, \sigma_{\theta\theta}^{(k)}$ muhit va qobiqning kuchlanish-deformatsiya holati komponentalari bilan o'zaro quyidagi

$$w_k = \frac{\partial \phi_k}{\partial r},\quad \sigma_{rr}^{(k)} = \frac{\partial w_k}{\partial r} + 2\chi_k \frac{w_k}{\partial r},\quad \sigma_{\theta\theta}^{(k)} = \chi_k \sigma_{rr}^{(k)} + (1 + \chi_k - 2\chi_k^2) w_k/r, \tag{6}$$

munosabatlar orqali bog'langan.

Yechish metodi. Masalani yechish uchun o'lchovsiz vaqtga nisbatan integral Laplas integral almashtirishlari [8] va o'zgaruvchilarni to'liq bo'lmagan ajratish usuli qo'llanilgan.

Tasvirlar fazosida berilgan va noma'lum funksiyalar Lejandr ortogonal ko'phadlari [9] bo'yicha cheksiz qatorlarga yoyilib, masala qatorlar koeffitsiyentlariga nisbatan cheksiz algebraik tenglamalar sistemasiga keltirilgan va u ikki matritsa tenglama ko'rinishda yozilgan. Sistemaning yechimi eksponentalar bo'yicha cheksiz qatorlar ko'rinishida izlanib, bu qatorlar koeffitsentlargan nisbatan rekurrent munosabatlar va boshlang'ich shartlar olingan. Bu munosabatlar qatorlar koeffitsentlari va izlanayotgan yechimni Laplas almashtirishlari parametrining ratsional funksiyalari ko'rinishda aniqlashga va ularning tasvirlaridan originalga o'tishda qoldiqlar nazariyasidan foydalanish imkonini beradi [10].

Tasvir fazosida kuchlanish tenzori va ko'chish vektori komponentalari uchun formulalar olingan.

$$w_{1n}^L(r,s) = -\,r^{-n-2}\,(\gamma_1 s)^{-n}\,D_{n1}(r\gamma_1 s), \qquad \sigma_{rrn}^{(1)L}(r,s) = r^{-n-3}\,(\gamma_1 s)^{-n}\,\beta_1 W_{n1}(r\gamma_1 s),$$

$$w_{0n}^L(r,s) = -\,\frac{R_{n1}(\zeta)\,e^{-\zeta}\,e^{z}}{r^2\,\zeta^{\,n}\,R_{n1}(z)}\,D_{n1}(r_2\gamma_1 s), \qquad \sigma_{rrn}^{(0)L}(r,s) = \frac{\beta_0 Q_{n1}(\zeta)\,e^{-\zeta}\,e^{z}}{r^3\,\zeta^{\,n}\,r_2^2\,Q_{n1}(z)}\,W_{n1}(r_2\gamma_1 s).$$

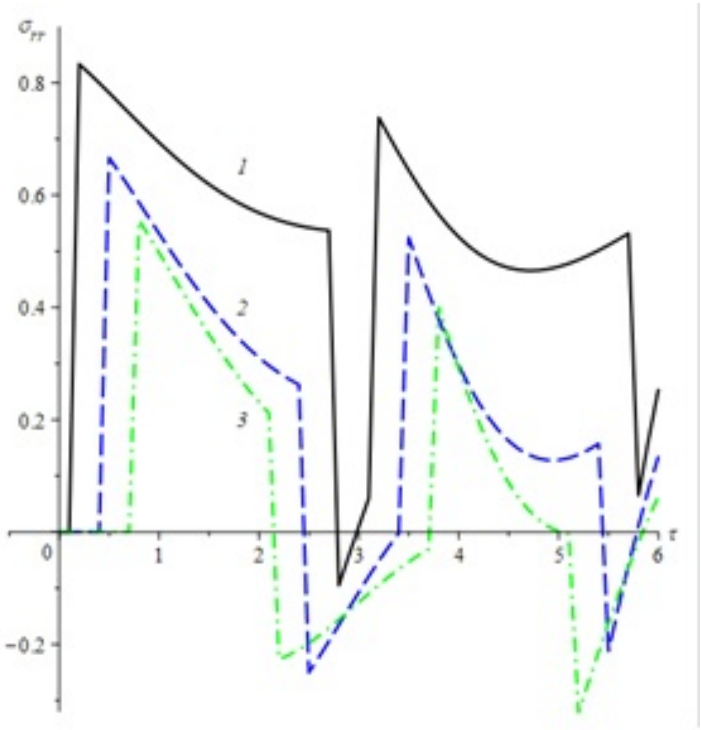
$$D_{n1}(\xi) = \sum_{i,j=0}^{\infty} \Big[R_{n1}(\xi)\,a_{ij}^{(n)}(s)\,e^{-\xi} + (-1)^n G_{n1}(\xi)\,b_{ij}^{(n)}(s) \Big]\,x^i y^{-j-1},$$

$$W_{n1}(\xi) = \sum_{i,j=0}^{\infty} \Big[Q_{n1}(\xi)\,a_{ij}^{(n)}(s)\,e^{-\xi} + (-1)^n E_{n1}(\xi)\,b_{ij}^{(n)}(s) \Big]\,x^i y^{-j-1}.$$

Sonli natijalar. Sonli eksperiment natijalari cheksiz granit ($E_0 = 5 \cdot 10^{10}\,\mathrm{Pa},\ \nu_0 = 0.15,\ \rho_0 = 3000\,\mathrm{kg/m^3}$) fazoda ichki va tashqi radiuslar mos ravishda$r_1 = 1.0,\ r_2 = 2.5$ bo'lgan qalin po'lat ($E_1 = 20.1 \cdot 10^{10}\,\mathrm{Pa},\ \nu_1 = 0.3,\ \rho_1 = 7800\,\mathrm{kg/m^3}$) sferik qobiqdan nostatsionar bo'ylama to'lqin tarqalishi jarayoni uchun olingan. U holda o'lchovsiz parametrlar $\gamma_0 = 0.71,\ \gamma_1 = 1.0,\ \chi_0 = 0.1765,\ \chi_1 = 0.43$ qiymatlar bilan aniqlanadi.

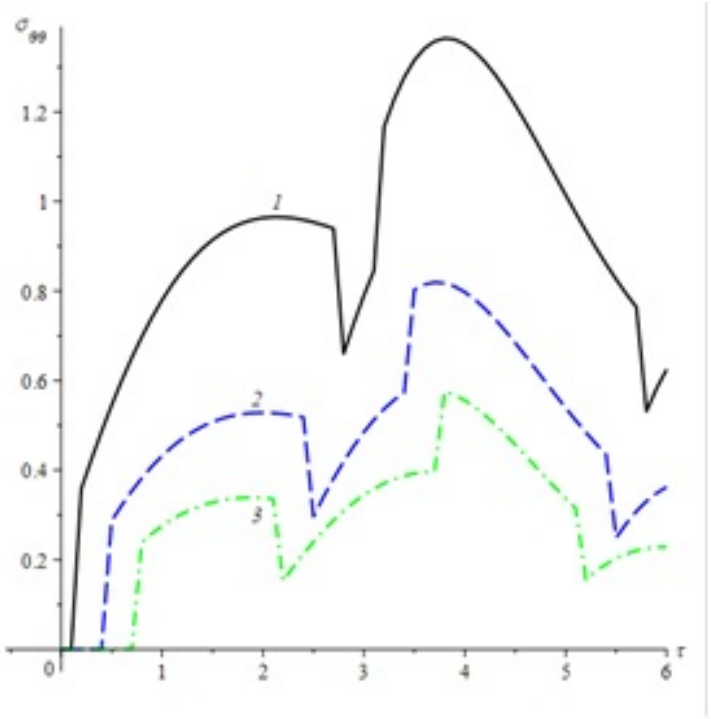
Qobiq ichki sirtiga esa $p_1(\tau,\theta) = f(\tau)H(\tau), \qquad f(\tau) = p_0, \qquad p_0 = 1$ normal kuch ta'sir etadi.

Quyidagi 2 va 3-rasmlarda qobiqning kuchlanish $\sigma_{rr}^{(1)}$, $\sigma_{\theta\theta}^{(1)}$ komponentalarining vaqt bo'yicha o'zgarishi grafiklari tasvirlangan. Bu yerda $r = 1.2$ (egri chiziq 1), $r = 1.5$ (egri chiziq 2) va $r = 1.8$ (egri chiziq 3).



Rasm 2.

Xulosa. Cheksiz bir jinsli izotropik fazoda joylashgan qalin sferik qobiqdan nostatsionar bo'ylama to'lqin tarqalishi masalasining yechish algoritmi ishlab chiqildi. Sonli natijalar olindi va ular grafiklar shaklda tasvirlangan. Sonli eksperimentlarning natijalarini geofizika, seysmologiya va yer osti inshootlari hamda omborlarini loyihalashtirishda foydalanish mumkin.



Rasm 3.

ADABIYOTLAR

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Tijorat banklarining ayollar tadbirkorligini moliyaviy qo'llab-quvvatlashdagi roli va uni takomillashtirish yo'nalishlari

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DOI 10.5281/zenodo.18217605

Annotatsiya

Mazkur ilmiy maqolada tijorat banklarining ayollar tadbirkorligini moliyaviy qo'llab-quvvatlashdagi o'rni va ahamiyati tadqiq etilgan. Xotin-qizlar tadbirkorligini rivojlantirishda bank kreditlari, imtiyozli moliyalashtirish dasturlari hamda kreditlash amaliyotidagi mavjud muammolar tahlil qilinib, bank-moliya mexanizmlarini takomillashtirish bo'yicha ilmiy-amaliy taklif va tavsiyalar ishlab chiqilgan. Shuningdek, tadqiqot natijalari ayollar tadbirkorligini rivojlantirishda tijorat banklari faoliyatini samarali tashkil etishga xizmat qiladi.

Kalit so'zlar: ayollar tadbirkorligi, tijorat banklari, moliyaviy qo'llab-quvvatlash, kreditlash mexanizmi, bank xizmatlari, kichik biznes.

Аннотация

В данной научной статье исследуется роль и значение коммерческих банков в финансовой поддержке женского предпринимательства. Проанализированы банковские кредиты, льготные программы финансирования, а также существующие проблемы в практике кредитования, направленные на развитие предпринимательской деятельности женщин. На основе проведённого анализа разработаны научно-практические предложения и рекомендации по совершенствованию банковско-финансовых механизмов. Результаты исследования способствуют повышению эффективности деятельности коммерческих банков в процессе развития женского предпринимательства.

Ключевые слова: женское предпринимательство, коммерческие банки, финансовая поддержка, механизм кредитования, банковские услуги, малый бизнес.

Abstract
This scientific article examines the role and significance of commercial banks in providing financial support for women's entrepreneurship. The study analyzes bank lending, preferential financing programs, and existing problems in credit practices aimed at developing women's entrepreneurial activities. Based on the analysis, scientific and practical proposals and recommendations are developed to improve banking and financial mechanisms. The research findings contribute to enhancing the effectiveness of commercial banks in supporting the development of women's entrepreneurship.

Keywords: women's entrepreneurship, commercial banks, financial support, lending mechanism, banking services, small business.

Kirish

Hozirgi globallashuv sharoitida iqtisodiy rivojlanishni ta'minlashda tadbirkorlik faoliyatining roli tobora ortib bormoqda. Xususan, ayollar tadbirkorligini rivojlantirish iqtisodiy o'sish, aholi bandligini ta'minlash va ijtimoiy barqarorlikni mustahkamlashning muhim omillaridan biri hisoblanadi. Jahon tajribasi shuni ko'rsatadiki, ayollar boshchiligidagi biznes subyektlarini qo'llab-quvvatlash yalpi ichki mahsulot hajmining oshishiga, kambag'allik darajasining pasayishiga va innovatsion faollikning kuchayishiga xizmat qiladi. O'zbekistonda ham ayollar tadbirkorligini rivojlantirish davlat siyosati darajasiga ko'tarilib, bu borada ko'plab islohotlar va qarorlar qabul qilindi hamda shu qarorlarga muvofiq ancha ishlar amalga oshirilmoqda. Ayollar tadbirkorligini qo'llab-quvvatlashda moliyaviy resurslarga bo'lgan ehtiyojni qondirish muhim ahamiyat kasb etib, bu jarayonda tijorat banklari yetakchi moliyaviy institut sifatida maydonga chiqadi.

Shu bois tijorat banklarining ayollar tadbirkorligini moliyaviy qo'llab-quvvatlashdagi faoliyatini ilmiy jihatdan o'rganish hamda uni takomillashtirish yo'nalishlarini aniqlash dolzarb ilmiy masalalardan biri hisoblanadi.

Adabiyotlar sharhi va tadqiqot metodologiyasi

Ayollar tadbirkorligini rivojlantirish va uni moliyaviy qo'llab-quvvatlash masalalari zamonaviy iqtisodiy tadqiqotlarda muhim yo'nalishlardan biri sifatida keng o'rganilgan. Xususan, tijorat banklarining ushbu jarayondagi o'rni moliyaviy vositachilik, kredit resurslariga kirish imkoniyatlarini kengaytirish hamda moliyaviy inklyuziyani ta'minlash nuqtai nazaridan ilmiy adabiyotlarda alohida e'tirof etilgan. Nazariy jihatdan, tadbirkorlikni moliyalashtirish masalalari J. Schumpeterning innovatsion rivojlanish nazariyasiga borib taqaladi. Unga ko'ra, banklar tadbirkorlik faoliyatini rivojlantirishda asosiy moliyaviy vositachi bo'lib, yangi g'oyalar va biznes tashabbuslarni iqtisodiy o'sish omiliga aylantiradi. Ushbu yondashuv ayollar tadbirkorligini moliyaviy qo'llab-quvvatlashda tijorat banklarining strategik rolini asoslab beradi.

Ayollar tadbirkorligini moliyalashtirishning institutsional jihatlari D. North va W.R. Scottning institutsional nazariyalarida keng yoritilgan. Mazkur nazariyalarga ko'ra, moliyaviy institutlar, jumladan tijorat banklari tomonidan yaratiladigan qoidalar, kredit siyosati va moliyaviy mexanizmlar ayollar tadbirkorligining rivojlanish darajasiga bevosita ta'sir ko'rsatadi. Scott (2001) ayollar tadbirkorligidagi cheklovlar ko'pincha moliyaviy institutlarning madaniy va ijtimoiy omillarni yetarlicha inobatga olmasligi bilan bog'liqligini ta'kidlaydi.

Xalqaro tadqiqotlarda ayollar tadbirkorligini moliyalashtirishda bank kreditlariga kirish muammolari alohida o'rganilgan. A. Demirg??-Kunt va R. Levine tomonidan olib

borilgan tadqiqotlarda moliyaviy tizim rivojlangan davlatlarda ayollar boshchiligidagi biznes subyektlarining kredit resurslaridan foydalanish imkoniyatlari kengroq ekani ilmiy jihatdan asoslab berilgan. Shu bilan birga, G. Brush, C. Carter va E. Gatewoodlarning ishlarida ayollar tadbirkorligi uchun moliyaviy resurslarga kirishda garov ta'minoti, kredit tarixi va banklar tomonidan riskni baholash mexanizmlarining muhim ahamiyatga egaligi ko'rsatib o'tilgan. Rossiyalik iqtisodchi olimlar E.N.Fetisov, T.Zaslavskaya va S.Suxanovlarning tadqiqotlarida ayollar tadbirkorligini moliyaviy qo'llab-quvvatlashda bank sektorining roli, ayniqsa imtiyozli kreditlash va davlat-bank hamkorligi mexanizmlarining samaradorligi tahlil qilingan. Ularning ilmiy qarashlariga ko'ra, tijorat banklari tomonidan maxsus kredit mahsulotlarini ishlab chiqish ayollar tadbirkorligi rivojiga ijobiy ta'sir ko'rsatadi.

O'zbekistonlik olimlar tomonidan ham mazkur yo'nalishda qator ilmiy izlanishlar olib borilgan. Jumladan, R.Ubaydullaeva, D.Raximova va M.Sobirovalarning ilmiy ishlarida kichik biznes va xususiy tadbirkorlikni rivojlantirishda tijorat banklari kreditlarining ahamiyati, ayollar tadbirkorligini moliyalashtirishdagi mavjud muammolar va ularni bartaraf etish yo'llari tahlil qilingan. Ushbu tadqiqotlarda ayollar tadbirkorligini qo'llab-quvvatlashda mikromoliyalash, imtiyozli kreditlar va bank xizmatlarini soddalashtirish mexanizmlarini takomillashtirish zarurligi asoslab berilgan.

Mazkur tadqiqot tijorat banklarining ayollar tadbirkorligini moliyaviy qo'llab-quvvatlashdagi rolini baholash hamda ushbu jarayonni takomillashtirish yo'nalishlarini aniqlashga qaratilgan. Tadqiqotning metodologik asosi institutsional iqtisodiyot, moliyaviy vositachilik va gender iqtisodiyoti nazariyalariga tayangan holda shakllantirildi. Tadqiqot jarayonida tahlil va sintez, induksiya va deduksiya, taqqoslash hamda tizimli yondashuv kabi umumiy ilmiy usullardan foydalanildi. Empirik tahlil O'zbekiston Respublikasi Markaziy banki, tijorat banklari, Davlat statistika qo'mitasi hamda xalqaro moliya institutlarining rasmiy statistik ma'lumotlari asosida amalga oshirildi.

Shuningdek, statistik va iqtisodiy-matematik usullar, jumladan korrelyatsion va regressiya tahlili yordamida bank kreditlari bilan ayollar tadbirkorligi rivoji o'rtasidagi o'zaro bog'liqlik baholandi. Tadqiqot natijalari asosida tijorat banklarining ayollar tadbirkorligini moliyaviy qo'llab-quvvatlash mexanizmlarini takomillashtirish bo'yicha amaliy taklif va tavsiyalar ishlab chiqildi.

Tahlil va natijalar

O'zbekistonda so'nggi yillarda ayollar tadbirkorligini rivojlantirish davlat siyosatining ustuvor yo'nalishlaridan biriga aylandi. Ushbu jarayonda tijorat banklari tomonidan ko'rsatilayotgan moliyaviy xizmatlar, xususan kreditlash mexanizmlarining ahamiyati ortib bormoqda. Tahlillar shuni ko'rsatadiki, tijorat banklari ayollar tadbirkorligini moliyaviy qo'llab-quvvatlashda asosiy moliyaviy vositachi sifatida faol ishtirok etmoqda. Markaziy bank va tijorat banklarining rasmiy ma'lumotlari asosida o'tkazilgan tahlil natijalariga ko'ra, ayollar boshchiligidagi tadbirkorlik subyektlariga ajratilayotgan kreditlar hajmi yildan-yilga oshib bormoqda. Ayniqsa, kichik biznes va oilaviy tadbirkorlikni moliyalashtirishga yo'naltirilgan imtiyozli kredit dasturlari ayollar tadbirkorligining kengayishiga ijobiy ta'sir ko'rsatgan. Biroq, kredit resurslariga kirishda garov ta'minoti yetishmasligi, kredit shartlarining murakkabligi va foiz stavkalarining nisbatan yuqoriligi ayollar tadbirkorligi rivojini cheklovchi omillar sifatida saqlanib qolmoqda.

Tijorat banklari kredit portfeli tarkibining tahlili shuni ko'rsatadiki, ayollar tadbirkorligi asosan savdo, xizmat ko'rsatish va qishloq xo'jaligi sohalarida jamlangan. Ishlab chiqarish va innovatsion loyihalarga yo'naltirilgan kreditlar ulushi esa nisbatan past bo'lib, bu holat banklar tomonidan riskni baholashda ehtiyotkorlik yuqori ekanligi bilan izohlanadi. Natijada, ayollar tadbirkorligining iqtisodiyotning yuqori qo'shilgan qiymatga ega tarmoqlaridagi ishtiroki yetarli darajada rivojlanmay qolmoqda.

Iqtisodiy-matematik tahlil natijalari tijorat banklari tomonidan ajratilgan kreditlar hajmi bilan ayollar tadbirkorligi subyektlari soni hamda bandlik darajasi o'rtasida ijobiy bog'liqlik mavjudligini ko'rsatdi. Bu holat bank kreditlarining ayollar tadbirkorligini rivojlantirishda muhim moliyaviy omil ekanini tasdiqlaydi. Shu bilan birga, kreditlardan foydalanish samaradorligi moliyaviy savodxonlik darajasi va bank xizmatlarining qulayligi bilan bevosita bog'liq ekani aniqlandi. O'tkazilgan tahlil natijalari shuni ko'rsatdiki, tijorat banklarining ayollar tadbirkorligini moliyaviy qo'llab-quvvatlashdagi faoliyati sezilarli darajada rivojlanayotgan bo'lsa-da, mazkur yo'nalishda bir qator tizimli muammolar mavjud. Xususan, ayollar tadbirkorlari uchun maxsus moslashtirilgan kredit mahsulotlari ulushining yetarli emasligi, bank va tadbirkor o'rtasidagi axborot nomutanosibligi hamda moliyaviy xizmatlarning hududlar kesimida notekis taqsimlanishi aniqlangan.

Yakuniy natijalarga ko'ra, tijorat banklari tomonidan ayollar tadbirkorligini moliyaviy qo'llab-quvvatlash mexanizmlarini takomillashtirish, kreditlash shartlarini soddalashtirish, imtiyozli moliyaviy instrumentlarni kengaytirish va moliyaviy savodxonlikni oshirish ayollar tadbirkorligining barqaror rivojlanishini ta'minlashda muhim ahamiyat kasb etadi.

Xulosa va takliflar

O'tkazilgan tadqiqot natijalari shuni ko'rsatadiki, tijorat banklari ayollar tadbirkorligini moliyaviy qo'llab-quvvatlashda muhim institutsional rol o'ynaydi. Bank kreditlari ayollar boshchiligidagi tadbirkorlik subyektlarining tashkil etilishi, ularning faoliyatini kengaytirish va bandlikni oshirishda asosiy moliyaviy manba hisoblanadi. Tadqiqot davomida bank kreditlari hajmi bilan ayollar tadbirkorligi rivoji o'rtasida ijobiy bog'liqlik mavjudligi ilmiy va empirik jihatdan asoslandi. Tahlil natijalari tijorat banklari tomonidan ayollar tadbirkorligini moliyalashtirish so'nggi yillarda faollashganini ko'rsatdi. Shunga qaramay, kredit resurslariga kirishdagi cheklovlar, xususan garov ta'minotining yetishmasligi, kreditlash shartlarining murakkabligi, foiz stavkalarining nisbatan yuqoriligi hamda moliyaviy savodxonlik darajasining pastligi ushbu yo'nalishdagi asosiy muammolar sifatida aniqlandi. Shuningdek, ayollar tadbirkorligi asosan past riskli va kam qo'shilgan qiymatga ega sohalarda jamlanib qolayotgani iqtisodiy samaradorlikni cheklamoqda.

Tadqiqot xulosalaridan kelib chiqib, tijorat banklarining ayollar tadbirkorligini moliyaviy qo'llab-quvvatlash tizimini takomillashtirish maqsadida quyidagi **ilmiy-amaliy takliflar** ishlab chiqildi:

Birinchidan, tijorat banklarida ayollar tadbirkorlari uchun maxsus moslashtirilgan kredit mahsulotlarini joriy etish zarur. Ushbu mahsulotlar garov talablarini yengillashtirish, moslashuvchan to'lov grafigi va imtiyozli foiz stavkalarini o'z ichiga olishi lozim.

Ikkinchidan, davlat va tijorat banklari o'rtasidagi hamkorlikni kuchaytirish orqali kafillik va sug'urta mexanizmlarini kengaytirish maqsadga muvofiq. Bu ayollar

tadbirkorlarining kredit resurslariga kirish imkoniyatlarini sezilarli darajada oshiradi.

Uchinchidan, ayollar tadbirkorligi uchun mikromoliyalash va raqamli bank xizmatlarini rivojlantirish tavsiya etiladi. Masofaviy kreditlash, onlayn ariza topshirish va tezkor qaror qabul qilish mexanizmlari hududiy tafovutlarni kamaytirishga xizmat qiladi.

To'rtinchidan, tijorat banklari huzurida moliyaviy savodxonlik va biznes-rejalashtirish bo'yicha maslahat xizmatlarini yo'lga qo'yish lozim. Bu kreditlardan foydalanish samaradorligini oshirish hamda kredit risklarini kamaytirishga yordam beradi.

Beshinchidan, ayollar tadbirkorligini yuqori qo'shilgan qiymatga ega sohalarga, xususan ishlab chiqarish, innovatsiya va xizmatlar eksportiga yo'naltirishni rag'batlantiruvchi moliyaviy instrumentlarni joriy etish maqsadga muvofiq.

Xulosa qilib aytganda, tijorat banklarining ayollar tadbirkorligini moliyaviy qo'llab-quvvatlashdagi rolini kuchaytirish iqtisodiy o'sishni jadallashtirish, bandlikni oshirish va ijtimoiy barqarorlikni ta'minlashda muhim omil hisoblanadi. Taklif etilgan chora-tadbirlarni amaliyotga tatbiq etish ayollar tadbirkorligining barqaror va samarali rivojlanishiga xizmat qiladi.

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Markaziy Osiyo mamalakatlarida sanoat rivojlanishining qiyosiy tahlili

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DOI 10.5281/zenodo.18217620

Annotatsiya:

Maqolada O'zbekiston, Qozog'iston va Qirg'izistonda 2015–2024-yillar davomida sanoat tarmog'ining rivojlanish tendensiyalari solishtirma statistik tahlil asosida o'rganilgan. Sanoat mahsuloti hajmi, sanoatda bandlik va mehnat unumdorligi ko'rsatkichlari tahlil qilingan. Valyuta kurslarining ta'siri hamda sanoat samaradorligidagi farqlar aniqlangan. Olingan natijalar sanoat siyosatini takomillashtirish bo'yicha ilmiy-amaliy xulosalar chiqarish imkonini beradi.

Kalit so'zlar: sanoat rivoji, sanoat mahsuloti, mehnat unumdorligi, bandlik,statistik tahlil, solishtirma tahlil, prognoz, Markaziy Osiyo.

So'nggi yillarda Markaziy Osiyo mamlakatlarida sanoat tarmog'ining rivojlanishi milliy iqtisodiy siyosatning ustuvor yo'nalishlaridan biri sifatida qaralmoqda. Xususan, O'zbekiston, Qozog'iston va Qirg'iziston respublikalarida sanoat ishlab chiqarishini kengaytirish, uning tarkibiy tuzilmasini diversifikatsiya qilish hamda sanoatda mehnat unumdorligini oshirishga qaratilgan tizimli islohotlar amalga oshirilmoqda. Ushbu jarayonlar sanoat tarmog'ining iqtisodiy o'sish, bandlikni ta'minlash va eksport salohiyatini oshirishdagi rolini yanada kuchaytirmoqda. Mazkur maqolaning asosiy maqsadi O'zbekiston, Qozog'iston va Qirg'iziston respublikalarida 2015–2024-yillar davomida sanoat tarmog'ining rivojlanish tendensiyalarini solishtirma statistik tahlil asosida baholashdan iborat. Tadqiqot doirasida sanoat mahsuloti hajmining dinamikasi, o'sish sur'atlari, sanoatda bandlik ko'rsatkichlari hamda mehnat unumdorligi darajasi tahlil qilinadi. Shuningdek, sanoat mahsuloti hajmining bandlar soniga nisbatan o'sish sur'atlari va ularning prognoz ko'rsatkichlari orqali sanoat rivojlanishining istiqbollari aniqlanadi [1]. Ushbu solishtirma statistik tahlil natijasida mamlakatlar o'rtasidagi sanoat rivojlanishidagi farqlar, ustun va zaif jihatlar aniqlanib, sanoat tarmog'ini rivojlantirish bo'yicha ilmiy-amaliy xulosalar chiqarish imkoniyati yaratiladi [2]. Olingan natijalar sanoat siyosatini takomillashtirish, mehnat unumdorligini oshirish va sanoatning iqtisodiy o'sishdagi rolini kuchaytirishga qaratilgan qarorlarni ilmiy asoslashda muhim ahamiyat kasb etadi [3].

1-jadval ma'lumotlari O'zbekiston, Qozog'iston va Qirg'izistonda 2015–2024- yillar davomida sanoat mahsuloti hajmining barqaror o'sish tendensiyasini ko'rsatadi.

Manba: www.stat.uz, www.stat.gov.kz, https://taldau.stat.gov.kz/ru/ www.stat.gov.kg saytlari ma'lumotlari asosida muallif tomonidan tuzilgan. [4], [5], [6]

Tahlil natijalariga ko'ra, milliy valyutada ifodalangan sanoat mahsuloti hajmi eng yuqori

1-jadval.O‘zbekiston, Qozog‘iston va Qirg‘izistonda 2015–2024-yillarda sanoat mahsuloti hajmi va sanoatda bandlar soni dinamikasi

Yillar	Sanoat ishlab chiqarish hajmi			Sanoatda bandlar soni		
	O‘zbekiston (mlrd so‘m)	Qozog‘iston (mlrd tenge)	Qirg‘iziston (mln som)	O‘zbekiston (ming kishi)	Qozog‘iston (ming kishi)	Qirg‘iziston (ming kishi)
2015	97 598,2	14 931,4	181 026,7	1768,7	1083,7	226,2
2016	111 869,4	19 026,8	209 812,0	1802,4	1087,2	238,7
2017	148 816,0	22 790,2	237 225,3	1826,8	1090,4	288,4
2018	235 340,7	27 218,1	257 348,5	1802,9	1097,8	344,0
2019	322 535,8	29 380,3	283 971,7	1821,5	1094,9	363,6
2020	368 740,2	27 028,5	325 090,1	1809,5	1089,2	344,0
2021	456 056,1	37 606,2	370 533,8	1863,2	1098,0	363,2
2022	553 265,0	48 777,1	435 815,6	1810,6	1121,2	360,0
2023	658 991,7	46 991,8	495 344,0	1835,9	1121,5	356,3
2024	880 198,5	51 469,1	585 275,8	1729,3	1157,8	375,7
O‘zgarish 2024/2015	9,02	3,45	3,23	0,98	1,07	1,66
O‘zgarish 2024/2019	2,73	1,75	2,06	0,95	1,06	1,03

o’sish sur’ati O’zbekistonda kuzatilgan bo’lib, 2024-yilda 2015-yilga nisbatan qariyb 9 barobar oshgan. Qozog’iston va Qirg’izistonda ham sanoat mahsuloti hajmi ijobiy dinamikaga ega bo’lsa-da, o’sish sur’atlari O’zbekistonga nisbatan pastroq bo’lgan. AQSh dollaridagi ifodada esa valyuta kurslari ta’siri tufayli ayrim yillarda o’sish sur’atlarining tebranishi kuzatiladi. Umuman olganda, sanoat tarmog’i barcha uch mamlakat iqtisodiyotida yetakchi o’sish manbalaridan biri bo’lib qolmoqda.

Ayniqsa, O’zbekistonda 2017-yilgacha amal qilgan qat’iy valyuta tartibi va 2017-yildan keyingi valyuta liberallashuvi sanoat ko’rsatkichlarining AQSh dollaridagi ifodasiga sezilarli ta’sir ko’rsatgan. Liberallashuvdan keyingi davrda so’m kursining bozor mexanizmlariga moslashuvi natijasida sanoat mahsuloti hajmining dollar ekvivalentida yanada realroq baholanishiga erishilgan. Bu holat sanoat rivojlanishini xalqaro miqyosda taqqoslash imkoniyatlarini kengaytirgan.

2-jadval. O’zbekiston, Qozog’iston va Qirg’izistonda 2015-2024-yillardagi sanoat mahsuloti hajmining o’sish sur’ati

Yillar	Mln. AQSh dollarida			O’zbekiston sanoat mahsuloti hajmining nisbati	
	O’zbekistonda	Qozog’istonda	Qirg’izistonda	Qozog’istonga nisbatan	Qirg’izistonga nisbatan
2015	38 005,3	67 258,6	2 878,0	0,57	13,21
2016	37 726,2	55 797,1	3 001,6	0,68	12,57
2017	29 100,3	69 908,6	3 448,0	0,42	8,44
2018	29 163,9	78 893,0	3 762,4	0,37	7,75
2019	36 502,5	76 911,8	4 068,4	0,47	8,97
2020	36 669,4	65 603,2	4 288,8	0,56	8,55
2021	42 983,6	88 485,2	4 400,6	0,49	9,77
2022	50 063,8	106 037,2	5 276,2	0,47	9,49
2023	56 152,3	104 426,2	5 693,6	0,54	9,86
2024	69 566,1	108 356,0	6 576,1	0,64	10,58
O’zgarish 2024/2015	1,83	1,61	2,28	1,14	0,80
O’zgarish 2024/2019	1,91	1,41	1,62	1,35	1,18

Manba: www.stat.uz, www.stat.gov.kz, https://taldau.stat.gov.kz/ru/ www.stat.gov.kg saytlari ma’lumotlari asosida muallif ishlanmasi. [4], [5], [6]

2–jadvalda sanoat mahsuloti hajmining AQSh dollaridagi qiymatylari mamlakatlar kesimida sezilarli farqlanishi kuzatiladi. O’zbekistonda 2015–2016- yillarda o’sish

sur'atlarining past yoki manfiy bo'lishi asosan valyuta kursining ma'muriy tartibga solinishi bilan izohlanadi; bu holat dollar ekvivalentida ishlab chiqarish natijalarini real aks ettirishni cheklagan. 2017-yildan keyin valyuta liberallashuvi, investitsiya muhiti yaxshilanishi va sanoatni kengaytirishga qaratilgan davlat dasturlari hisobiga o'sish sur'atlari barqarorlashib, tezlashgan. Natijada, O'zbekistonning sanoat o'sishi dollar ifodasida ham izchil ijobiy trayektoriyaga o'tdi.

Qozog'istonda o'sish sur'atlari nisbatan yuqori tebranishlarga ega bo'lib, ayrim yillarda keskin pasayishlar qayd etiladi. Buning asosiy sababi — sanoat tuzilmasining xomashyo sektoriga yuqori darajada bog'liqligi va jahon bozorlaridagi narx kon'yunkturasining ta'siri. Shuning uchun, Qozog'istonda o'sish sur'atlari ayrim yillarda O'zbekistondan yuqori bo'lsa-da, barqarorlik darajasi pastroq bo'lib qolmoqda.

Qirg'izistonda sanoat o'sish sur'atlari ko'pincha ijobiy, ammo past va o'zgaruvchan xarakterga ega. Bunga sanoat bazasining cheklanganligi, ishlab chiqarish hajmlarining kichikligi va ayrim yirik loyihalarga qaramlik sabab bo'lmoqda. Natijada, dollar ifodasidagi o'sish sur'atlari mavjud bo'lsa-da, ularning iqtisodiyotga umumiy ta'siri nisbatan cheklangan.

2-jadvalning 5- va 6-ustunlarida O'zbekistonda sanoat mahsuloti hajmining AQSh dollaridagi qiymati Qozog'iston va Qirg'iziston ko'rsatkichlariga nisbatan ifodalangan.

3-jadvalda sanoatda bandlar soni hamda bir bandga to'g'ri keladigan sanoat mahsuloti hajmi ko'rsatkichlari orqali sanoat rivojlanishining miqdoriy emas, balki sifat jihatlari tahlil qilinadi. Ushbu yondashuv sanoat o'sishining faqat bandlar soni hisobiga emas, balki mehnat unumdorligi orqali ta'minlanayotganini aniqlash imkonini beradi.

Tahlil natijalari O'zbekistonda sanoat mahsuloti hajmi bandlar soniga nisbatan tezroq oshib borayotganini ko'rsatadi, bu esa sanoatda samaradorlikning ortib borayotganidan dalolat beradi. Qozog'istonda bir bandga to'g'ri keladigan sanoat mahsuloti yuqori bo'lsa-da, uning o'sish sur'ati nisbatan barqaror emas. Qirg'izistonda esa bandlar soni va mahsulot hajmi o'rtasidagi nomutanosiblik sanoat samaradorligining pastligidan dalolat beradi. Umuman olganda, mazkur tahlil sanoat rivojlanishining ekstensiv yoki intensiv xarakterga ega ekanini aniqlashga xizmat qiladi. Olingan natijalar sanoat siyosatini takomillashtirishda mehnat unumdorligini oshirishga qaratilgan choralar muhimligini asoslab beradi.

3-jadval.O'zbekiston, Qozog'iston va Qirg'izistonda 2015–2024-yillarda sanoatda bandlar soni hamda sanoatda bandlarga mos sanoat mahsuloti hajmi va nisbati

Yillar	Sanoatda bandlar soni (ming kishi)			Sanoat mahsuloti hajmi (1000 $)			O'zbekistonda sanoatda bandlarga mos mahsulot	
	O'zbekistonda	Qozog'istonda	Qirg'izistonda	O'zbekistonda	Qozog'istonda	Qirg'izistonda	Qozog'istonga nisbatan	Qirg'izistonga nisbatan
2015	1768,7	1083,7	226,2	21,488	62,064	12,723	0,35	1,69
2016	1802,4	1087,2	238,7	20,931	51,322	12,575	0,41	1,66
2017	1826,8	1090,4	288,4	15,930	64,113	11,956	0,25	1,33
2018	1802,9	1097,8	344,0	16,176	71,865	10,937	0,23	1,48
2019	1821,5	1094,9	363,6	20,040	70,246	11,189	0,29	1,79
2020	1809,5	1089,2	344,0	20,265	60,231	12,467	0,34	1,63
2021	1863,2	1098,0	363,2	23,070	80,588	12,116	0,29	1,90
2022	1810,6	1121,2	360,0	27,650	94,575	14,656	0,29	1,89
2023	1835,9	1121,5	356,3	30,586	93,113	15,980	0,33	1,91
2024	1729,3	1157,8	375,7	40,228	93,588	17,504	0,43	2,30
O'zgarish 2024/2015	0,98	1,07	1,66	1,87	1,51	1,38	1,24	1,36
O'zgarish 2024/2019	0,95	1,06	1,03	2,01	1,33	1,56	1,51	1,28

Manba: www.stat.uz, www.stat.gov.kz, https://taldau.stat.gov.kz/ru/ www.stat.gov.kg saytlari ma'lumotlari asosida muallif tomonidan tuzilgan. [4], [5], [6]

3-jadvalning birinchi uch ustunida sanoatda bandlar soni mamlakatlar kesimida aks ettirilgan bo'lib, O'zbekistonda bandlar sonining 2015–2024-yillarda deyarli barqaror saqlanib, ayrim yillarda qisqarishi sanoat o'sishining ekstensiv emasligini ko'rsatadi. Qozog'istonda bandlar sonining sekin, ammo izchil oshishi sanoat rivojlanishining nisbatan barqaror mehnat bazasiga tayanganini anglatadi. Qirg'izistonda esa bandlar sonining tezroq o'sishi sanoat ishlab chiqarishining ko'proq ekstensiv omillarga bog'liqligidan dalolat beradi. Keyingi uch ustunda keltirilgan bir bandga to'g'ri keladigan sanoat mahsuloti hajmi O'zbekistonda keskin oshib, mehnat unumdorligining sezilarli yaxshilanganini ko'rsatadi. Qozog'istonda ushbu ko'rsatkich yuqori bo'lsa-da, o'sish sur'atlarining tebranishi xomashyo sektoriga bog'liqlik bilan izohlanadi. Umuman olganda, 3-jadval sanoat rivojlanishining mamlakatlar kesimida intensivlik darajasi turlicha ekanini va O'zbekiston sanoati samaradorlik bosqichiga o'tayotganini ko'rsatadi. Jadvalning dastlabki uch ustuni mehnat unumdorligi o'sish sur'atlarining mamlakatlar kesimidagi farqlarini ko'rsatib, O'zbekistonda 2017-yildan keyin mazkur ko'rsatkichning sezilarli tezlashganini tasdiqlaydi. Bu holat sanoatni modernizatsiya qilish, texnologik yangilanish va kapital sig'imining ortishi bilan izohlanadi. 3-jadvalning 8-ustunida O'zbekistonda bir bandga to'g'ri keladigan sanoat mahsuloti hajmining Qozog'iston ko'rsatkichiga nisbati keltirilgan bo'lib, ushbu nisbat 2015–2024-yillarda 0,23–0,43 oralig'ida shakllangan. Bu natija Qozog'istonda mehnat unumdorligi yuqori ekanini ko'rsatsa-da, nisbatning vaqt o'tishi bilan oshib borishi O'zbekiston sanoatida samaradorlikning tezroq o'sayotganidan dalolat beradi. 9-ustunda esa O'zbekistonning bir bandga to'g'ri keladigan sanoat mahsuloti hajmi Qirg'iziston ko'rsatkichiga nisbatan ifodalangan bo'lib, ushbu nisbat 1,3–2,3 barobar atrofida shakllangan. Bu O'zbekiston sanoatining mehnat unumdorligi bo'yicha Qirg'izistondan ancha ustun ekanini ko'rsatadi. Shu bilan birga, nisbatning vaqt o'tishi bilan ortib borishi O'zbekistonda sanoat samaradorligining oshib borayotganini, Qirg'izistonda esa texnologik va kapital cheklovlar saqlanib qolayotganini anglatadi. Mazkur tadqiqotda 3-jadval sanoatda bandlar soniga to'g'ri keladigan sanoat mahsuloti hajmining kelgusi davr uchun prognoz ko'rsatkichlarini aniqlash maqsadida keltirilgan. Oldingi jadvallarda sanoat rivojlanishining joriy va o'tgan davrdagi holati tahlil qilingan bo'lsa, 3-jadval ushbu tendensiyalarning barqarorligi va davomiyligini baholash imkonini beradi. 3-jadvaldagi 5-6-7-ustunlar asosida prognoz qilish imkoniyati mavjud. Prognozlar MS Excel dasturida oddiy vaqtga bog'liq trend modellar orqali hisoblangan.Prognoz ko'rsatkichlari sanoat rivojlanishining intensiv omillarga tayangan holda shakllanayotganini aniqlashga xizmat qiladi. Shuningdek, jadval mamlakatlar kesimida sanoat samaradorligining kelgusidagi nisbiy holatini solishtirish imkonini beradi. Bu yondashuv sanoat siyosati uchun oldindan baholash va rejalashtirish nuqtayi nazaridan muhimdir.

4-jadval natijalari O'zbekistonda sanoatda bandlar soniga to'g'ri keladigan sanoat mahsuloti hajmi kelgusi yillarda tez sur'atlarda oshib borishini ko'rsatadi. Prognozlarga ko'ra, mazkur ko'rsatkichning o'sish sur'ati Qozog'iston va Qirg'izistonga nisbatan yuqoriroq bo'lib, bu O'zbekistonda sanoat samaradorligining oshib borayotganidan dalolat beradi.

Qozog'istonda prognoz ko'rsatkichlari yuqori mutlaq qiymatlarda shakllansada, o'sish sur'atlarining nisbatan pastligi sanoat rivojlanishining yetuk bosqichga kirganini

4-jadval. Sanoatda bandlar soniga mos sanoat mahsuloti hajmi prognozi

Yillar	Sanoat mahsuloti hajmining prognozi (1000 dollar)			O‘zbekiston sanoat mahsuloti hajmining nisbati	
	O‘zbekistonda	Qozog‘istonda	Qirg‘izistonda	Qozog‘istonga nisbatan	Qirg‘izistonga nisbatan
2024	39,101	98,334	17,719	0,40	2,21
2025	46,730	106,060	20,056	0,44	2,33
2026	55,498	114,429	22,758	0,49	2,44
2027	65,409	123,440	25,824	0,53	2,53
2028	76,462	133,094	29,253	0,57	2,61
Farq (+/–) 2028–2024	37,361	34,760	11,534	0,18	0,41
O‘zgarish 2028/2024	95,54	35,35	65,10	44,47	18,44

anglatadi. Qirg’izistonda esa prognoz natijalari sanoat samaradorligi o’sishining sekinroq kechishini ko’rsatadi. Umuman olganda, 4-jadval O’zbekiston sanoatining o’rta muddatda raqobatbardoshligini kuchaytirish imkoniyatlari mavjudligini tasdiqlaydi.

Olib borilgan tadqiqot natijalari O’zbekiston, Qozog’iston va Qirg’iziston respublikalarida sanoat rivojlanishining miqyosi, sur’atlari va samaradorligi bo’yicha sezilarli farqlar mavjudligini ko’rsatdi. Tahlillar O’zbekistonda sanoat mahsuloti hajmi va mehnat unumdorligining so’nggi yillarda barqaror o’sib borayotganini tasdiqlaydi. Valyuta liberallashuvi, investitsiya muhitining yaxshilanishi va sanoatni diversifikatsiya qilishga qaratilgan siyosat sanoat rivojlanishiga ijobiy ta’sir ko’rsatgan. Qozog’istonda sanoat rivoji yuqori mutlaq ko’rsatkichlar bilan tavsiflansada, uning xomashyo sektoriga bog’liqligi barqarorlikka ma’lum darajada salbiy ta’sir ko’rsatmoqda. Qirg’izistonda esa sanoat rivojlanishi cheklangan ishlab chiqarish bazasi va kapital yetishmasligi bilan izohlanadi.

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Mintaqalarda yashil innovatsion boshqaruv tizimini modernizatsiya qilishning iqtisodiy asoslari

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DOI 10.5281/zenodo.18217644

Annotatsiya:

Ushbu maqola O'zbekistonning "yashil"iqtisodiyotga o'tish jarayonini va uning kelajakdagi prognozlarini chuqur o'rganishga bag'ishlangan. Maqolada O'zbekiston iqtisodiyotining barqaror rivojlanishi va xalqaro hamkorlikning roli, "yashil"iqtisodiyotning zarurligi, energetika sohalarini "yashillashtirish"hamda muqobil energiya manbalarini rivojlantirishning ahamiyati keltiriladi. Shuningdek, "yashil"iqtisodiyotda samaradorlik va inklyuzivlikni oshirish, mintaqaviy innovatsiyalarni boshqarishning ahamiyati, va bu o'zgarishlarning O'zbekiston iqtisodiyotiga, xususan, uning asosiy loyihalariga kelajakdagi ta'siri tahlil qilinadi. Maqola, shuningdek, mintaqaviy barqaror innovatsiyalarni boshqarish orqali O'zbekistonning "yashil"iqtisodiyotga o'tish yo'li va uning iqtisodiy, ijtimoiy hamda ekologik ta'sirlarini tahliliy o'rganildi.

Keywords: Yashil iqtisodiyot, ijtimoiy muhit, ekologik innovatsiyalar, yashil siyosat, ekologik investitsiyalar, muqobil energiya manbalari, ekologik toza transport, sanoat 4.0 . O'zbekistonda iqtisodiy o'zgarishlar .

Kirish.

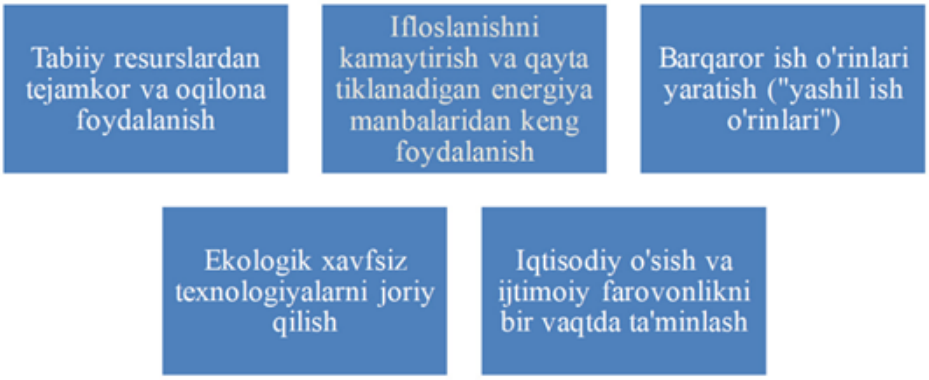
Jahon banki ma'lumotlariga ko'ra, dunyo umumiy boyligining 66 foizi inson kapitaliga, ya'ni asosan shaxsning ilm darajasiga to'g'ri kelmoqda. Shuning uchun davlatimiz rahbari 2020 yil boshida e'lon qilgan Murojaatida "Eng katta boylik - bu aql-zakovat va ilm, eng katta meros - bu yaxshi tarbiya, eng katta qashshoqlik - bu bilimsizlikdir degan fikrni alohida keltirib o'tgan edi . Barqaror rivojlanish o'ta dolzarb muammo hisoblanib, uni "ijtimoiy "iqtisodiy "ekologik"uchlikning o'zaro bog'liqlikda rivojlanishiga asoslangan sinergetik samara sifatida tasavvur etish to'g'ri bo'ladi. Barqaror rivojlanishni ta'minlashdagi iqtisodiy yondashuv yashil iqtisodiyot asosida cheklangan resurslardan va mavjud elektr energiyasidan oqilona foydalanish kabi muammolarni hal qilishni nazarda tutuvchi munosabatlarni yuzaga chiqaradi.

Barqaror rivojlanish tamoyillari mintaqaviy iqtisodiy o'sishning asosiy harakatlantiruvchi kuchi bo'lib, bugungi kunda nafaqat global, balki milliy va hududiy miqyosda ham dolzarb masala hisoblanadi. Insoniyat taraqqiyoti bosqichida iqtisodiy o'sishni ijtimoiy barqarorlik va ekologik xavfsizlik bilan uyg'unlashtirish zaruriyati kundan-kunga ortib bormoqda. Barqaror iqtisodiy o'sishni ta'minlash, bu jarayonda resurslardan oqilona foydalanish va innovatsion texnologiyalarni joriy etish orqali

raqobatbardoshlikni oshirish. Shuningdek, Aholi turmush darajasini yaxshilash, ta'lim, sog'liqni saqlash va bandlikni ta'minlash orqali ijtimoiy tenglikni mustahkamlash. Atrof-muhitni muhofaza qilish, tabiiy resurslarni tejash va iqlim o'zgarishlariga qarshi kurashish bo'yicha samarali choralarni amalga oshirish orqali, mintaqaviy rivojlanish strategiyalarini uch asosiy ustun iqtisodiyot, ijtimoiy soha va ekologiya o'zaro bog'liq holda ko'rib chiqishni talab qiladi. Shu yo'l bilan uzoq muddatli va inklyuziv taraqqiyotga erishish mumkin. Shu sababli BMT tomonidan 2015-yilda qabul qilingan Barqaror rivojlanish maqsadlari (SDGs) 2030-yilgacha bo'lgan davr uchun barcha mamlakatlarning strategik yo'nalishlarini belgilab berdi [1].

O'zbekiston ushbu global tashabbusni faol qo'llab-quvvatlab, o'zining 'O'zbekiston–2030' strategiyasini ishlab chiqdi [2]. Mazkur strategiya mamlakatning iqtisodiy salohiyatini oshirish, ijtimoiy sohalarni rivojlantirish va ekologik muammolarni bartaraf etishga yo'naltirilgan. Iqtisodiy barqarorlik hududiy iqtisodiyotning samaradorligini, investitsion jozibadorlikni va innovatsion faoliyatni oshirish bilan bog'liq. Ijtimoiy barqarorlik esa aholining bandligi, turmush darajasi, ta'lim va sog'liqni saqlash sohalaridagi islohotlar samaradorligi bilan belgilanadi. Ekologik barqarorlik esa tabiiy resurslardan oqilona foydalanish, atrof-muhitni muhofaza qilish va ekologik xavfsizlikni mustahkamlashni nazarda tutadi [3]. Mamlakatimizda "yashil iqtisodiyot"ga o'tish zarurati milliy iqtisodiyotda iste'mol qilinayotgan energiyaning aksariyat qismi tiklanmaydigan tabiiy resurslardan foydalanilayotganligi, ushbu resurslar zahirasining cheklanganligi, atrof-muhitning ifloslanishi bilan bog'liq ekologik muammolarning keskinlashib borayotganligi bilan ham izohlanadi.

O'zbekiston milliy iqtisodiyotini barqaror rivojlantirish, tarkibiy o'zgarishlarning uzoq muddatli strategiyasini ishlab chiqish "yashil iqtisodiyot"ga o'tish muammolarini hisobga olishni taqozo etadi, zero, tabiiy resurslar zahiralarining cheklanganligi, takror ishlab chiqarilmasligi bilan bog'liq muammolar insoniyatning barcha faoliyat yo'nalishlari, jumladan, iqtisodiy faoliyatiga jiddiy ta'sir ko'rsatishi han nazardan chetda qolmaydi. Bugungi kunda global miqyosda iqlim o'zgarishi, tabiiy resurslarning qisqarishi va ekologik muammolarning keskinlashuvi iqtisodiyotning yangi rivojlanish modeliga o'tishni taqozo etmoqda. Ana shunday dolzarb yondashuvlardan biri "yashil iqtisodiyot" konsepsiyasidir. Ushbu model nafaqat iqtisodiy o'sishni ta'minlash, balki resurslardan samarali foydalanish, atrof-muhitni muhofaza qilish va inson kapitalini rivojlantirishga qaratilgan. Xalqaro Mehnat Tashkiloti (ILO) ma'lumotlariga ko'ra [1], yashil iqtisodiyotga o'tish jarayoni 2030-yilgacha dunyo bo'yicha 24 milliondan ortiq yangi ish o'rinlarini yaratish imkonini beradi. Shu nuqtai nazardan, O'zbekiston ham milliy strategik hujjatlar orqali yashil transformatsiyani ustuvor yo'nalish sifatida belgilab oldi.



Rasm 1. Yashil iqtisodiyotning asosiy tamoyillari

Ushbu maqolada mintaqaviy iqtisodiy o'sishda barqaror rivojlanish tamoyillarining nazariy asoslari, ularning mintaqada qo'llanishi hamda xorijiy mamlakatlarning ilg'or tajribasi tahlil qilinadi. Shuningdek, mavjud muammolar va ularni hal etish yo'llari bo'yicha ilmiy asoslangan takliflar ishlab chiqiladi.

Barqaror rivojlanish tushunchasi XX asrning ikkinchi yarmida shakllanib, 1987-yilda BMTning Brundtland hisobotida "bizning umumiy kelajagimiz"konsepsiyasi asosida keng e'tirof etildi [1]. Bu konsepsiya insoniyatning hozirgi ehtiyojlarini qondirish bilan birga, kelajak avlodlarning o'z ehtiyojlarini qondirish imkoniyatlariga zarar yetkazmaslikni ta'minlashga qaratilgan. Barqaror rivojlanish o'zaro bog'liq bo'lgan uchta asosiy komponentga tayanadi, ular ko'pincha "Uch P"(People, Planet, Profit) deb nomlanadi va muvozanatli yondashuvni talab qiladi:

-Uzoq muddatli iqtisodiy o'sishni ta'minlash, resurslarni samarali boshqarish va barcha uchun adolatli iqtisodiy imkoniyatlarni yaratish.

-Jamiyatda tenglik, adolat, inson huquqlari va farovonlikni ta'minlash. Ta'lim, sog'liqni saqlash va ijtimoiy xizmatlarga universal kirishni o'z ichiga oladi.

-Atrof-muhitni himoya qilish, tabiiy resurslarni saqlash, ifloslanishni kamaytirish va iqlim o'zgarishiga qarshi kurashish.

O'zbekistonda barqaror rivojlanish siyosati davlatning uzoq muddatli strategik hujjatlarida aks etgan. Xususan, 2023-yilda qabul qilingan "O'zbekiston–2030"strategiyasi mamlakatni iqtisodiy, ijtimoiy va ekologik yo'nalishlarda kompleks rivojlantirishga qaratilgan [2]. Shuningdek, "Qishloq xo'jaligini rivojlantirish strategiyasi 2020–2030"ham mintaqaviy barqaror rivojlanishda asosiy hujjatlardan biri hisoblanadi [3].

2021–2024 yillar mobaynida O'zbekiston iqtisodiy ko'rsatkichlari barqaror o'sish tendensiyasini namoyon qilmoqda. 2021 yilda pandemiyadan keyingi tiklanish davrida YAIM 7,4 foiz ga oshgan bo'lsa, keyingi yillarda o'sish 6 foiz atrofida barqarorlashdi. Bandlik darajasi ham har yili asta-sekin ko'tarilib, 2024 yilga kelib 57,8 foizga yetti. Qishloq xo'jaligining YAIMdagi ulushi esa pasayib bormoqda (2021 yildagi 25,6 foizdan 2024 yilda 23,8 foizgacha), bu esa iqtisodiyotda sanoat va xizmatlar sohasining tezroq rivojlanayotganidan dalolat beradi. Shu bilan birga, investitsiyalar va eksport hajmining muntazam ortib borishi O'zbekistonning xalqaro bozorlar bilan integratsiyalashuvini kuchaytirayotganini ko'rsatadi. [5].

1-jadval

O‘zbekistonda barqaror rivojlanish indikatorlari (2020–2024)

Ko‘rsatkichlar	2020	2021	2022	2023	2024
YaIM o‘sishi (%)	1,6	7,4	5,7	6,0	6,1
Bandlik darajasi (%)	54,1	55,3	56,2	57,1	57,8
Qishloq xo‘jaligi ulushi YaIMda (%)	26,1	25,6	24,9	24,1	23,8
Investitsiyalar hajmi (mlrd. $)	19,3	22,5	25,7	27,1	28,6
Eksport hajmi (mlrd. $)	13,0	14,3	15,7	18,2	19,5

Manba: O‘zbekiston Respublikasi Davlat statistika qo‘mitasi

Rasm 1. Yashil iqtisodiyotning asosiy tamoyillari

Barqaror rivojlanish bo’yicha xalqaro tajribalar O’zbekiston uchun muhim ahamiyat kasb etadi. Yevropa Ittifoqi "Green Deal"dasturi orqali karbon neytrallikka erishishni maqsad qilgan [6]. Janubiy Koreya "Yashil o’sish"modeli orqali iqtisodiyotning innovatsion rivojlanishiga erishgan [7]. Turkiya va Qozog’iston esa mintaqaviy rivojlanishda suv resurslarini boshqarish bo’yicha tajribaga ega. Ushbu tajribalarni mamlakatimiz sharoitiga moslashtirish barqaror rivojlanishga xizmat qilishi mumkin.

Tadqiqot natijalari shuni ko’rsatadiki, mintaqaviy iqtisodiy o’sishda barqaror rivojlanish tamoyillari muhim ahamiyat kasb etadi. Ushbu tamoyillar iqtisodiy, ijtimoiy va ekologik barqarorlikni ta’minlash orqali mintaqaning umumiy farovonligini oshirishga xizmat qiladi. Iqtisodiy barqarorlik investitsion jozibadorlikni oshirish, innovatsion platformalarni joriy etish va ishlab chiqarish samaradorligini ta’minlash orqali namoyon bo’lmoqda. Bu, o’z navbatida, yangi ish o’rinlari yaratish va mintaqaviy daromadlarni oshirishga yordam beradi. Ijtimoiy barqarorlik aholining bandligi, ta’lim va sog’liqni saqlash tizimlarining rivojlanishi bilan chambarchas bog’liq. Sifatli ta’lim va sog’liqni saqlash xizmatlaridan foydalanish imkoniyatini kengaytirish inson kapitalini mustahkamlaydi.

Ekologik barqarorlik esa suv resurslaridan oqilona foydalanish, bioxilma-xillikni asrash va ekologik xavfsizlikni mustahkamlash orqali amalga oshirilmoqda. Tabiiy resurslarni saqlash kelajak avlodlar uchun sog’lom muhitni ta’minlashning asosidir. Keyingi yillarda barqaror rivojlanishni ta’minlash, "sof texnologiyalar"ga nisbatan innovatsion yondashishni keltirib chiqardi va xalqaro investitsion forumlarda muntazam ravishda muhokama etib kelinmoqda. "Yashil iqtisodiyot"konsepsiyasini amaliyotga tadbiq etish masalalari ushbu muhokamalarning negizini tashkil etadi. O’zbekistonda yashil iqtisodiyot va barqaror rivojlanish strategiyasini ishlab chiqish masalasiga e’tibor kuchaydi. Mamlakatimiz 2015-yildan keyingi davrda BMT tomonidan 2030-yilgacha mo’ljallangan 17 ta maqsadlar va 169 ta vazifalardan iborat barqaror rivojlanish dasturini qo’llab-quvvatlab, barqaror rivojlanish sohalari bo’yicha kompleks ishlarni olib borishini ma’lum qildi.

Mamlakatda 2022-2026 yillarga mo’ljallangan Yangi O’zbekistonning taraqqiyot strategiyasida inson qadrini yuksaltirish va erkin fuqarolik jamiyatini yanada rivojlantirish

2-jadval

Mintaqada barqaror innovatsiyalarni joriy etish orqali eksport salohiyatini optimal rivojlantirishning uslubiy asoslari.

Bosqich	Maqsad	Uslubiy tamoyillar	Amaliy choralari
Tahlil va diagnostika	Hududning ayni eksport salohiyatini aniqlash	SWOT tahlili (kuchli tomonlar, zaif tomonlar, imkoniyatlar, tahdidlar) Resurslar va infra imkoniyatlar inventari: tabiiy resurslar, ishchi kuchi, transport, kommunikatsiya, moliyaviy xizmatlar Bozor tadqiqoti: mahsulot qaysi bozorlar uchun raqobatbardosh, talab qanday, narx va transport xarajatlari qanday	Hududdagi asosiy sektorlarda eksport bo‘yicha imkoniyatlar jadvalini tuzish; mahsulot-bozor matritsasi yaratish; talab va narx tendensiyalarini o‘rganish; logistika imkoniyatlarini baholash (masalan, transport yo‘llari, chegara o‘tish punktlari, bojxona posti).
Mahsulot diversifikatsiyasi va qo‘shilgan qiymat oshirish	Oddiy xom ashyodan tayyor yoki yarim tayyor mahsulotlarga o‘tish, yuqori qo‘shilgan qiymat yaratish	Innovatsiyalarni jalb qilish, Texnologik yangilanishlar va ishlab chiqarish jarayonini modernizatsiya qilish, Sifat nazorati va sertifikat talablarini oshirish (ISO, global GAP, ekologik, gigiyena va hokazo)	Mahalliy mahsulotni eksportga mos dizayn, qadoqlash, brend yaratish; kichik va o‘rta korxonalarda texnologik vositalarga investitsiya; sanoat zonalarini, eksportga qaratilgan litsenziyalar va soliq imtiyozlarini joriy etish; “klaster” shakllarini rag‘batlantirish.
Logistika va infratuzilma	Tashish, saqlash, bojxona, infratuzilma to‘siqlarini kamaytirish	Raqobatbardosh transport-marşrutlar tashkil qilish, Saqlash omborlari, sovutgichli/invoqatorli infratuzilma rivojlantirish, Raqobatbardosh bojxona va chek o‘tkazish punktlari, Raqamli tizimlar, axborot texnologiyalari orqali jarayonlarni soddalashtirish	Hududiy logistika markazlarini tashkil etish; transport yo‘llari, yo‘l yaxshilanishi; tashqi bozorlarga chiqishda jarayonlarni tezlashtiradigan elektron hujjatlar tizimini kengaytirish; chegaradagi savdo infratuzilmasini rivojlantirish; tarif va notarif to‘siqlarni kamaytirish.
Moliyaviy –iqtisodiy rag‘batlar va siyosiy qo‘llab-quvvatlash	Eksportch ilarga imtiyozlar, subsidiyalar, kreditlar orqali yordam berish	Ex-ante va ex-post yordam shakllari (kredit, kafolat, sug‘urtalash, ekspor-garantiyalar), Soliqqa oid preferensiyalar, bojxona chegirmalari, Eksportni rag‘batlantirish strategiyasini davlat darajasida qo‘llab-quvvatlash, Hududiy boshqaruv organlarini jalb qilish	Eksport kreditlarini kengaytirish; banklarning pre-export moliyalashtirish imkoniyatlarini yaratish; davlat tomonidan eksportga oid treninglar, maslahat xizmatlarini tashkil etish; soliq, bojxona tartib-qoidalarini soddalashtirish; eksport markazlari, bir nechta davlat-xususiy hamkorlik loyihalarini amalga oshirish.
Bozorlar bilan aloqalar, marketing va brending	Mahsulotn i xalqaro bozorda tanitish va raqobatbardoshlikni oshirish	Mahsulot brendi va sifat imidjini shakllantirish, Sertifikat va standartlarga muvofiqlik orqali ishonch hosil qilish, Tashqi bozorlarga mos mahsulot ishlab chiqish (adaptatsiya), Raqamli savdo, elektron savdo platformalaridan foydalanish	Tashqi bozor ko‘rgazmalari, yarmarkalar; eksport agentliklari yoki plagin markazi tashkiloti; marketing kampaniyalari; mahsulot qadoqlash, etiketka, til va talabga moslab dizayn ishlarini
Inson kapitali va institutiv salohiyat	Eksportni amalga oshira oladigan malakali kadrlar, institutlar va siyosiy barqarorlikni ta’minlash	Kasbiy tayyorgarlik va malaka oshirish, Innovatsion tadqiqot-laboratoriyalar, texnologik parklari, ilmiy-ishlab chiqarish hamkorliklar, Transparent va barqaror siyosat, korruptsiyani kamaytirish	Hududiy darajadagi eksport bo‘yicha o‘quv markazlari; chet el ekspertlarini jalb qilish; hukumat siyosatini stabil va oldindan aytilgan qilish; qonun va normativ bazalarni takomillashtirish; davlat-xususiy sherikliklar orqali institutlar barqarorligini oshirish.
Monitoring va baholash	Eksport siyosatining samaradorligini doimiy o‘lchash va takomillashtirish	Indikatorlar (eksport hajmi, eksport qiluvchi korxona soni, mahsulotlar diversifikatsiyasi, bozorlar soni, qo‘shilgan qiymat foizi, logistika xarajatlari va vaqt), Raqamli ma’lumotlar tizimlari orqali real vaqt nazorat Qayta aloqa va korrigatsion chora-tadbirlarni joriy etish	Har yil eksport salohiyati bo‘yicha hisobotlar; hududiy eksport monitoring markazlari; eksportga oid loyihalar samaradorligi bo‘yicha tahlil va audit; eksport siyosatini takomillashtirish yuzasidan takliflar chiqarish.

orqali xalqparvar davlat barpo etish, milliy iqtisodiyotni jadal rivojlantirish va yuqori o'sish sur'atlarini ta'minlash, milliy manfaatlardan kelib chiqqan holda umumbashariy muammolarga yondashish kabi dolzarb masalalar ko'ndalang qilib belgilab berilgan [8].Bu strategiyasining qabul qilinishi "yashil iqtisodiyot"ni rivojlantirish barqaror iqtisodiy o'sishni ta'minlash omillaridan biri ekanligini tasdiqladi.

Kelajak istiqbolida O'zbekistonning eksport siyosati hududiy farqlarni hisobga olgan holda differensial rivojlanish yo'lidan borishi lozim. Har bir viloyatning o'z tabiiy resurslari, sanoat ixtisoslashuvi va infratuzilma imkoniyatlariga tayangan holda eksport strategiyasi ishlab chiqilsa, milliy darajada barqaror o'sish modeli shakllanadi. hududiy eksport strategiyalarini ishlab chiqish har bir viloyat yoki hudud eksport salohiyati, resurslari va infratuzilmasiga mos reja tuzishi kerak. Har bir hududda kuchli bo'lgan sektorlar va mahsulotlarga urg'u berish; eksport uchun markazlar yaratilishi hududiy eksport markazlari, logistika va qadoqlash inshootlari, sovutgichli saqlash omborlari, tovarni tashqi bozorlarga tayyorlash bo'limlari; Respublikasida barqaror rivojlanishga erishish uchun agrobiznesni modernizatsiya qilish, suv resurslaridan oqilona foydalanish, ekologik muammolarni hal qilish va xorijiy tajribalarni qo'llash zarurligi aniqlandi. Bundan tashqari, davlat siyosatini yanada takomillashtirish, investitsiya va innovatsion loyihalarni qo'llab-quvvatlash hududiy rivojlanishda asosiy omil bo'lib xizmat qilmoqda.

Xulosa.

Maqolada mintaqaviy iqtisodiy o'sishda barqaror rivojlanish tamoyillari nazariy va amaliy jihatdan o'rganildi. Xalqaro hamda mintaqaviy tajribalar shuni ko'rsatadiki, iqtisodiy o'sishni barqaror rivojlanish bilan uyg'unlashtirish davlat siyosati samaradorligini oshiradi. Bu jarayonda quyidagi xulosalarga kelindi:

1. Barqaror rivojlanish tamoyillari iqtisodiy o'sishni uzoq muddatli va izchil tarzda rivojlantirishning asosiy omilidir;

2. O'zbekistonda qabul qilingan strategik hujjatlar, jumladan "O'zbekiston–2030"strategiyasi va "Qishloq xo'jaligini rivojlantirish strategiyasi 2020–2030"barqaror rivojlanishning huquqiy va institutsional asosini yaratmoqda;

3. Respublikada barqaror rivojlanishga erishish uchun agrobiznesni modernizatsiya qilish, suv resurslaridan oqilona foydalanish va ekologik muammolarni bartaraf etish zarur;

4. Xorijiy tajribalarni, xususan Yevropa Ittifoqi, Janubiy Koreya va Qozog'iston tajribasini mahalliy sharoitga moslashtirish barqaror rivojlanish maqsadlariga erishishda muhim ahamiyat kasb etadi.

Shunday qilib, mintaqaviy iqtisodiy o'sishda barqaror rivojlanish tamoyillarini izchil amalga oshirish orqali iqtisodiy samaradorlik, ijtimoiy farovonlik va ekologik muvozanat ta'minlanadi. Shuningdek, mahalliy ishlab chiqarishda "yashil iqtisodiyot"ni samarali joriy etilishi o'ziga xos mintaqaviy profilni shakllantirish imkoniyatini yaratadi, resurslarning ustuvor yo'nalishlarda konsentratsiyasi esa resurslardan foydalanishda va ishlab chiqarishda samaradorlikni hamda raqobatbardoshlikning yuqori darajasini, atrof-muhitning maksimal muhofaza qilinishini ta'minlaydi.

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O'zbekistonda kreativ iqtisodiyotni qo'llash istiqbollari: 2025-2030 uchun imkoniyatlar va ustuvor yo'nalishlar

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DOI 10.5281/zenodo.18217654

Abstract. The topic of developing the creative economy in Uzbekistan has become a very active agenda in recent years within the framework of legal, institutional and international cooperation. This literature review summarizes the sources of normative and legal acts, international reports and national initiatives (Creative Park, incubator, IT ecosystem) that are the basis for the article, and serves to show their main ideas, strengths and limitations. It thereby reveals the evidence on which conclusions about the prospects of the creative economy in Uzbekistan are based.

Keywords: creative economy, IT park, Creative park, IT ecosystem, incubator

Anotatsiya. O'zbekistonda kreativ iqtisodiyotni rivojlantirish mavzusi so'nggi yillarda huquqiy, institutsional va xalqaro hamkorlik doirasida ancha faol kun tartibiga chiqdi. Mazkur adabiyotlar sharhi maqolada tayanch bo'lgan normativ-huquqiy hujjatlar, xalqaro hisobotlar hamda milliy tashabbuslar (Creative Park, inkubator, IT ekotizimi) haqidagi manbalarni jamlab, ularning asosiy g'oyalari, kuchli tomoni va cheklovlarini ko'rsatishga xizmat qiladi. Shu orqali kreativ iqtisodiyotning O'zbekistondagi istiqbollari bo'yicha xulosalar qanday dalillarga tayanganini ochib beradi.

Kalit so'zlar: kreativ iqtisodiyot, IT park, Kreativ park, IT ekotizimi, incubator

Kirish

Kreativ iqtisodiyot — bu g'oya, ijodiy mehnat va intellektual kapitalga tayanib qiymat yaratuvchi tarmoqlar majmui: dizayn, moda, kino, musiqa, nashriyot, reklama, IT va o'yinlar (game development), animatsiya, hunarmandchilik, madaniy merosga bog'liq xizmatlar va boshqalar. Bunday tarmoqlar "nomoddiy"ko'ringani bilan iqtisodiy natijasi juda moddiy: bandlik, xizmatlar eksporti, turizm daromadlari, brend qiymati, shaharlar jozibadorligi, innovatsiya va mahsulot raqobatbardoshligi.

So'nggi yillarda O'zbekistonda bu yo'nalish institutsional darajada kun tartibiga chiqdi: 2024-yil 3-oktabrda "Kreativ iqtisodiyot to'g'risida"gi qonun qabul qilinib, 2024-yil 4-oktabrdan kuchga kirgani qayd etilgan. Bu, kreativ tarmoqlarni alohida siyosat obyekti sifatida ko'rish va ularga xizmat qiladigan infratuzilma hamda moliyalashtirish mexanizmlarini tizimli qurish uchun muhim huquqiy poydevor demakdir.

Adabiyotlar sharxi

Kreativ iqtisodiyot bo'yicha eng ko'p uchraydigan muammo — **ta'riflarning turlicha bo'lishi** va natijada statistik solishtirishning qiyinlashishidir. UNCTAD'ning *Creative Economy Outlook 2024* hisobotida kreativ iqtisodiyotning mamlakatlar bo'yicha YAIM va bandlikdagi ulushi keng diapazonda ekanligi, ammo ma'lumotlar bir-biriga to'liq taqqoslanmasligi (turli ta'riflar, turli o'lchovlar va turli yil kesimi) qayd etiladi. Maqoladagi eng asosiy tayanch manba — 2024-yil 3-oktabrda qabul qilingan "Kreativ iqtisodiyot to'g'risida"gi Qonun (O'RQ-970).

Metodologiya

Ushbu tadqiqotda mavzu bo'yicha muhim ma'lumotlarni berish uchun tarixiy, induktiv va deduktiv usullardan foydalanildi.

Tahlil va natijalar: Nega aynan O'zbekiston uchun kreativ iqtisodiyot istiqbolli?

O'zbekistonning kreativ iqtisodiyotga tayanchi bo'ladigan uchta kuchli omili bor:

1. Madaniy kapital va meros: tarixiy shaharlar, hunarmandchilik maktablari, milliy naqsh, to'qimachilik va zargarlik an'analari — bularning barchasi mahsulot dizayni, suvenir, interyer, moda, turizm tajribasi va kontent (film/multimedia) uchun "xomashyo"vazifasini bajaradi.

2. Raqamli xizmatlar orqali eksport imkoniyati: kreativ xizmatlar (dizayn, animatsiya, o'yinlar, multimedia, dasturiy mahsulot) geografiyaga kamroq bog'liq, ya'ni "masofadan"turib ham eksport qilinadi.

3. Yoshlar resursi: kreativ kasblar, odatda, yoshlar uchun tez o'zlashtiriladigan, mustaqil daromad topishga imkon beradigan yo'nalishlar bo'lib, O'zbekistonda aynan yoshlar bandligini kengaytirish siyosati bilan uyg'unlashadi.

Siyosiy-institutsional baza: qonun, parklar va inkubatsiya

O'zbekistonda kreativ iqtisodiyotning "ekotizimi"shakllanishi bir nechta parallel yo'nalishda ketmoqda:

1) Maqsadli ko'rsatkich: YAIMda ulushni oshirish

IT Park va hamkorlarining Creative Incubator ochilishiga bag'ishlangan rasmiy xabarida 2024-yil 3-oktabrdagi Kreativ iqtisodiyot haqidagi qonun ijrosi doirasida kreativ industriyalarning YAIMga hissasini **2030-yilgacha 5% ga yetkazish** maqsadi tilga olingan. Bu ko'rsatkich — kreativ iqtisodiyotni "madaniyat"gina emas, balki iqtisodiy diversifikatsiya va eksportning real drayveri sifatida ko'rish belgisi.

2) Hududiy infratuzilma: Creative Parklar

Toshkentdagi Yoshlar ijod saroyida "model Creative Park"tashkil etilib, keyin barcha hududlarda shunday markazlarni barpo etish rejalari haqida xabar berilgan. Muhimi, park rezidentlari uchun preferensial soliq rejimi ko'zda tutilgani qayd etiladi (jumladan, JShDS va ijtimoiy soliq bo'yicha 50% chegirma, aylanmadan soliq rejimi).

3) "Soft"infratuzilma: inkubator, ta'lim va biznes ko'nikmalari

Creative Incubator loyihasi dizayner, animator, dasturchi, tadbirkor va boshqa kreativ mutaxassislar uchun ta'lim, master-klass, pitch va mentorlik kabi amaliy resurslar berishini ta'kidlaydi.

O'zbekistonda eng tez o'sishi mumkin bo'lgan kreativ yo'nalishlar

UNCTADning Creative Economy Outlook 2024 hujjatida O'zbekiston eksport bo'yicha e'tibor qaratayotgan kreativ mahsulot va xizmatlar sifatida quyidagilar sanaladi: **dasturiy ta'minot, madaniy-tarixiy meros va hunarmandchilik, zargarlik, to'qimachilik, kinematografiya**. Bu ro'yxat O'zbekiston uchun tabiiy "prioritet portfel"bo'lib, ularni qiymat zanjiri nuqtayi nazaridan shunday kuchaytirish mumkin:

1) Raqamli kreativ xizmatlar: dizayn-animatsiya-o'yinlar-multimedia

Raqamli yo'nalishning ustunligi — eksportga chiqish tezligi va nisbatan kam kapital talab qilishi. 2025-yilning dastlabki 7 oyida IT Parkka qo'shilgan eksportga yo'naltirilgan yangi kompaniyalar tarkibida game development bilan shug'ullanuvchilar ham borligi (21 ta) qayd etilgan, shuningdek IT Park a'zolari xizmatlarni 90 dan ortiq mamlakatga eksport qilayotgani aytiladi.

2) Hunarmandchilik va madaniy meros: "storytelling"+ dizayn + turizm

Hunarmandchilikni faqat suvenir sifatida emas, balki: dizayn mahsuloti (interyer, dekor, moda aksessuarlari), turizm tajribasi (workshop, "ustaxonaga sayohat festival), premium sovg'a bozoriga mos brendli kolleksiya sifatida qayta qadoqlash zarur. Bunda autentiklik va sifat standarti muhim: sertifikatlash, geografik ko'rsatkichlar, brend himoyasi, qadoqlash, logistika va onlayn savdo (cross-border e-commerce) kabi bo'g'inlar qo'shilsa, bitta buyumning qo'shilgan qiymati bir necha barobar oshadi.

3) Moda va tekstil: "xomashyo"dan "brend"ga

To'qimachilik O'zbekistonda an'anaviy kuchli tarmoq. Kreativ iqtisodiyot nuqtayi nazaridan esa asosiy vazifa — "ishlab chiqarish"dan "brend va dizayn"ga ko'tarilish: dizaynerlar bilan kapsula-kolleksiyalar, mahalliy naqsh va mato identifikatsiyasi, elektron tijorat orqali eksport, xalqaro ko'rgazmalar va "Made in Uzbekistan"brendi ostida targ'ibot.

4) Kino, serial, animatsiya va media

UNCTAD hujjatida O'zbekiston kinematografiyani ham kreativ eksport

prioritetlaridan biri sifatida ko'rsatadi. Bu yo'nalishda iqtisodiy samaradorlik "bitta film"dan ko'ra kengroq: post-prodakshn (montaj, VFX), animatsiya va motion design, musiqiy mahsulot, lokatsiya turizmi (film tourism),intellektual mulkdan licenziya va merchandising daromadlari.

Asosiy muammolar: "g'oya"dan "barqaror biznes"ga o'tishdagi to'siqlar

Kreativ sektorda odatda quyidagi iqtisodiy muammolar uchraydi: norasmiy bandlik, daromadning mavsumiyligi, moliyalashtirishga kirish qiyinligi, intellektual mulk himoyasi va adolatli haq to'lash mexanizmlarining zaifligi.

2025-2030 uchun amaliy "yo'l xaritasi"taklifi

Quyidagi yondashuv O'zbekistonda kreativ iqtisodiyotning tez o'sishini ta'minlaydi:

1) Statistika va "o'lchash"tizimi: nimani rivojlantirayotganimizni aniq bilish

- Kreativ tarmoqlar tasnifini milliy statistika bilan uyg'unlashtirish (subsektorlar kesimida).
- Bandlik, eksport, daromad, startaplar, festival/event iqtisodiyoti kabi KPIlarni muntazam yuritish.

UNCTADning O'zbekiston bo'yicha harakat rejasi ishlab chiqishga qaratilgan loyihasi ham aynan data va capacity buildingga urg'u berishini ko'rsatadi.

2) Moliyalashtirish: grant + inkubatsiya + bozorga chiqish

- Mikrograntlar (prototip, kolleksiya, pilot epizod, demo o'yin).
- Inkubatsiya orqali investor/potensial xaridor bilan bog'lash.
- Kreativ bizneslar uchun kredit kafolatlari va "revenue-based"(daromadga bog'langan) moliyalashtirish mexanizmlarini sinovdan o'tkazish.

3) Creative Park va inkubatorlarni "hududiy klaster"ga aylantirish

Creative Parklar nafaqat bino, balki xizmatlar to'plami bo'lishi kerak: studiyalar, uskunalar, yuridik/IP klinikalari, eksport bo'yicha maslahat, marketplace, foto/video kontent laboratoriyalari. Preferensial soliq rejimi esa rezidentlar uchun "xavfni kamaytiruvchi"omil bo'lib xizmat qiladi.

4) Eksport strategiyasi: "raqamli xizmatlar + madaniy mahsulotlar"

UNCTAD hujjatida O'zbekistonda eksportni rag'batlantirish bo'yicha milliy choralar va 2018-yilda "Made in Uzbekistan"brendi ostida ko'rgazmalar orqali targ'ibot tajribasi tilga olinadi. Bu tajribani kuchaytirish uchun: dizayn/multimedia/game dev bo'yicha "export studio"lar, xalqaro platformalarda (marketplace) savdo ko'nikmalari, turizm bilan bog'langan kreativ mahsulot paketlari (festival + craft + gastro + retail) kerak.

5) Intellektual mulk va shaffof shartnomalar

Kreativ sektorda "adolatli bozor"ning asosi — IP. WIPOning 2024-yil oktabrida Toshkentdagi Kreativ iqtisodiyot konferensiyasida ishtiroki ham intellektual mulk masalasi global kun tartibida ekanini ko'rsatadi. Amaliy choralar: mualliflik huquqi/brand/trademark bo'yicha arzon va tezkor maslahat xizmatlari, standart shartnoma shablonlari (dizayn buyurtmasi, video prodakshn, musiqiy licenziya), platformalarda monetizatsiya va huquqiy savodxonlik.

6) Raqamli infratuzilma: AI va data markazlar — kontent iqtisodiyotining tayanchi

Raqamli kreativ xizmatlar sifatli internet, data markazlar, hisoblash quvvatiga tayanadi. Reuters 2025-yil noyabrida Qoraqalpog'istonda AI va data-markaz loyihalari uchun soliqdan ozod zona yaratish tashabbusi haqida yozgan.

Xulosa

O'zbekistonda kreativ iqtisodiyot "qo'shimcha yo'nalish"emas, balki iqtisodiy diversifikatsiya, yoshlar bandligi, xizmatlar eksporti va milliy brendni kuchaytirishning real instrumentiga aylanmoqda.

Eng katta istiqbol — O'zbekistonning ikki ustunini birlashtirishda: madaniy meros (craft, dizayn, turizm) va raqamli kreativ xizmatlar (dizayn, animatsiya, o'yinlar, multimedia). Agar data, moliyalashtirish, IP va bozorga chiqish mexanizmlari izchil qurilsa, 2025-2030 yillarda kreativ iqtisodiyot O'zbekistonning eng tez o'suvchi, eksportbop va yoshlarni jalb qiluvchi tarmoqlaridan biriga aylanishi mumkin.

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Algebraik-giperbolok natural splayn va D^m splayn

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DOI 10.5281/zenodo.18217663

Ma'lumki, splaynlar nazariyasi rivojlanishi I.J.Schonbergning [1] maqolasida keltirilgan kubik splaynlarning $L_2^{(2)}$ fazosidagi funksiya normasining kvadratiga minimum beruvchi variatsion masala yechimi bilan bog'liqligi ko'rsatuvchi D.Holladay [2] maqolasiga borib taqaladi. Dastlabki sistematik splaynlar nazariyasi J.H.Ahlberg, E.Nilson, J.Walsh lar tomonidan [3] adabiyotda keltirilgan.

Ushbu ishda $L_2^{(m)}(0,1)$ fazosidagi normani minimallashtiruvchi D^m splayn va $K_{2,y}^m$ fazolaridagi normalarni minimallashtiruvchi algebraik-giperbolik natural splaynlarning funksyalarni yaqinlashtirishdagi xatoliklarini $m = 2, 3, 4$ hollarida taqqoslaymiz.

Dastlab, $L_2^{(m)}(0,1)$ va $K_{2,y}^m$ Gilbert fazolari ta'riflari va undagi normalarni minimallashtiruvchi spalyn funksyalarni keltiramiz.

Ushbu $L_2^{(m)}(0,1)$ $-m-1$ tartibgacha hosilalalari uzluksiz va m tartibli hosilasi kvadarati bilan integralanuvchi funksyalar sinfi quyidagi skalayar ko'paytma bilan faktorizatsya qilingan Gilbert fazosini tashkil qiladi.

$$(\varphi,\psi)_{L_2^{(m)}(0,1)} = \int_0^1 \varphi^{(m)}\,\psi^{(m)}\,dx$$

Ushbu fazoda norma skalyar ko'paytma yordamida quyidagicha kiritiladi

$$\|f\|_{L_2^{(m)}(0,1)} = \Big(\int_0^1 (f^{(m)}(x))^2\,dx\Big)^{1/2} \tag{1}$$

Ushbu (1) normaga minimum beruvchi spalynni D^m spalyn deb nomlaymiz. D^m splayn $P_{m-1}(x)$ funkisylarga aniq va quyidagi ko'rinishda aniqlanadi

$$S_m(x) = \sum_{i=0}^{N} C_i G(x-x_i) + \sum_{\alpha=0}^{m-1} p_\alpha x^\alpha$$

Bu yerda, $C_i (i = 0, 1, ..., m-1), p_\alpha (\alpha = 0, 1, ..., m-1)$ lar D^m spalynning koeffissientlari va ular [4] ishda berilgan.

II. Ushbu $K_{2,y}^m(0,1)$ $m-1$ tartibgacha hosilalalari uzluksiz va m tartibli hosilasi kvadarati bilan integralanuvchi funksyalar sinfi quyidagi skalyar ko'paytma bilan fazktorizatsya qilingan Gilbert fazo tashkil qiladi.

$$(\varphi,\psi)_{K_{2,y}^m(0,1)} = \int_0^1 (\varphi^{(m)} - \nu^2\varphi^{(m-2)})(\psi^{(m)} - \nu^2\psi^{(m-2)})\,dx \tag{2}$$

ushbu fazoda norma skalyar ko 'paytma orqali kiritiladi.

$$\|\varphi\|_{K^m_{2,y}(0,1)} = (\int_0^1 (\varphi^{(m)} - \nu^2\varphi^{(m-2)})^2 dx)^{1/2} \tag{3}$$

Ushbu (2) normaga minimum beruvchi spalynni algebraik-giperbolik splayn deb ataymiz. Algebraik-giperbolik splayn $sinh\nu x, cosh\nu x,$

$$P_{m-3}(x)$$

funksyalarga aniq va quyidagi ko'rinishda aniqlanadi.

$$S_{\Delta m}(x) = \sum_{i=0}^{N} C_i G(x - x_i) + d_1 sinh(\nu x) + d_2 cosh(\nu x) + \sum_{\alpha=0}^{m-3} p_\alpha x^\alpha$$

Bu yerda, $C_i (i = 0, 1, ..., N), p_\alpha (\alpha = 0, 1, ..., m-3)$ lar algebraik giperbolik natural spalynning koeffisientalri va ular $m = 2, 3, 4$ hollarida mos ravishda [5,6,7] ishlarda berilgan. 1. $f_1 = tan(x)$ funksyani algebraik-giperbolik natural spalayn va D^m splayn orqali yaqinlashtirgandagi xatoliklar.

$f_1(x) = \tan(x)$							
	Taranglik koeffisienti	N=10			N=50		
		m=2	m=3	m=4	m=2	m=3	m=4
$R_n(f_1)_{K^{(m)}_{2,y}}$	$\nu = 0{,}1$	5,0(-3)	1,0(-3)	3,0(-4)	2,0(-4)	1,0(-5)	9,0(-7)
	$\nu = 1$	4,0(-3)	1,0(-3)	3,0(-4)	1,6(-4)	1,0(-5)	9,0(-7)
	$\nu = 2$	2,0(-3)	8,0(-4)	3,0(-4)	8,0(-5)	8,0(-6)	8,0(-7)
	$\nu = 3$	1.6(-3)	4,0(-4)	2,0(-4)	6,0(-5)	5,0(-6)	7,0(-7)
	$\nu = 4$	7,0(-3)	3,0(-4)	1,0(-4)	2,0(-4)	3,0(-6)	5,0(-7)
	$\nu = 10$	6,0(-2)	6,0(-3)	8,0(-4)	2,0(-3)	6,0(-4)	1,0(-6)
	$\nu = 15$	1,5(-1)	1,6(-2)	2,0(-3)	6,0(-3)	1,5(-4)	5,0(-6)
$R_n(f_1)_{L_2^{(m)}}$		5,0(-2)	1,0(-2)	3,0(-4)	2,0(-4)	1,0(-5)	9,0(-7)

2. $f_2 = e^5 x$ funksyani algebraik-giperbolik natural spalayn va D^m splayn orqali yaqinlashtirgandagi xatoliklar.

$f_2(x)=e^{5x}$							
	Taranglik koeffisienti	N=10			N=50		
		m=2	m=3	m=4	m=2	m=3	m=4
$R_n(f_2)_{K_{2,\nu}^{(m)}}$	$\nu=0,1$	1,6	0,4	1,0(-1)	7,0(-1)	3,0(-3)	2,0(-4)
	$\nu=1$	1,6	0,4	1,0(-1)	6,0(-2)	3,0(-3)	2,0(-4)
	$\nu=2$	1,0	0,3	8,0(-2)	6,0(-2)	3,0(-3)	1,8(-4)
	$\nu=3$	1,0	0,2	6,0(-2)	4,0(-2)	2,0(-3)	1,0(-4)
	$\nu=4$	0,6	0,1	3,0(-2)	2,0(-2)	1,0(-3)	8,0(-5)
	$\nu=5$	1,0(-42)	1,0(-36)	3,0(-32)	7,0(-38)	8,0(-30)	4,0(-22)
	$\nu=6$	7,0(-1)	1,8(-1)	4,0(-2)	3,0(-2)	1,6(-3)	1,0(-4)
	$\nu=10$	5,0	1,0	3,0(-1)	2,0(-1)	1,0(-2)	6,0(-4)
	$\nu=15$	10,0	3,0	7,0(-1)	5,0(-1)	3,0(-2)	1,8(-3)
$R_n(f_2)_{L_2^{(m)}}$		1,6	0,4	0,1	0,7	3,0(-3)	2,0(-4)

Berilgan jadvallarda, $R_n(f)$ bu $[0,1]$ oraliqda berilgan funksyalar bilan D^m ,algebraik-giperbolik splaynlar orasidagi xatoliklaring maximalidir. Yuqoridagi jadvallardan shuni xulosa qilamizki: ma'lum bir oraliqda uzluksiz bo'lgan funksyalarni algebraik-giperbolik natural spalyn bilan yaqinlashtirgandagi absolyut xatoliklarni taranglik koeffisientini o'zgartirish yordamida yaxshilash mumkin. Natijada, funksyalarni algebraik- giperbolik natural splayn bilan yaqinlashtirganda yaqinlashish xatoligi taaranglik koeffissientining ma'lum qiymatlarida, D^m splayn, xususan $m = 3$ bo'lganda kubik splayn, bilan yaqinlashtirgandagi absolyut xatoliklaridan kichkina ekanligini kuzatamiz. Ko'plab amaliy masalalarda kubik splayn orqali yaqinlashtirishdan foydanalinadi, yuqoridagi natijalardan ko'rinadiki, algebraik-giperbolik natural splaynni amaliy masalalarga qo'llash orqali olingan natijalrni yanada yaxshilash mumkin.

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Google Earth Engine asosida suv omborlari monitoringi (Talimarjon suv ombori misolida)

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DOI 10.5281/zenodo.18217674

Annotatsiya. Mazkur maqolada Google Earth Engine (GEE) platformasi asosida suv omborlarining masofadan monitoringini amalga oshirish hamda suv yuzasi va suv hajmi o‘rtasidagi bog‘lanishni baholash masalalari tadqiq etilgan. Tadqiqot obyekti sifatida Talimarjon suv ombori tanlandi. Sentinel-1 SAR va Landsat sun’iy yo‘ldosh ma’lumotlari asosida suvli yuzalar aniqlanib, vaqt qatorlari shakllantirildi. SAR tasvirlaridan foydalanish bulutlilik muammosini bartaraf etish imkonini berdi va suv yuzasini aniqlash aniqligini oshirdi. Suv yuzasining o‘zgarishi va suv hajmi o‘rtasidagi bog‘lanishni aniqlash maqsadida 2023 va 2025 yillarda o‘tkazilgan batimetrik o‘lchash natijalari asosida regressiya modellar tuzildi. Chiziqli va darajali regressiya modellari tahlil qilinib, yuqori aniqlik ko‘rsatkichlariga ($R^2 = 0,98 - 0,99$) erishildi. Olingan bog‘lanish tenglamalari asosida suv hajmi va suv sathi balandligining oylik qiymatlari hisoblandi hamda ularning vaqt bo‘yicha o‘zgarish tendensiyalari tahlil qilindi.
Tadqiqot natijalari Google Earth Engine platformasidan foydalanib suv omborlari holatini muntazam kuzatish, suv resurslarini baholash va boshqarish jarayonlarida amaliy ahamiyatga ega ekanligini ko‘rsatadi.
Kalit so‘zlar: Google Earth Engine, masofadan zondlash, suv ombori, Sentinel-1, SAR, suv yuzasi, suv hajmi, regressiya modeli.

1.Kirish

Suv resurslarini samarali boshqarish va ularning holatini doimiy monitoring qilish global iqlim o‘zgarishi, aholi sonining o‘sishi hamda suvga bo‘lgan talabning ortishi sharoitida dolzarb vazifalardan biri hisoblanadi. Ayniqsa, sun’iy suv omborlarining suv sathi, suvli yuzasi va suv hajmining vaqt bo‘yicha o‘zgarishini aniq baholash gidrotexnik inshootlardan oqilona foydalanish va xavflarni kamaytirishda muhim ahamiyat kasb etadi. So‘nggi yillarda masofadan zondlash texnologiyalari va sun’iy yo‘ldosh ma’lumotlari asosida suv havzalarini monitoring qilish usullari keng qo‘llanilmoqda. Bu yo‘nalishda katta hajmdagi geofazoviy ma’lumotlarni qayta ishlash imkoniyatiga ega bo‘lgan Google Earth Engine (GEE) platformasi alohida ahamiyatga ega. U global sun’iy yo‘ldosh

ma'lumotlar bazalari bilan ishlash, fazoviy-vaqt qatorlarini tahlil qilish hamda hisoblash jarayonlarini yuqori aniqlikda amalga oshirish imkonini beradi.
Suv omborlarini monitoring qilishda asosiy muammolardan biri — bulutlilik ta'siri va ma'lumotlarning uzluksiz emasligidir. Ushbu muammoni hal etishda radar tasvirlariga asoslangan Sentinel-1 sun'iy yo'ldosh ma'lumotlaridan foydalanish samarali hisoblanadi. SAR (Synthetic Aperture Radar) texnologiyasi orqali olingan tasvirlar bulutli va yog'ingarchilik sharoitida ham suv yuzasini aniqlash imkonini beradi.

Mazkur tadqiqotda Google Earth Engine platformasi asosida Talimarjon suv omborining suv yuzasini masofadan turib aniqlash, uning vaqt bo'yicha o'zgarishini tahlil qilish hamda suv yuzasi va suv hajmi o'rtasidagi bog'lanishni regressiya modellari yordamida baholash maqsad qilib olingan. Olingan natijalar suv resurslarini baholash va boshqarish tizimlarini takomillashtirishda amaliy ahamiyatga ega.

2.Masalaning qo'yilishi

Suv havzalarini masofadan monitoring qilish masalalari ko'plab tadqiqotlarda ko'rib chiqilgan. Ilmiy adabiyotlarda sun'iy yo'ldosh tasvirlari asosida suv yuzasini aniqlash uchun spektral indekslar, tasvirlarni tasniflash va porog usullari keng qo'llaniladi. Ayniqsa, Landsat va Sentinel sun'iy yo'ldoshlari ma'lumotlari suv obyektlarini kuzatishda asosiy manbalar hisoblanadi.
Optik diapazonda olingan tasvirlar asosida suv obyektlarini aniqlashda NDWI, MNDWI kabi suv indekslaridan foydalanish keng tarqalgan. Biroq, bunday usullar bulutlilik va yorug'lik sharoitlariga bog'liq bo'lib, ma'lumotlarning uzluksizligini ta'minlashda cheklovlarga ega. Shu sababli, so'nggi yillarda SAR texnologiyalariga asoslangan tadqiqotlar soni ortmoqda.
Sentinel-1 sun'iy yo'ldoshi ma'lumotlari asosida suv yuzasini aniqlashda orqaga tarqalish koeffitsientlaridan foydalanish samarali usul sifatida qayd etilgan. Ilmiy ishlarda suv yuzasi odatda past tarqalish qiymatlari bilan xarakterlanishi va quruqlik fonida aniq ajralib turishi ta'kidlanadi. Bu esa porog qiymatlari orqali suvli piksellarni tasniflash imkonini beradi.
Suv yuzasi va suv hajmi o'rtasidagi bog'lanishni baholashda regressiya modellari keng qo'llaniladi. Ayrim tadqiqotlarda chiziqli modellar yetarli aniqlik bersa, boshqa ishlarda darajali yoki ko'phadli regressiya modellari yuqori natijalarni ko'rsatgan. Bunday modellar, ayniqsa, batimetrik o'lchashlar bilan birgalikda qo'llanilganda, suv ombori hajmini baholashda ishonchli hisoblanadi.
Shu bilan birga, Google Earth Engine platformasidan foydalangan holda suv omborlarini uzluksiz monitoring qilish bo'yicha tadqiqotlar hali yetarli darajada emas. Ayniqsa, mahalliy suv omborlari uchun suv yuzasi, suv hajmi va suv sathi o'rtasidagi bog'lanishni kompleks tahlil qilish masalasi dolzarbligini saqlab qolmoqda. Mazkur tadqiqot ushbu bo'shliqni to'ldirishga qaratilgan.
Mazkur tadqiqot obyekti sifatida O'zbekiston Respublikasi hududida joylashgan Talimarjon suv ombori tanlandi. U mamlakatning yirik gidrotexnik inshootlaridan biri bo'lib, energetika va qishloq xo'jaligini suv bilan ta'minlashda muhim ahamiyatga ega. Suv omboridagi suv sathi va hajmining o'zgarishi mavsumiy va yillik gidrologik omillar, shuningdek, inson faoliyati bilan bog'liq bo'lgan omillar ta'sirida shakllanadi.

Tadqiqotda quyidagi ma'lumotlar manbalaridan foydalanildi:

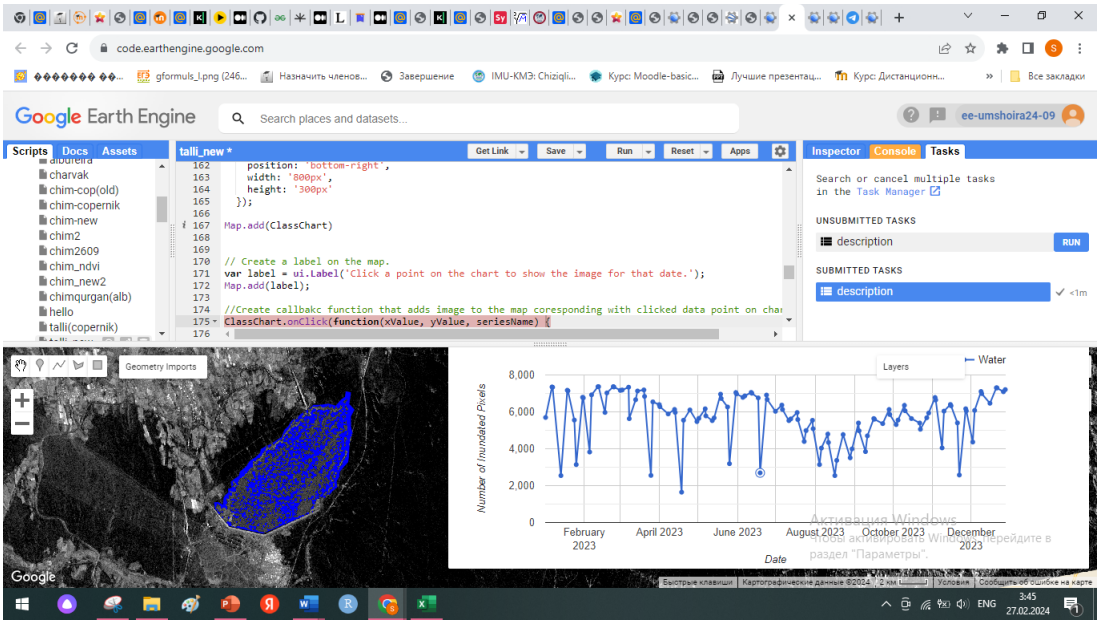
- Sentinel-1 sun'iy yo'ldoshining SAR tasvirlari (2024–2025 yillar),

- Landsat missiyasi doirasida olingan optik tasvirlar,
- suv ombori uchun 2023 va 2025 yillarda olib borilgan batimetrik o‘lchashlar natijalari,
- Global Reservoir and Dam ma’lumotlar bazasidan olingan geofazoviy ma’lumotlar.

Sentinel-1 ma’lumotlari suv yuzasini aniqlashda asosiy manba sifatida qo‘llanildi, chunki SAR tasvirlari bulutlilik va yorug‘lik sharoitlariga bog‘liq emas. Landsat tasvirlari esa qo‘shimcha tekshirish va taqqoslash maqsadida foydalanildi. Batimetrik ma’lumotlar suv yuzasi va suv hajmi o‘rtasidagi bog‘lanishni baholashda etalon ma’lumot sifatida xizmat qildi.

3. Tadqiqot metodologiyasi

Tadqiqot Google Earth Engine platformasining hisoblash imkoniyatlaridan foydalangan holda amalga oshirildi. Metodologiya bir nechta bosqichlarni o‘z ichiga oladi: sun’iy yo‘ldosh tasvirlarini tanlash va filtrlash, suv yuzasini aniqlash, vaqt qatorlarini shakllantirish hamda regressiya modellari orqali suv hajmini baholash.
Birinchi bosqichda Sentinel-1 sun’iy yo‘ldoshining SAR tasvirlari tadqiqot hududi va belgilangan vaqt oralig‘i bo‘yicha filtrlandi. Tasvirlarni qayta ishlash jarayonida tasvir sifatini yaxshilash maqsadida median filtr qo‘llanildi. Bu usul SAR tasvirlariga xos bo‘lgan dog‘lar shovqinini kamaytirish imkonini berdi.
Ikkinchi bosqichda suvli piksellarni aniqlash uchun orqaga tarqalish koeffitsientlari asosida porog usuli qo‘llanildi. Suv yuzasi past tarqalish qiymatlari bilan xarakterlanishi hisobga olinib, belgilangan porog qiymatdan past bo‘lgan piksellar suv sifatida tasniflandi. Har bir tasvir uchun suvli piksellar yig‘indisi hisoblanib, suv yuzasining vaqt bo‘yicha o‘zgarish qatorlari shakllantirildi.



1-rasm. GEE platformasida suv ombori monitoringi

Uchinchi bosqichda suv yuzasi va suv hajmi o‘rtasidagi bog‘lanishni aniqlash uchun regressiya tahlili amalga oshirildi. Bu maqsadda batimetrik o‘lchashlar asosida olingan suv hajmi qiymatlari suv yuzasi bilan bog‘landi. Chiziqli va darajali regressiya modellari tuzilib, ularning aniqlik darajasi determinatsiya koeffitsienti orqali baholandi.

Olingan regressiya tenglamalari asosida suv yuzasining vaqt qatorlari orqali suv hajmi va suv sathi balandligining hisoblangan qiymatlari aniqlandi. Ushbu yondashuv suv omboridagi gidrologik jarayonlarni uzluksiz monitoring qilish imkonini beradi hamda an'anaviy o'lchash usullarini to'ldiruvchi samarali vosita sifatida xizmat qiladi.
Suv ombori yuzasini aniqlash Sentinel-1 sun'iy yo'ldoshining SAR tasvirlari asosida amalga oshirildi. SAR tasvirlarida suv yuzasi boshqa yer qoplamalariga nisbatan past orqaga tarqalish koeffitsientlari bilan tavsiflanadi. Shu sababli suvli hududlarni ajratib olish uchun orqaga tarqalish qiymatlariga asoslangan porog (threshold) usulidan foydalanildi.
Har bir tasvir uchun belgilangan porog qiymatdan past bo'lgan piksellar suv sifatida tasniflandi. Natijada binar suv–quruqlik xaritalari hosil qilindi. Ushbu xaritalar asosida suvli piksellar soni hisoblandi va ularning umumiy maydoni quyidagi ifoda orqali aniqlandi:

$$A_t = N_t \cdot S_p,$$

bu yerda

- A_t — t-vaqtdagi suv yuzasining maydoni (m^2),
- N_t — suv sifatida tasniflangan piksellar soni,
- S_p — bir piksel maydoni (Sentinel-1 uchun 10 000 m^2).

Har bir tasvir uchun hisoblangan suv yuzasi qiymatlari asosida suv omborining vaqt qatorlari shakllantirildi. Vaqt qatorlari suv yuzasining mavsumiy va yillik o'zgarish tendensiyalarini aniqlash imkonini berdi. Natijalarga ko'ra, suv yuzasi qiymatlarida yoz va kuz oylarida kamayish, qish va bahor oylarida esa nisbatan o'sish tendensiyasi kuzatildi. Bu holat gidrologik rejimi va suvdan foydalanish xususiyatlari bilan izohlanadi.
Suv ombori hajmini baholash uchun suv yuzasi va suv hajmi o'rtasidagi funksional bog'lanish regressiya tahlili yordamida aniqlandi. Bu maqsadda 2023 va 2025 yillarda o'tkazilgan batimetrik o'lchashlar natijalari asosida suv hajmi (V) va suv yuzasi (A) o'rtasidagi bog'lanish tahlil qilindi.

Dastlab chiziqli regressiya modeli ko'rib chiqildi:

$$V = \alpha A + \beta, \tag{19}$$

bu yerda

- V — suv hajmi (m^3),
- A — suv yuzasi maydoni (m^2),
- α, β — regressiya koeffitsientlari.

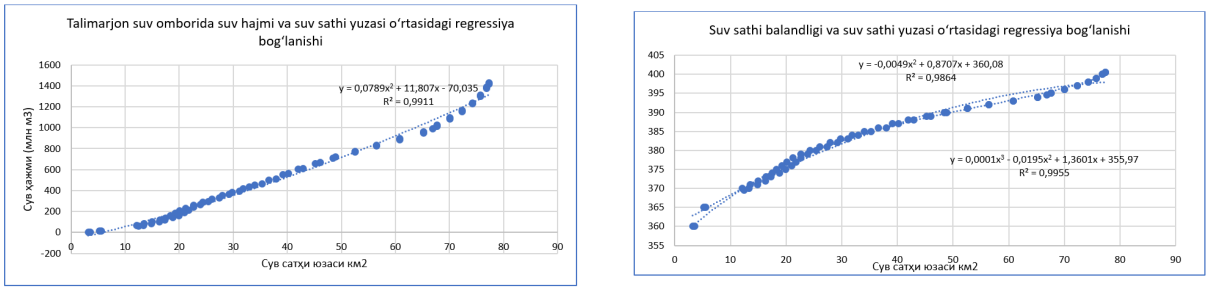
Natijalar chiziqli model suv hajmini yetarlicha aniqlikda ifodalashini ko'rsatdi va determinatsiya koeffitsienti $R^2 = 0{,}98$ ga teng bo'ldi. Biroq, suv ombori geometriyasining murakkabligi sababli darajali regressiya modeli yanada yuqori aniqlikni ta'minladi:

$$V = \gamma A^{\delta}, \tag{20}$$

bu yerda

- γ, δ — empirik regressiya koeffitsientlari.

Darajali regressiya modeli uchun determinatsiya koeffitsienti $R^2 = 0{,}99$ ni tashkil etdi. Bu esa suv hajmining o‘zgarishini suv yuzasi orqali 99 % aniqlik bilan izohlash mumkinligini ko‘rsatadi.



2-rasm.Suv ombori ko’rsatkichlari uchun regressiya modellari

Shuningdek, suv yuzasi va suv sathi balandligi (H) o‘rtasidagi bog‘lanish ham tahlil qilindi. Ikkinchi va uchinchi tartibli regressiya modellari sinovdan o‘tkazilib, eng yuqori aniqlik uchinchi tartibli modelda kuzatildi:

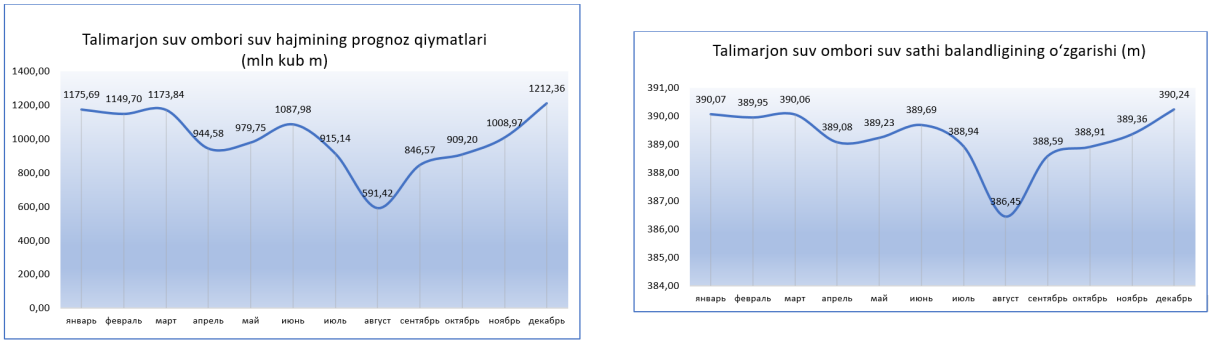
$$H = aA^3 + bA^2 + cA + d, \tag{21}$$

bu yerda

- H — suv sathi balandligi (m),
- a, b, c, d — regressiya koeffitsientlari.

Mazkur model uchun determinatsiya koeffitsienti $R^2 = 0{,}9955$ ga teng bo‘lib, suv sathi balandligini suv yuzasi orqali ishonchli baholash imkonini berdi.

Olingan regressiya tenglamalari asosida GEE platformasidan olingan suv yuzasi vaqt qatorlari yordamida 2025 yil uchun suv hajmi va suv sathi balandligining oylik qiymatlari hisoblandi. Hisoblangan natijalar suv omborining mavsumiy ish rejimini va suv resurslarining taqsimlanish xususiyatlarini chuqurroq tahlil qilish imkonini berdi.



3-rasm. Suv ombori ko’rsatkichlarining prognoz qiymatlari

4. Natijalar va muhokama

Google Earth Engine платформаси асосида олинган натижалар Talimarjon suv omborida suv yuzasining vaqt bo‘yicha sezilarli o‘zgarishlarga ega ekanligini ko‘rsatdi. Sentinel-1 SAR tasvirlari yordamida shakllantirilgan vaqt qatorlari suv yuzasining mavsumiy dinamikasini aniqlash imkonini berdi. Kuzatuvlarga ko‘ra, bahor va qish oylarida suv yuzasi maydonining nisbatan ortishi, yoz va kuz oylarida esa kamayishi qayd etildi. Bu holat daryo oqimi, bug‘lanish darajasi hamda suvdan xo‘jalik ehtiyojlari uchun foydalanish bilan izohlanadi.

Suv yuzasi va suv hajmi o‘rtasidagi regressiya modellari tahlili shuni ko‘rsatdiki, darajali regressiya modeli chiziqli modelga nisbatan yuqoriroq aniqlikka ega. Determinatsiya koeffitsientlarining yuqori qiymatlari ($R^2 = 0,98 - 0,99$) suv hajmining asosiy qismi suv yuzasi orqali izohlanishini tasdiqlaydi. Bu natijalar masofadan zondlash ma’lumotlari yordamida suv hajmini bilvosita baholash imkoniyati mavjudлигини кўрсатади.

Shuningdek, suv yuzasi va suv sathi balandligi o‘rtasidagi bog‘lanish uchun tuzilgan regressiya modellari ham yuqori aniqlikni namoyon etdi. Uchunchi tartibli regressiya modeli suv sathi balandligini ishonchli baholash imkonini berdi ($R^2 = 0,9955$). Olingan natijalar suv omborining ish rejimini tahlil qilish va suv resurslarini boshqarish jarayonlarida muhim axborot manbai bo‘lishi mumkin.

Natijalarni an’anaviy batimetrik o‘lchashlar bilan taqqoslash shuni ko‘rsatdiki, Google Earth Engine asosida olingan baholashlar amaliy jihatdan maqbul aniqlikka ega. Bu esa vaqt va moliyaviy xarajatlarni kamaytirgan holda suv omborlari holatini doimiy kuzatish imkonini yaratadi.

5. Xulosa

Mazkur tadqiqotda Google Earth Engine platformasi asosida suv omborlarini masofadan monitoring qilish va suv hajmini baholash usullari ishlab chiqildi. Sentinel-1 SAR va Landsat sun’iy yo‘ldosh ma’lumotlari yordamida Talimarjon suv omborining suv yuzasi aniqlanib, uning vaqt bo‘yicha o‘zgarishlari tahlil qilindi.

Batilmetrik o‘lchashlar asosida suv yuzasi, suv hajmi va suv sathi balandligi o‘rtasidagi bog‘lanishlar regressiya modellari orqali aniqlanib, yuqori aniqlik ko‘rsatkichlariga erishildi. Olingan natijalar masofadan zondlash ma’lumotlari yordamida suv omborlarining hajmiy ko‘rsatkichlarini baholash mumkinligini tasdiqlaydi.

Tadqiqot natijalari suv resurslarini boshqarish, gidrotexnik inshootlar faoliyatini rejalashtirish va suv tanqisligi sharoitida qarorlar qabul qilish jarayonlarida amaliy ahamiyatga ega. Taklif etilgan yondashuv an’anaviy o‘lchash usullarini to‘ldiruvchi samarali vosita bo‘lib, kelgusida boshqa suv omborlari uchun ham qo‘llanilishi mumkin.

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Eksport salohiyatini raqamlashtirish innovatsion iqtisodiy o'sishning drayveri sifatida: xalqaro tajriba va O'zbekiston

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DOI 10.5281/zenodo.18217683

Keywords: Export potential, digitization, innovative economic growth, digital economy, international trade, artificial intelligence (AI), Big Data, digital export platforms, global value chain, economy of Uzbekistan.

Kirish.

Bugungi kunda jahon iqtisodiyoti keskin raqobat sharoitida kechayotgan "global raqamli poyga"bosqichiga qadam qo'ydi. An'anaviy xomashyo eksportiga tayanuvchi iqtisodiy modellar o'zining uzoq muddatli barqarorligini yo'qotib bormoqda. Jahon banki prognozlariga ko'ra, 2025-yilga kelib jahon savdosining qariyb 25-30 foizi raqamli platformalar orqali amalga oshiriladi, bu esa eksportni tashkil etishning mutlaqo yangi yondashuvlarini talab etadi [1, 18]. Bunday sharoitda eksport salohiyatini raqamlashtirish davlatlar uchun shunchaki tanlov emas, balki global bozorda raqobatbardoshlikni saqlab qolish va iqtisodiy "tirik qolish"ning muhim sharti hisoblanadi.

Eksportni raqamlashtirish jarayoni faqatgina bojxona va davlat tartib-taomillarini avtomatlashtirish bilan cheklanib qolmaydi. Aksincha, ushbu jarayon eksport qiluvchi korxonalar va firmalarning ichki biznes jarayonlarini raqamli transformatsiya qilish, marketing, savdo, logistika va mijozlar bilan ishlash tizimlarini zamonaviy texnologiyalar asosida qayta tashkil etishni ham o'z ichiga oladi. Bugungi kunda Big Data tahlili, sun'iy intellekt va raqamli savdo platformalari eksportyorlarga masofadan turib yangi bozorlarni aniqlash, iste'molchi xulq-atvorini prognoz qilish va mahsulotni talabga moslashtirish imkonini bermoqda. Bu esa eksportning samaradorligi va qo'shilgan qiymatini sezilarli darajada oshiradi [2, 42].

Eksport faoliyatini raqamlashtirishning dolzarbligi, shuningdek, logistika va ma'muriy xarajatlarni keskin kamaytirish zarurati bilan ham bog'liq. Xalqaro tajriba shuni ko'rsatadiki, "aqlli"bojxona tizimlari, raqamli sertifikatlash, elektron hujjat aylanishi va blokcheyn asosidagi ta'minot zanjirlari tovarlarning chegaradan o'tish vaqtini 40 foizgacha qisqartiradi. Bu, ayniqsa, dengiz yo'llariga chiqish imkoniyati cheklangan (land-locked) davlatlar uchun transport xarajatlarini kamaytirish va eksport narxlarining raqobatbardoshligini ta'minlashda hal qiluvchi ahamiyat kasb etadi.

XXI-asrda eksport salohiyati endilikda ishlab chiqarish quvvatlari yoki zavodlar soni bilan emas, balki ma'lumotlarni tezkor qayta ishlash, ularni raqamli mahsulot va xizmatlarga aylantirish qobiliyati bilan o'lchanmoqda. Dasturiy ta'minot, biznes-jarayonlar autsorsingi (BPO), onlayn konsalting, dizayn va kreativ industriya kabi yo'nalishlar jismoniy chegaralarni tan olmaydi va global bozorga bevosita chiqish

imkonini yaratadi [2, 42]. Shu nuqtayi nazardan, O'zbekiston uchun eksport tarkibini diversifikatsiya qilish, ya'ni paxta va metall kabi an'anaviy mahsulotlardan yuqori qo'shilgan qiymatli raqamli mahsulot va xizmatlar eksportiga o'tish innovatsion iqtisodiy sakrashning muhim omili hisoblanadi.
Shu o'rinda ta'kidlash joizki, eksport salohiyatini raqamlashtirish borasida erishilgan eng muvaffaqiyatli xalqaro tajribalardan biri Singapur misolida namoyon bo'ladi. Mazkur mamlakat qisqa vaqt ichida davlat institutlari, xususiy sektor va raqamli texnologiyalar o'rtasidagi samarali integratsiya orqali global savdo tizimida yetakchi o'rinlarni egallashga muvaffaq bo'lgan. Singapur tajribasi eksportni raqamlashtirishning nafaqat bojxona va logistika bosqichlarida, balki korxona va firmalarning ichki biznes jarayonlarida ham chuqur transformatsiyani talab etishini yaqqol ko'rsatadi.
Shuningdek, Singapur va O'zbekistonni taqqoslash imkonini beruvchi muhim jihatlardan biri - har ikki mamlakatda eksportni raqamlashtirish davlat siyosatining ustuvor yo'nalishi sifatida e'tirof etilganidir. Biroq, Singapurda ushbu jarayon asosan biznes tashabbuslari va innovatsion korxonalarning faolligi bilan qo'llab-quvvatlangan bo'lsa, O'zbekistonda raqamlashtirish jarayoni ko'proq institutsional islohotlar bosqichida amalga oshirilmoqda. Bu farq eksportni raqamlashtirishda xususiy sektor rolini chuqurroq o'rganishni taqozo etadi.
O'zbekistonda 2030-yilgacha eksport hajmini 45 milliard AQSh dollariga yetkazish strategik maqsad qilib belgilangan bo'lib, ushbu natijaga an'anaviy usullar orqali erishish deyarli imkonsizdir. Shu sababli, eksport jarayonlarini "yagona darcha"tamoyili asosida raqamlashtirish bilan bir qatorda, korxonalar va firmalarning elektron tijorat (E-commerce) platformalariga integratsiyasini kuchaytirish, IT-xizmatlar va raqamli xizmatlar eksportini rag'batlantirish bugungi kunning eng dolzarb vazifalaridan biri hisoblanadi. Mazkur tadqiqot aynan eksport salohiyatini kompleks raqamlashtirish orqali innovatsion iqtisodiy o'sishni ta'minlash mexanizmlarini ochib berishga qaratilgani bilan ilmiy va amaliy jihatdan dolzarbdir.

Adabiyotlar tahlili.

Indermit Gill (Jahon banki) ta'kidlaganidek, raqamli texnologiyalarni eksport jarayonlariga integratsiya qilgan mamlakatlar global qiymat zanjirlarida tezroq va barqarorroq rivojlanadi. Brynjolfsson va Makafi esa iqtisodiy ustunlik ma'lumotlarni qayta ishlash va ularni iqtisodiy qiymatga aylantirish qobiliyati bilan belgilanadi. UNCTAD va JST tadqiqotlari raqamli bojxona, "yagona darcha"va platformalar eksport jarayonlarini soddalashtirib, tranzaksiya xarajatlarini kamaytirishini ko'rsatadi. Singapur tajribasi TradeNet va NTP tizimlari orqali davlat va xususiy sektor o'rtasida samarali raqamli hamkorlikni ta'minlagan. O'zbekiston kontekstida esa ilmiy manbalar raqamli infratuzilma, elektron tijorat va IT-xizmatlar eksportini rivojlantirish mamlakatning innovatsion iqtisodiy o'sishiga asos bo'lishini ta'kidlaydi. Shu bois, eksport salohiyatini raqamlashtirishni Singapur tajribasi bilan qiyosiy tahlil qilish ilmiy bo'shliqni to'ldiradi.

Tadqiqot metodologiyasi.

Mazkur tadqiqot eksport salohiyatini raqamlashtirishning innovatsion iqtisodiy o'sishga ta'sirini aniqlashga qaratilgan bo'lib, tadqiqot jarayonida kompleks va tizimli yondashuv qo'llanildi. Tadqiqot metodologiyasi xalqaro tajriba va milliy amaliyotni uyg'unlashtirgan holda, nazariy va amaliy tahlilni o'z ichiga oladi.
Tadqiqot davomida, avvalo, eksportni raqamlashtirish, raqamli iqtisodiyot va innovatsion

o'sish tushunchalarining nazariy asoslari o'rganildi. Ushbu bosqichda ilmiy adabiyotlar, xalqaro tashkilotlar (Jahon banki, JST, UNCTAD) hisobotlari hamda yetakchi iqtisodchi olimlarning qarashlari tahlil qilindi. Bu yondashuv tadqiqot muammosini chuqurroq anglash va ilmiy asoslangan gipotezalarni shakllantirish imkonini berdi.
Tadqiqotda qiyosiy tahlil (comparative analysis) usuli muhim o'rin egalladi. Ushbu usul orqali Singapur va O'zbekistonning eksportni raqamlashtirish tajribalari taqqoslanib, ularning institutsional mexanizmlari, davlat siyosati va xususiy sektor ishtiroki o'rtasidagi farqlar va o'xshashliklar aniqlab berildi. Qiyosiy tahlil Singapurda muvaffaqiyatli joriy etilgan raqamli eksport modellarini O'zbekiston sharoitiga moslashtirish imkoniyatlarini baholashga xizmat qildi.
Shuningdek, tadqiqotda statistik tahlil usuli qo'llanilib, eksport hajmi, eksport tarkibi, raqamli savdo ulushi va logistika samaradorligi kabi ko'rsatkichlar dinamikasi o'rganildi. Statistik ma'lumotlar asosida eksportni raqamlashtirishning iqtisodiy samaradorlikka ta'siri tahlil qilinib, umumiy tendensiyalar aniqlashtirildi.
Tadqiqot jarayonida institutsional tahlil usulidan ham foydalanildi. Ushbu usul orqali eksportni raqamlashtirish jarayonida ishtirok etuvchi davlat organlari, bojxona tizimi, raqamli platformalar va xususiy sektor subyektlari o'rtasidagi o'zaro aloqalar tahlil qilindi. Bu yondashuv eksport jarayonlarida mavjud institutsional muammolarni aniqlash va ularni bartaraf etish bo'yicha amaliy takliflar ishlab chiqishga imkon berdi.
Bundan tashqari, mantiqiy va strukturaviy tahlil usullari asosida eksport salohiyatini raqamlashtirishning innovatsion iqtisodiy o'sishga ta'sir mexanizmi bosqichma-bosqich ochib berildi. Ushbu usullar yordamida raqamli texnologiyalar, korxonalar faoliyati va makroiqtisodiy natijalar o'rtasidagi sabab-oqibat bog'liqligi aniqlandi.
Tadqiqot natijalarini umumlashtirishda induksiya va deduksiya usullaridan foydalanilib, xalqaro tajribadan kelib chiqib umumiy xulosalar chiqarildi hamda ular O'zbekiston sharoitiga tatbiq etildi. Natijada, eksport salohiyatini raqamlashtirish orqali innovatsion iqtisodiy o'sishni ta'minlashga qaratilgan ilmiy asoslangan xulosalar va amaliy tavsiyalar shakllantirildi.
So'nggi yillarda iqtisodiyotning raqamli transformatsiyasi global miqyosda muhim paradigma darajasiga ko'tarildi. Raqamli iqtisodiyot atamasi an'anaviy ishlab chiqarish va savdo mexanizmlaridan farqli o'laroq, axborot texnologiyalari, tarmoq infratuzilmasi va elektron xizmatlar orqali yaratilayotgan iqtisodiy qiymatni ifodalaydi. Brynjolfsson va boshqalar raqamli iqtisodiyotda ma'lumotlar, algoritmlar va avtomatlashtirish ishlab chiqarish samaradorligini tubdan oshirib, iqtisodiy o'sishning yangi manbalarini yaratishini ta'kidlaydi[2.42]. Eksportni raqamlashtirish eksport jarayonining barcha bosqichlarini - buyurtma olish, bojxona protseduralari, logistika, to'lov tizimlari va mijozlar bilan aloqani - raqamli vositalar orqali tashkil etish demakdir. Bu jarayon raqamli platformalar, elektron tijorat (e-commerce), ma'lumotlarni tahlil qilish (Big Data), sun'iy intellekt (AI) kabi texnologiyalarga tayanadi. Jahon banki hisob-kitoblariga ko'ra, raqamli savdo infratuzilmasi eksport operatsiyalarini sezilarli darajada tezlashtiradi, tranzaksiya xarajatlarini qisqartiradi va eksportyorlar uchun yangi bozorlarni ochadi[3.27].
Jahon banki (World Bank) raqamli transformatsiya iqtisodiy o'sish uchun strategik omil ekanini ta'kidlaydi. Ularning hisobotida eksport jarayonlarini raqamlashtirmaslik global qiymat zanjirlaridan chetda qolish xavfini kuchaytirishi mumkinligi qayd etilgan. Bu strukturaviy o'zgarishlar mamlakatlar iqtisodiyotini texnologik raqobatga moslashtirishni talab qiladi.
Jahon Savdo Tashkiloti (JST/WTO) ham raqamli savdoning o'sishini global savdo dinamikasining muhim elementi sifatida ko'radi. JST hisobotida elektron tijoratning

global savdo hajmidagi o'sish sur'ati sezilarli ekanligi va davlatlar raqamli mexanizmlarni joriy etishni strategik ustuvor vazifa sifatida belgilashlari kerakligi ta'kidlanadi.
UNCTAD (Birlashgan Millatlar Tashkiloti Savdo va Taraqqiyot konfederatsiyasi) raqamli iqtisodiyot va eksportni raqamlashtirish bo'yicha integratsiyalashgan yondashuvni qo'llab-quvvatlaydi. Tashkilot hisob-kitoblariga ko'ra, raqamli savdo jarayonlari ishlab chiqarish tarmoqlari bilan birlashtirilganda mamlakatlarning eksport potensialini yanada mustahkamlaydi va ichki bozorni raqamli xizmatlar bilan boyitadi.
Eksportni raqamlashtirish va raqamli iqtisodiyot o'rtasidagi aloqa innovatsion o'sish kontseptsiyasida mujassamlangan. Raqamli iqtisodiyot eksport jarayonlarining samaradorligini oshiradi, bu esa eksport salohiyatining kengayishiga olib keladi. Natijada:
- Korxonalar darajasida: raqamli vositalar mahsulotni global bozorlarga tezroq va samaraliroq eksport qilish imkonini beradi, mijoz talabini o'rganish va unga moslashishni soddalashtiradi.
- Milliy iqtisodiyot darajasida: raqamli eksport infratuzilmasi nafaqat eksport hajmini oshiradi, balki innovatsion iqtisodiy o'sishni rag'batlantiradi.
Singapur - butun dunyo bo'yicha eng yuqori savdo-iqtisodiy ko'rsatkichlarga ega davlatlardan biri bo'lib, uning tashqi savdo hajmi rasmiy statistikaga ko'ra juda katta. Eksportning yuqori ko'rsatkichlari Singapurning re-eksport salohiyati, raqamli savdo tizimlari va logistika samaradorligi bilan bevosita bog'liq bo'lib, bu ko'rsatkichlar mamlakat iqtisodiyotining umumiy eksport hajmiga ijobiy ta'sir ko'rsatdi.



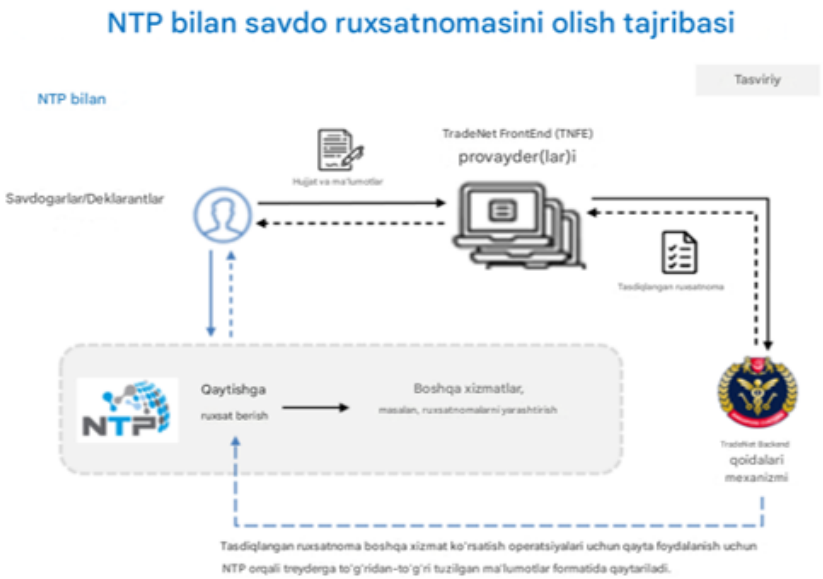
1-rasm. Global raqamli xizmatlar savdosining o'sish trendi
Manba: Reuters (Thomson Reuters)
https://www.reuters.com/markets/eu-singapore-agree-digital-trade-deal-2024-07-25/utm-source

Ushbu diagramma global miqyosda raqamli xizmatlar savdosining qanchalik tez sur'atlarda rivojlanganini yaqqol ko'rsatib beradi. Grafikda 2005-yil bazaviy nuqta (100 indeks) sifatida olingan bo'lib, undan keyingi yillarda raqamli xizmatlar savdosi barqaror va uzluksiz o'sib borganini ko'rish mumkin. Ayniqsa, 2015-yildan keyin o'sish chizig'ining keskin yuqorilashi raqamli texnologiyalar, internet xizmatlari, moliyaviy va IT-xizmatlarga bo'lgan global talabning keskin ortganini anglatadi.
2022-yilga kelib, raqamli xizmatlar savdosi hajmi 2005-yilga nisbatan bir necha barobar

oshgan, bu esa uning tovarlar va an'anaviy xizmatlar savdosiga qaraganda tezroq rivojlanayotgan segment ekanini tasdiqlaydi. Aynan shu global tendensiya Yevropa Ittifoqi va Singapur o'rtasidagi xizmatlar savdosida ham namoyon bo'lib, 2022-yilda 43 yevro milliardlik savdoning yarmidan ko'pi raqamli xizmatlar hissasiga to'g'ri kelgan.Singapurda raqamli savdo tizimlari eksport jarayonini soddalashtirgan va bu orqali eksport hajmini oshirgan. Bojxona protseduralarining raqamlashtirilishi, ma'lumot almashinuvi va hujjatlarni elektron tarzda topshirish imkoniyatlari korxonalarni global bozorga tezroq integratsiya qilishga xizmat qiladi.

Savdo va logistika jarayonlaridagi tranzaksiya xarajatlari tobora raqamlashtirilayotgan bozorlar uchun muhim indikator hisoblanadi. Raqamli tizimlar TradeNet va Networked Trade Platform (NTP) kabi platformalar orqali hujjat topshirish, bojxona rasmiylashtiruvi va boshqa eksport jarayonlarining vaqtini va xarajatlarini sezilarli darajada qisqartirdi. Xalqaro ekspertlar ta'kidlashicha, mamlakatlarda tashqi savdoni raqamlashtirish jarayoni orqali tranzaksiya xarajatlarini sezilarli darajada kamaytirish mumkin, bu esa savdoni yo'lga qo'yish xarajatini pasaytiradi va korxonalar uchun raqobatbardoshlikni oshiradi [4,36]

TradeNet - bu Singapur tashqi savdo deklaratsiyalari uchun yaratilgan birinchi va eng keng qamrovli elektron "Yagona darcha"(National Single Window) tizimi bo'lib, u 1989-yilda joriy etilgan. Bu platforma orqali importyorlar, eksportyorlar va logistika xizmat ko'rsatuvchi subyektlar barcha savdo hujjatlarini yagona elektron interfeys orqali topshiradilar, ko'p agentliklarga alohida murojaat qilish shart emas. TradeNet export-import jarayonlarini birlashtirib, hujjat topshirish vaqtini qisqartiradi va rasmiylashtirish xarajatlarini sezilarli darajada kamaytiradi. Bu tizim bojxona va boshqa regulyatorlar tomonidan tovar harakati monitoringi va xavfsizlik nazoratini soddalashtirishga xizmat qiladi. Natijada, eskicha qog'ozli hujjatlar orqali amalga oshiriladigan murakkab jarayonlar o'rnini samarali raqamli protseduralar egallaydi, bu esa savdo operatsiyalarini tejamkor va tezkor qiladi.



2-rasm. NTP orqaali savdo ruxsotnomasini olish tartibi
Manba: Singapur hukumati agentligi. https://www.customs.gov.sg/

Networked Trade Platform (NTP) esa TradeNet tizimidan keyingi bosqich bo'lib,

2018–2019–yillarda joriy etilgan innovatsion raqamli ekotizimdir. NTP - bu nafaqat davlat xizmatlarini yagona platformada birlashtiruvchi tizim, balki biznes, logistika provayderlari, banklar va xalqaro hamkorlarni o'zaro bog'lovchi savdo va logistika ma'lumotlar bazasi bo'lib xizmat qiladi. NTP orqali foydalanuvchilar bojxona deklaratsiyalarini topshirishdan tashqari, hujjatlarni boshqa tizimlarga qayta yuborish, eksport statistikasi, bojxona ruxsatnomalari, logistika tafsilotlari va hatto xalqaro regulyatorlar bilan o'zaro ma'lumot almashish imkoniyatiga ega bo'lishadi.

NTPning eng muhim afzalliklaridan biri - raqamli ma'lumotlar almashinuvi va qayta foydalanish imkoniyati bo'lib, bu orqali korxonalar eksport jarayonlarini avtomatlashtirilgan, xavfsiz va tezkor ko'rinishda amalga oshiradi. Shu bilan birga, NTP ochiq arxitekturaga ega bo'lib, IT ishlab chiquvchilar va uchinchi tomonlar uchun innovatsion xizmatlar yaratishga platforma taqdim etadi, bu esa Singapur korxonalarining raqamli xizmatlar eksportini ham kengaytirishga yordam beradi.
Singapurda raqamli logistika jarayonlari - bu eksport jarayonining boshqa muhim qismi bo'lib, u ma'lumotlar uzatish, konteynerlar harakati kuzatuvi, bojxona rasmiylashtiruvi va transport ta'minotining barcha pog'onalarini inklyuziv tarzda qamrab oladi. NTP bu jarayonlarni birlashtirgan savdo-logistika axborot ekotizimi bo'lib, eksportchilarga to'liq avtomatlashtirilgan va shaffof bosqichlardan o'tish imkonini beradi. Bu o'z navbatida logistika xarajatlarini kamaytiradi, tovarlarni eksport qilish vaqtini qisqartiradi va xalqaro ta'minot zanjirida samarali raqobatni ta'minlaydi.
Raqamli eksport tizimining joriy etilishi Singapurda biznes muhitining yaxshilanishiga ham sezilarli hissa qo'shdi. Raqamli iqtisodiyot bo'yicha hisobotlar shuni ko'rsatadiki, Singapurda raqamli iqtisodiyot umumiy ichki mahsulotning 17,7 foizidan ortig'ini tashkil etadi, bu esa davlat infratuzilmasining eksport va biznes faoliyatini qo'llab-quvvatlashdagi rolini kuchaytiradi. Raqamli tizimlar korxonalarga nafaqat hujjat topshirish jarayonlarini soddalashtirishga, balki ta'minot zanjiri, xarajatlarni boshqarish, bozorda raqobatlashish kabi ko'plab biznes jarayonlarini yanada samarali olib borishga imkon beradi.
Bundan tashqari, Singapur raqamli iqtisodiyot investitsiyasini jalb etish va innovatsion bizneslarni qo'llab-quvvatlash orqali biznes muhitini yanada mustahkamladi. Raqamli transformatsiyani rag'batlantiruvchi siyosatlar, korxonalar uchun raqamli xizmatlar va global savdo tizimlariga kirish imkoniyatlari biznes uchun qulay shartlar yaratdi, bu esa iqtisodiy faollikni oshirishga yordam berdi.

Muhokama.

Singapur va O'zbekiston eksport tizimlarini qiyosiy tahlil qilish shuni ko'rsatadiki, ikki davlat o'rtasidagi asosiy farq raqamlashtirishning darajasi emas, balki uning tizimliligi va muvofiqlashtirilganligida namoyon bo'ladi. Singapurda eksportni raqamlashtirish uzoq yillik strategik siyosat natijasida davlat boshchiligida, biroq xususiy sektor bilan uzviy hamkorlikda amalga oshirilgan. TradeNet va Networked Trade Platform (NTP) kabi tizimlar bojxona, logistika, bank, sug'urta va eksportyorlarni yagona raqamli muhitda birlashtirgan.
O'zbekistonda esa eksportni raqamlashtirish jarayoni asosan fragmentar va sektorlar kesimida olib borilmoqda. Ayrim jarayonlar (bojxona deklaratsiyasi, sertifikatlash) raqamlashtirilgan bo'lsa-da, ular o'zaro to'liq integratsiyalashmagan. Natijada eksportyorlar uchun jarayonlar murakkabligicha qolmoqda va tranzaksiya xarajatlari yuqori bo'lib qolmoqda.

Eksportni raqamlashtirish innovatsion iqtisodiy o'sishga bevosita va bilvosita mexanizmlar orqali ta'sir ko'rsatadi. Bevosita mexanizm sifatida eksport jarayonlarining tezlashuvi, xarajatlarning kamayishi va bozorlar bilan bog'lanishning soddalashuvi keltiriladi. Raqamli platformalar eksportyorlarga yangi bozorlarga tez kirish, talabni Big Data asosida tahlil qilish va mahsulotni moslashtirish imkonini beradi. Bilvosita mexanizm esa raqamli eksport infratuzilmasi orqali innovatsion muhit shakllanishi bilan bog'liq. Raqamlashtirilgan savdo muhitida IT-xizmatlar, elektron tijorat, raqamli logistika va kreativ industriya kabi yuqori qo'shilgan qiymatli sohalar rivojlanadi. Bu esa iqtisodiyotda innovatsion multiplikativ ta'sirni yuzaga keltirib, umumiy iqtisodiy o'sishni jadallashtiradi. Singapur tajribasi shuni ko'rsatadiki, eksportni raqamlashtirish oddiy samaradorlik vositasi emas, balki innovatsion o'sishning institutsional drayveri hisoblanadi.

Singapur tajribasini O'zbekistonga to'liq ko'chirish amaliy jihatdan mumkin emas, biroq uni moslashtirish va bosqichma-bosqich joriy etish real va maqsadga muvofiqdir. Avvalo, O'zbekistonning geografik joylashuvi, iqtisodiy tuzilmasi va eksport tarkibi inobatga olinishi zarur. Dengizga chiqish imkoniyati bo'lmagan mamlakat sifatida O'zbekiston uchun raqamli logistika, elektron hujjatlashtirish va oldindan deklaratsiya tizimlari alohida ahamiyat kasb etadi. Singapur modelining eng muhim jihati - "yagona darcha"tamoyilining to'liq ishlashi va davlat organlari o'rtasida real vaqt rejimidagi ma'lumot almashinuvidir. Ushbu elementlar O'zbekistonda mavjud islohotlar bilan uyg'unlashtirilsa, eksport jarayonlarining shaffofligi va tezligi sezilarli darajada oshadi.

O'zbekistonda eksportga oid barcha jarayonlarni qamrab oluvchi yagona raqamli eksport platformasini shakllantirish zarur. Ushbu platforma bojxona, soliq, sertifikatlash, logistika va bank xizmatlarini yagona ekotizimga birlashtirishi lozim. Bu eksportyorlar uchun ma'muriy yukni kamaytirib, eksport jarayonlarini tezlashtiradi. Sun'iy intellekt va Big Data texnologiyalaridan foydalanish eksport bozorlarini prognozlash, talabni tahlil qilish va risklarni boshqarishda muhim ahamiyatga ega. AI asosidagi tizimlar eksportyorlarga qaysi bozorda qaysi mahsulotga talab yuqori ekanini aniqlashga yordam beradi va davlat siyosatini ma'lumotlarga asoslangan tarzda olib borish imkonini yaratadi. Eksportni raqamlashtirish jarayonida davlatning roli strategik bo'lsa-da, texnologik innovatsiyalar asosan xususiy sektor tomonidan yaratiladi. Shu sababli, davlat-xususiy sektor hamkorligi (PPP) mexanizmlarini kengaytirish, startaplar va IT-kompaniyalarni eksport infratuzilmasini rivojlantirishga jalb qilish zarur.

Mazkur tadqiqot natijalari shuni ko'rsatadiki, eksportni raqamlashtirish O'zbekiston iqtisodiy siyosatining alohida yo'nalishi sifatida qaralishi lozim. Bu jarayon faqat texnik modernizatsiya emas, balki institutsional islohotlar, kadrlar tayyorlash va innovatsion ekotizim yaratish bilan chambarchas bog'liq. Tadqiqot xulosalari davlat eksport strategiyasini ishlab chiqishda, raqamli savdo siyosatini shakllantirishda va innovatsion iqtisodiy o'sishni jadallashtirishda amaliy ahamiyatga ega bo'lib, O'zbekistonning global qiymat zanjirlaridagi o'rnini mustahkamlashga xizmat qiladi.

Xulosa.

Mazkur tadqiqot natijalari shuni ko'rsatdiki, eksport salohiyatini raqamlashtirish zamonaviy sharoitda innovatsion iqtisodiy o'sishni ta'minlovchi asosiy omillardan biri hisoblanadi. Jahon tajribasi, xususan Singapur misolida olib borilgan tahlil raqamli savdo infratuzilmasining izchil va tizimli joriy etilishi eksport jarayonlarining samaradorligini keskin oshirishini isbotladi. TradeNet va Networked Trade Platform kabi raqamli

platformalar bojxona, logistika va moliyaviy xizmatlarni yagona ekotizimda birlashtirgan holda tranzaksiya xarajatlarini kamaytirib, eksport hajmining barqaror o'sishiga xizmat qilgan.

Tadqiqot davomida aniqlanishicha, eksportni raqamlashtirish faqat texnik jarayon emas, balki institutsional muvofiqlashtirish, biznes muhitini yaxshilash va kadrlar salohiyatini rivojlantirish bilan chambarchas bog'liq kompleks tizimdir. O'zbekiston sharoitida olib borilgan tahlil esa eksport jarayonlarida texnologik infratuzilmaning yetarli emasligi, davlat organlari o'rtasida muvofiqlashtirishdagi kamchiliklar va raqamli ko'nikmalarga ega mutaxassislar yetishmovchiligi asosiy to'siqlar sifatida namoyon bo'layotganini ko'rsatdi. Shu bilan birga, tadqiqot natijalari Singapur tajribasining O'zbekistonga to'liq ko'chirilishi emas, balki uning moslashtirilgan modeli orqali eksportni raqamlashtirish strategiyasini ishlab chiqish maqsadga muvofiqligini asoslab berdi.

Tadqiqot natijalari O'zbekiston eksport siyosatini shakllantirish va uni raqamli transformatsiya sharoitlariga moslashtirishda muhim amaliy ahamiyatga ega. Xususan, taklif etilgan yagona raqamli eksport platformasi modeli eksportyorlar uchun ma'muriy yukni kamaytirish, bojxona va logistika jarayonlarini soddalashtirish hamda tashqi bozorlarga chiqish imkoniyatlarini kengaytirishga xizmat qiladi. Sun'iy intellekt va Big Data texnologiyalaridan foydalanish bo'yicha ishlab chiqilgan takliflar davlat organlari va xususiy sektor uchun ma'lumotlarga asoslangan qarorlar qabul qilish imkonini yaratadi. Bu esa eksport tarkibini diversifikatsiya qilish, yuqori qo'shilgan qiymatli raqamli mahsulotlar va xizmatlar ulushini oshirish orqali iqtisodiyotning innovatsion rivojlanishini jadallashtiradi.

Kelgusida olib boriladigan ilmiy izlanishlar eksportni raqamlashtirishning ayrim yo'nalishlarini yanada chuqurroq o'rganishga qaratilishi lozim. Jumladan, sun'iy intellekt asosida eksport risklarini prognozlash, blokcheyn texnologiyalarining ta'minot zanjiridagi samaradorligi va raqamli xizmatlar eksportining iqtisodiy o'sishga uzoq muddatli ta'sirini baholash istiqbolli tadqiqot yo'nalishlari hisoblanadi.

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Umumiy o'rta ta'lim maktablari o'quvchilarining geometrik tasavvurini rivojlantirish

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DOI 10.5281/zenodo.18217687

Annotatsiya

Ushbu maqola umumiy o'rta ta'lim maktablarida o'quvchilarning geometrik tasavvurini rivojlantirishga bag'ishlangan. Geometrik tasavvur, ya'ni fazoviy tafakkurning muhim komponenti sifatida, nafaqat matematika fanini o'zlashtirish, balki STEAM (Science, Technology, Engineering, and Mathematics) sohalarida muvaffaqiyatga erishish uchun ham fundamental ahamiyatga ega. Maqolada PISA va TIMSS kabi xalqaro baholash dasturlarining natijalari tahlil qilinib, umumiy o'rta ta'lim o'quvchilarining matematik savodxonlik, xususan, geometriyaga oid ko'nikmalaridagi mavjud holat ochib berilgan. Tadqiqot Van Hiele modeli, konstruktivizm va faoliyat yondashuvi kabi nazariy asoslarga tayanib, geometrik tasavvurni rivojlantirishning zamonaviy metodlari вЂ“ dinamik geometriya dasturlari (masalan, GeoGebra), manipulyativ vositalar, muammoli ta'lim va STEAM-integratsiyasining samaradorligini o'rganadi. Natijalar shuni ko'rsatadiki, an'anaviy yodlashga asoslangan usullardan voz kechib, interaktiv, amaliy va texnologiyaga asoslangan yondashuvlarni joriy etish o'quvchilarning fazoviy tafakkurini, mantiqiy fikrlashini va muammoni yechish qobiliyatini sezilarli darajada yaxshilaydi. Maqola yakunida o'quv dasturlarini modernizatsiya qilish, o'qituvchilar malakasini oshirish va baholash tizimini takomillashtirish bo'yicha amaliy tavsiyalar keltirilgan.

Kalit so'zlar: geometrik tasavvur, fazoviy tafakkur, matematika ta'limi, van Hiele modeli, PISA, TIMSS, STEAM, dinamik geometriya.

Abstract This article is devoted to the development of students' geometric imagination in general education schools. Geometric imagination, that is, as an important component of spatial thinking, is fundamental not only for mastering mathematics, but also for achieving success in the fields of STEAM (Science, Technology, Engineering, and Mathematics). The article analyzes the results of international assessment programs such as PISA and TIMSS, revealing the current state of mathematical literacy, in particular, geometry skills of general secondary school students. The research, based on such theoretical foundations as the Van Hiele model, constructivism, and the activity approach, examines the effectiveness of modern methods for the development of geometric imagination - dynamic geometry programs (for example, GeoGebra), manipulative tools, problem-based learning, and STEAM integration. The results show

that abandoning traditional memorization-based methods and implementing interactive, practical, and technology-based approaches significantly improves students' spatial thinking, logical thinking, and problem-solving abilities. At the end of the article, practical recommendations are given for modernizing curricula, improving the system of professional development and assessment of teachers.

Keywords: geometric imagination, spatial thinking, mathematics education, van Hiele model, PISA, TIMSS, STEAM, dynamic geometry.

Kirish

XXI asr raqamli texnologiyalar va innovatsiyalar davri bo'lib, unda fan, texnologiya, muhandislik va matematika (STEAM) sohalari iqtisodiy rivojlanishning asosiy harakatlantiruvchi kuchiga aylandi [1]. Bu sohalarning barchasi uchun fundamental asos bo'lib xizmat qiladigan ko'nikmalardan biri bu вЂ“ fazoviy tafakkur, uning muhim tarkibiy qismi esa geometrik tasavvurdir. Shu bilan birga o'quvchilarga berilayotgan axborotlar hajmi, o'quvchilar bu axborotlarni mushohada qilish imkoniyati va undan kelajakda kerakli maqsadlarda foydalanish imkoniyatlari ancha past ekanligi kuzatilmoqda. Bu esa o'z navbatida o'qitish вЂ“ o'rgatishni takomillashtirishni talab etmoqda. Mavjud o'quv qo'llanmalar, darsliklar, qo'shimcha materiallar o'zining mazmuni, tuzilishi va funktsiyasi jihatidan yuqorida keltirilgan fikrlarga to'la javob bermaydi. Bundan tashqari o'quvchilarning mantiqiy fikrlashini, ularning geometrik tasavvurlarini rivojlantiruvchi va shu tasavvur ichida mushohada qilishni ta'minlaydigan o'quv materiallari yetarli emas. Ikkinchi tomondan mustaqil fikrlay oladigan o'quvchilarni tarbiyalab voyaga yetkazish hozirgi kunning dolzarb muammolaridan biridir. Bunday shaxsni tarbiyalash jamiyatimizning rivojini ta'minlovchi, mamlakatimizda fan va texnikaning yangi texnologiyalarining rivojlanishini, insoniyatning ilm olish madaniyatining takomil topishi kabi sifatlarning barchasi inson tafakkuri va tasavvurisiz sodir bo'lmaydi yoki yuzaga kelmaydi. Geometrik tasavvur вЂ“ bu shunchaki shakllarni tanish emas, balki ob'ektlarni aqlan vizualizatsiya qilish, ularni turli burchaklardan ko'ra olish, o'zgartirish va fazodagi munosabatlarini tushunish qobiliyatidir. Bu ko'nikma arxitektura, dizayn, tibbiyot va hatto kundalik hayotda (masalan, xaritadan foydalanish yoki mebelni joylashtirish) ham muhim rol o'ynaydi.

Shu bilan birga, xalqaro tadqiqotlar ko'plab mamlakatlarda, jumladan, O'zbekistonda ham o'quvchilarning matematik savodxonligi, ayniqsa, amaliy geometriya va fazoviy masalalarni yechish bo'yicha ko'nikmalari yetarli darajada emasligini ko'rsatmoqda. Iqtisodiy hamkorlik va taraqqiyot tashkiloti (OECD) tomonidan o'tkazilgan PISA-2022 tadqiqotida O'zbekiston ilk bor ishtirok etdi va 15 yoshli o'quvchilarning matematika bo'yicha o'rtacha bali 364 ni tashkil etdi, bu OECD mamlakatlari o'rtacha ko'rsatkichidan pastdir. O'quvchilarning atigi 19 foizi matematikadan minimal (2-daraja) malaka darajasiga erishgan, OECD bo'yicha bu ko'rsatkich 69 foizni tashkil etadi. Bu natijalar ta'lim tizimida, xususan, abstrakt va mantiqiy fikrlashni talab qiluvchi geometriya o'qitish metodikasini qayta ko'rib chiqish zarurligini ko'rsatadi.

An'anaviy ta'limda geometriya ko'pincha formulalar va teoremalarni yodlashga asoslangan holda o'qitiladi, bu esa o'quvchilarda geometriya faniga nisbatan qiziqishning so'nishiga va tasavvur qilish qobiliyatining sust rivojlanishiga olib keladi. Zero, zamonaviy didaktika o'qitish jarayonini o'quvchining faol ishtirokiga, amaliy mashg'ulotlarga va texnologiyalardan unumli foydalanishga qurishni talab etadi [2]. Van Hiele tomonidan ishlab chiqilgan geometrik tafakkur darajalari nazariyasi o'quvchilarning geometriyani

qanday o'zlashtirishini bosqichma-bosqich tushuntirib, har bir daraja uchun mos o'qitish strategiyalarini taklif qiladi [3].

Ushbu tadqiqotning maqsadi вЂ“ O'zbekiston umumiy o'rta ta'lim maktablarida o'quvchilarning geometrik tasavvurini rivojlantirishning dolzarb muammolarini tahlil qilish, xalqaro ilg'or tajribalar va nazariy yondashuvlar asosida samarali metodik yechimlarni taklif etishdan iborat. Maqolada geometrik tasavvurning mohiyati, uni shakllantiruvchi komponentlar, zamonaviy o'qitish metodlari va baholash usullari atroflicha ko'rib chiqiladi. Tadqiqot natijalari o'qituvchilar, metodistlar va ta'lim siyosatchilari uchun amaliy qo'llanma bo'lib xizmat qilishi mumkin.

Metodlar

Ushbu tadqiqot deduktiv-analitik yondashuvga asoslangan bo'lib, unda umumiy o'rta ta'limda geometrik tasavvurni rivojlantirishga oid mavjud adabiyotlarni tizimli tahlil qilish (systematic literature review) metodi qo'llanilgan. Tadqiqot doirasida nazariy va amaliy manbalar tanlab olinib, ulardagi asosiy g'oyalar, metodlar va natijalar umumlashtirildi va O'zbekiston ta'lim tizimi kontekstida tahlil qilindi. Tadqiqot metodologiyasi bir necha bosqichdan iborat bo'ldi.

Birinchi bosqichda O'quvchilar bilimini faollashtirish va qayta ishlash, ma'lumotlar manbalarini aniqlash va yig'ish ishlari olib borildi. Buning uchun quyidagi turdagi hujjatlar o'rganildi: 1) Xalqaro baholash dasturlari hisobotlari, xususan, Iqtisodiy hamkorlik va taraqqiyot tashkilotining (OECD) PISA-2022 va Ta'limdagi yutuqlarni baholash xalqaro assotsiatsiyasining (IEA) TIMSS-2019 va TIMSS-2023 [4, 5] natijalari; 2) Nufuzli ilmiy jurnallarda (masalan, Springer, ScienceDirect, Frontiers, PMC NCBI) chop etilgan fazoviy tafakkur, geometrik fikrlash va matematika ta'limi metodikasiga oid 2020вЂ“2025-yillardagi maqolalar [3, 6]; 3) UNESCO va Jahon Banki kabi xalqaro tashkilotlarning ta'lim sifatini oshirishga qaratilgan tahliliy hisobotlari va tavsiyalari [7]; 4) O'zbekiston Respublikasining ta'lim sohasidagi me'yoriy-huquqiy hujjatlari, jumladan, Davlat ta'lim standartlari, milliy o'quv dasturlari va matematika fanini o'qitishga oid metodik qo'llanmalar [8, 9].

Ikkinchi bosqichda to'plangan yangi ko'nikma va malakani rivojlantirish, ma'lumotlarni tahlil qilish uchun nazariy asoslar tanlandi. Asosiy nazariy model sifatida van Hiele geometrik tafakkur darajalari (vizualizatsiya, tahlil, tartiblash, deduksiya, qat'iylik) modeli olindi [3, 10]. Ushbu model o'quvchilarning geometrik tushunchalarni o'zlashtirish jarayonini tushunish va o'qitish strategiyalarini har bir darajaga moslashtirish uchun qulay asos yaratadi. Shuningdek, tahlilda konstruktivizm (o'quvchi bilimni o'zi faol yaratadi) va faoliyat yondashuvi (bilim amaliy faoliyat orqali egallanadi) tamoyillaridan foydalanildi [2].

Uchinchi bosqichda yangi ko'nikma va malakaning amaliyotga tatbiq etish, tahlil natijalari sintez qilindi. Turli manbalardan olingan ma'lumotlar, statistik ko'rsatkichlar va metodik yondashuvlar o'zaro taqqoslanib, geometrik tasavvurni rivojlantirishning samarali modelini yaratishga xizmat qiluvchi asosiy komponentlar ajratib olindi. Jumladan, rivojlantirish metodlari (dinamik geometriya, manipulyativlar, STEAM), baholash vositalari (rubrikalar, portfoliolar) va amaliyotdagi cheklovlar (resurs, o'qituvchi malakasi) tizimlashtirildi. Tadqiqotning ishonchliligi va xolisligini ta'minlash maqsadida faqat nufuzli va tasdiqlangan manbalarga tayanildi, statistik ma'lumotlar manbasi aniq ko'rsatildi.

Natijalar

Tahlil natijalar shuni ko'rsatadiki, o'quvchilarning geometrik tasavvurini rivojlantirish ko'p qirrali jarayon bo'lib, u ham nazariy, ham amaliy jihatdan chuqur yondashuvni talab etadi. Natijalar bir necha asosiy yo'nalishlarda umumlashtirildi: xalqaro va milliy baholashlar konteksti, geometrik tasavvurning tarkibiy tuzilmasi va uni rivojlantirishning zamonaviy metodlari.

Xalqaro baholash dasturlari o'quvchilarning amaliy ko'nikmalarini, jumladan, geometrik materialni real hayotiy vaziyatlarda qo'llay olish qobiliyatini o'lchaydi. PISA-2022 natijalariga ko'ra, O'zbekistonlik o'quvchilarning matematik savodxonlik bo'yicha ko'rsatkichlari (364 ball) OECD o'rtacha ko'rsatkichidan (472 ball) sezilarli darajada past. Topshiriqlar tahlili shuni ko'rsatadiki, o'quvchilar ko'proq standart algoritmlarni bajarishga qiynalmasa-da, nostandart, vizualizatsiya va modellashtirishni talab etuvchi masalalarda jiddiy qiyinchiliklarga duch keladi. Masalan, PISA topshiriqlaridan biri bo'lgan "Ferma tomi"(Farmhouse Roof) masalasida o'quvchidan piramida shaklidagi tomning matematik modelini tahlil qilib, uning elementlarini hisoblash talab etiladi, bu esa kuchli fazoviy tasavvurni nazarda tutadi [11].

IEA tomonidan o'tkazilgan TIMSS-2023 tadqiqoti ham shunga o'xshash manzarani ko'rsatadi. 4-sinf o'quvchilarining matematika bo'yicha natijasi 443 ball (58 davlat orasida 50-o'rin), 8-sinf o'quvchilariniki esa 421 ball (44 davlat orasida 32-o'rin) bo'lgan [5]. TIMSS baholash mezonlarida "Geometriya va o'lchashlar"(Geometry and Measurement) alohida blok sifatida ajratilgan bo'lib, 8-sinf uchun bu yo'nalish umumiy ballning 20%ini tashkil etadi [4]. Bu natijalar milliy o'quv dasturlari va o'qitish amaliyotida geometrik tasavvurni rivojlantirishga yetarlicha e'tibor qaratilmayotganidan dalolat beradi.

Geometrik tasavvurni shakllantirishning tarkibiy qismlari va metodlari

Ilmiy adabiyotlar tahlili geometrik tasavvurni bir nechta o'zaro bog'liq komponentlarga ajratish mumkinligini ko'rsatadi. Bular: fazoviy vizualizatsiya, mental aylantirish, fazoviy munosabatlarni anglash va shakl-zaminni ajratish kabi qobiliyatlardir [8, 11]. Ushbu komponentlarning har birini tizimli rivojlantirish uchun maxsus metodik yondashuvlar talab etiladi. Quyidagi jadvalda geometrik tasavvurning asosiy komponentlari, ularni rivojlantirish metodlari va baholash mezonlari umumlashtirilgan.

1-jadval. Geometrik tasavvurni rivojlantirishning komponentlari, metodlari va baholash mezonlari

Rivojlantirishning zamonaviy yondashuvlari

Tadqiqotlar shuni ko'rsatadiki, an'anaviy "doska-bo'r"formatidan voz kechib, o'quv jarayoniga interaktiv texnologiyalar va amaliy faoliyatni integratsiya qilish geometrik tasavvurni rivojlantirishda yuqori samara beradi. Masalan, Tunisda 1-sinf o'quvchilari bilan o'tkazilgan tajribada "Xazina o'yini"(Treasure Game) deb nomlangan harakatli o'yin an'anaviy darslarga qaraganda geometrik fikrlash darajasini va o'quvchilarning kayfiyatini sezilarli darajada oshirgan [12]. Bu o'yin davomida bolalar jamoa bo'lib geometrik shakllarni o'z tanalari yordamida "chizishgan bu esa ularning fazoni his qilishini va jismoniy tajriba orqali tushunchalarni o'zlashtirishini ta'minlagan.

Dinamik geometriya dasturlari (DGS), xususan, GeoGebra, o'qitish jarayonida inqilobiy o'zgarish yasadi. Ular o'quvchilarga geometrik ob'ektlarni shunchaki chizish emas, balki ularni "jonli"tarzda o'zgartirish, nuqtalarni siljitish orqali xossalar

в„–	Komponent	Indikator	Rivojlantirish metodlari	Baholash vositalari
1	Fazoviy vizualizatsiya	2 va 3 o'lchovli fazoda shakllarni tasavvur qilish; yoyilma va yig'ma holatni anglash.	GeoGebra, Desmos; kompyuter modellar; qog'ozdan shakl yasash.	Yoyilmadan shaklni aniqlash testlari; turli ko'rinishlarini chizish.
2	Mental aylantirish	Ob'ektni xayolan o'qlar atrofida aylantirib yangi holatini aniqlash.	Kubiklar, konstruktorlar; Rubik kubi; 3D o'yinlar.	Aylantirilgan shakllarni taqqoslash testlari (VandenbergвЂ“Kuse).
3	Shakl-tuzilma tahlili	Murakkab shaklni sodda geometrik qismlarga ajratish.	Konstruksiya masalalari; shakl tahlili; STEAM loyihalari.	Yuza va perimetrni topish; вЂњortiqchasini topвЂќ topshiriqlari.
4	Mantiqiy dalillash	Geometrik mulohazalarni mantiqiy zanjir asosida tuzish.	Van Hiele (3вЂ“4 bosqich); вЂњagarвЂќundavЂќ mulohazalari.	Ochiq masalalar; isbotdagi xatoni topish.

Таблица 2: Fazoviy tafakkurni rivojlantirish komponentlari

qanday o'zgarishini kuzatish imkonini beradi. Bu jarayon o'quvchilarda gipotezalar yaratish va ularni shu zahotiyoq tekshirib ko'rish malakasini shakllantiradi, bu esa van Hiele modelining yuqori bosqichlariga o'tishni tezlashtiradi [132]. Tadqiqotlar DGSdan foydalanish o'quvchilarning geometriyadan o'zlashtirishini va fazoviy tafakkurini yaxshilashini tasdiqlaydi [4].

STEAM (Science, Technology, Engineering, Art, Mathematics) yondashuvi fanlarni alohida emas, balki o'zaro integratsiyada, real hayotiy loyihalar orqali o'rganishni taklif etadi [1]. Geometriya STEAM loyihalari uchun tabiiy platforma hisoblanadi. Masalan, o'quvchilar arxitektura elementlarini o'rganib, geometrik printsiplar asosida bino yoki ko'prik modelini yaratishlari, bunda mustahkamlikni hisoblash uchun matematikadan, dizayn uchun san'atdan va materiallarni tanlash uchun texnologiyadan foydalanishlari mumkin. Bu jarayon geometriyani zerikarli fandan amaliy va ijodiy faoliyatga aylantiradi.

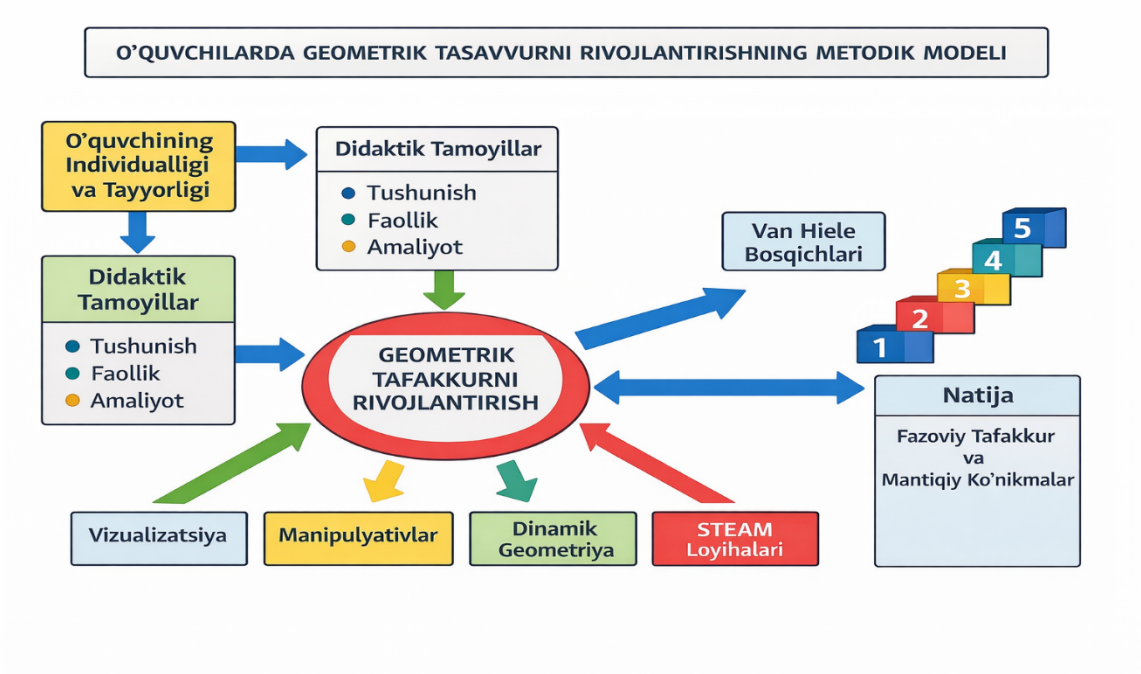
Muhokama

Tadqiqot natijalari shuni ko'rsatadiki, O'zbekiston umumta'lim maktablarida o'quvchilarning geometrik tasavvurini rivojlantirish masalasi tizimli va kompleks yondashuvni talab qiladi. PISA va TIMSS kabi xalqaro baholashlardagi past natijalar shunchaki bilimlar yetishmasligini emas, balki o'quvchilarning fikrlash ko'nikmalari, xususan, bilimlarni nostandart vaziyatlarda qo'llash va fazoviy mulohaza yuritish qobiliyati yetarli shakllanmaganini ko'rsatadi [5]. Bu muammoning ildizi an'anaviy o'qitish metodlarida, ya'ni tayyor bilimlarni uzatish va yodlashga asoslangan yondashuvda yotadi.

Geometrik tasavvurni rivojlantirish uchun taklif etilayotgan metodik model (2-rasm) o'quv jarayonini o'quvchining individual xususiyatlari va tayyorgarlik darajasidan kelib chiqqan holda, aniq didaktik tamoyillar asosida qurishni nazarda tutadi. Bu modelda asosiy urg'u faoliyatga, vizualizatsiyaga va amaliyotga qaratilgan. Van Hiele nazariyasi ko'rsatganidek, o'quvchi geometrik tafakkurning bir darajasidan keyingisiga o'tishi uchun maxsus tashkil etilgan, yo'naltirilgan faoliyat bosqichlaridan o'tishi kerak [3, 10]. Dinamik geometriya dasturlari, manipulyativlar va STEAM loyihalari aynan shunday faoliyatni tashkil etish uchun samarali vositalardir.

Biroq, bu zamonaviy metodlarni amaliyotga joriy etish bir qator cheklovlar va qiyinchiliklar bilan bog'liq. Birinchidan, bu o'qituvchilardan yuqori darajadagi kasbiy

mahorat va yangi texnologiyalarni egallashni talab qiladi. Ko'plab o'qituvchilarning o'zlari an'anaviy tizimda ta'lim olgan va ularning geometrik tafakkur darajasi van Hiele shkalasi bo'yicha yuqori bo'lmasligi mumkin [14]. Shu sababli, o'qituvchilarning malakasini oshirish kurslarini tubdan qayta ko'rib chiqish, ularni amaliyotga yo'naltirilgan, loyihaga asoslangan va zamonaviy vositalardan foydalanishga o'rgatadigan formatda tashkil etish lozim [9, 15].



2-rasm. O'quvchilarda geometrik tasavvurni rivojlantirishning metodik modeli. Model o'qitish jarayonini tizimli loyihalashga yordam beradi.

Ikkinchidan, maktablarning moddiy-texnik bazasi ham muhim omil. Har bir sinfni kompyuterlar, interaktiv doskalar va dinamik dasturlar bilan ta'minlash, shuningdek, amaliy mashg'ulotlar uchun yetarli miqdorda manipulyativ vositalar (geometrik jismlar to'plamlari, konstruktorlar) bilan jihozlash zarur. Uchinchidan, mavjud o'quv dasturlari va darsliklar ko'pincha nazariy materialga haddan tashqari yuklangan bo'lib, amaliy va ijodiy topshiriqlar uchun yetarli vaqt va o'rin qoldirmaydi [16]. Dasturlarni qayta ko'rib chiqishda geometrik tasavvurni rivojlantirishga alohida urg'u berish, mavzularni spiral prinsipida (boshlang'ich sinflardan boshlab bosqichma-bosqich murakkablashtirib borish) o'qitish maqsadga muvofiqdir.

Xulosa qilib aytganda, O'zbekiston maktablarida geometriya o'qitishni isloh qilish вЂ“ bu shunchaki darsliklarni yangilash emas, balki butun bir pedagogik falsafani o'zgartirish demakdir. Bu o'zgarish o'quvchini passiv tinglovchidan faol tadqiqotchiga, o'qituvchini esa ma'lumot uzatuvchidan o'quv jarayonining muhandisi va yo'naltiruvchisiga aylantirishni talab qiladi [2]. Geometrik tasavvurni rivojlantirish orqali biz nafaqat matematikani yaxshi o'zlashtiradigan, balki tanqidiy va ijodiy fikrlaydigan, murakkab muammolarga nostandart yechim topa oladigan avlodni tarbiyalashga erishamiz. Bu esa, o'z navbatida, mamlakatning innovatsion salohiyatini yuksaltirish uchun mustahkam poydevor bo'ladi.

Amaliy tavsiyalar

Yuqoridagi tahlillar asosida maktablar va o'qituvchilar uchun quyidagi bosqichma-bosqich tavsiyalarni berish mumkin:

1. **Diagnostika va rejalashtirish:** O'quv yili boshida o'quvchilarning geometrik tafakkur darajasini van Hiele modeliga asoslangan sodda testlar yordamida aniqlash. Natijalarga ko'ra sinfni shartli guruhlarga ajratib, har bir guruh uchun tabaqalashtirilgan topshiriqlar rejasini tuzish.

2. **Metodlarni integratsiya qilish:** Har bir darsda nazariya va amaliyotni uyg'unlashtirish. An'anaviy masala yechish bilan birga, kamida 10-15 daqiqa vaqtni amaliy faoliyatga (masalan, GeoGebra'da yasash, model yasash, guruhda loyiha ishlash) ajratish.

3. **Texnologiyalardan foydalanish:** GeoGebra, Desmos kabi bepul va qulay dasturlardan darsda va uy vazifalarida keng foydalanish. O'qituvchilar o'zlari kichik interaktiv dars ishlanmalarini yaratib, o'quvchilar bilan bo'lishishlari mumkin.

4. Baholashni o'zgartirish: Yakuniy nazorat ishlaridan tashqari, formativ baholash usullarini (kuzatish, savol-javob, kichik loyihalar) faol qo'llash. O'quvchilarning geometriyaga oid ijodiy ishlaridan (chizmalar, modellar, taqdimotlar) iborat portfolioni yuritish va uni baholash.

5. Kasbiy rivojlanish: O'qituvchilar o'zaro tajriba almashish uchun "kasbiy o'rganish jamoalari"(professional learning communities) tashkil etishlari, zamonaviy metodikalar bo'yicha onlayn kurslar va seminarlarda muntazam ishtirok etib borishlari lozim.

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Xalqaro jarayonlarni tahlil qilishning miqdoriy usullari

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DOI 10.5281/zenodo.18217692

Annotatsiya: Ushbu maqolada xalqaro jarayonlarni tahlil qilishning miqdoriy usullari O'zbekiston misoli orqali o'rganilgan. Tadqiqotda 2025 yilga oid tashqi savdo aylanmasi, eksport-import ko'rsatkichlari, chet el investitsiyalari (FDI), sayyohlar va mehnat migrantlari tomonidan yuborilgan mablag'lar kabi raqamli indikatorlar tahlil qilingan. Shu ma'lumotlar yordamida O'zbekistonning global iqtisodiy tizimdagi o'rni, xalqaro savdo va migratsiya jarayonidagi roli hamda mamlakatning iqtisodiy barqarorligi va rivojlanish sur'ati miqdoriy jihatdan baholangan. Maqola natijalari O'zbekistonning xalqaro jarayonlarga integratsiyasi va dunyo bilan o'zaro aloqalarini statistik tahlil qilishga imkon beradi.

Kalit so'zlar: Xalqaro jarayonlar, miqdoriy usullar, O'zbekiston, tashqi savdo, chet el investitsiyalari (FDI), mehnat migratsiyasi, xalqaro turizm.

Abstract: This article examines the quantitative methods of analyzing international processes using the example of Uzbekistan. The study analyzes digital indicators for 2025, such as foreign trade turnover, export-import statistics, foreign direct investments (FDI), international tourism, and remittances sent by labor migrants. Using this data,

Uzbekistan's role in the global economic system, participation in international trade and migration processes, and the country's economic stability and growth rate are evaluated quantitatively. The results allow statistical analysis of Uzbekistan's integration into international processes and its interactions with the world.

Keywords: International processes, quantitative methods, Uzbekistan, foreign trade, foreign direct investment (FDI), labor migration, international tourism.

Аннотация: В данной статье рассматриваются количественные методы анализа международных процессов на примере Узбекистана. В исследовании проанализированы цифровые показатели за 2025 год, такие как объем внешней торговли, статистика экспорта и импорта, прямые иностранные инвестиции (FDI), международный туризм и переводы денежных средств трудовых мигрантов. С помощью этих данных оценивается роль Узбекистана в глобальной экономической системе, участие в международной торговле и миграционных процессах, а также экономическая стабильность и темпы роста страны.

Ключевые слова: Международные процессы, количественные методы, Узбекистан, внешняя торговля, прямые иностранные инвестиции (ПИИ), трудовая миграция, международный туризм.

So‘nggi yillarda xalqaro munosabatlar tizimida yuz berayotgan jadal o‘zgarishlar davlatlarning global jarayonlardagi o‘rnini chuqur va ilmiy asosda tahlil qilish zaruratini kuchaytirmoqda. Xususan, O‘zbekistonning ochiq va pragmatik tashqi siyosat olib borishi, xalqaro savdo hajmining kengayishi, to‘g‘ridan-to‘g‘ri xorijiy investitsiyalar oqimining ortishi hamda mintaqaviy va global tashkilotlar bilan hamkorligining faollashuvi xalqaro jarayonlarni zamonaviy tadqiqot usullari asosida o‘rganishni talab etadi.

Xalqaro jarayonlarni tahlil qilishning miqdoriy usullari statistik ko‘rsatkichlar, indekslar va iqtisodiy modellashtirish yondashuvlariga asoslanib, davlatlar o‘rtasidagi iqtisodiy va siyosiy munosabatlardagi tendensiyalarni aniqlash, ta’sir omillarini baholash hamda rivojlanish istiqbollarini prognoz qilish imkonini beradi.

Mazkur maqolada xalqaro jarayonlarni o‘rganishda qo‘llaniladigan asosiy miqdoriy usullar tahlil qilinib, ularning O‘zbekistonning xalqaro maydondagi pozitsiyasini baholashdagi ahamiyati yoritiladi.

Empirik tahlil natijalari: 2025 yilning yanvar–oktabr oylarida O‘zbekistonning tashqi savdo aylanmasi 66,5 milliard AQSh dollariga yetib, 2024 yilga nisbatan 21,5% ga oshdi. Eksport 29 milliard dollar, import esa 37,5 milliard dollarni tashkil etdi.

Turizm sohasida 2025 yilning yanvar–oktabr oylarida 9,7 milliondan ortiq chet ellik sayyoh tashrif buyurdi.

Chet eldan jo‘natilgan pul mablag‘lari 2025 yilning birinchi yarmida 8,2 milliard dollarni tashkil etdi.

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Aholi farovonligiga ta'sir etuvchi omillarning ekonometrik tahlili (Namangan viloyati misolida)

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DOI 10.5281/zenodo.18217695

Annotatsiya: Ushbu tezisda Namangan viloyati aholisining farovonligiga ta'sir etuvchi asosiy omillarni aniqlash va ularning o'zaro bog'liqliklarini ekonometrik tahlil qilishga qaratilgan. Tadqiqot 2015–2024 yillar davri uchun viloyatga xos 12 ta ko'rsatkichni qamrab oladi. Asosiy komponentlar tahlili (AKT) va ko'p o'lchovli regressiya tahlili usullari qo'llangan. AKT ko'p o'lchovli ma'lumotlarni soddalashtirish va multikollinearlik muammolarini bartaraf etish uchun qo'llanildi. Tadqiqot Namangan viloyatida amalga oshirilayotgan iqtisodiy islohotlar samaradorligini baholash va strategik rejalashtirish uchun empirik asos yaratildi hamda ijtimoiy holatni yaxshilash uchun tavsiyalar berildi.

Tayanch so'zlar: Namangan viloyati, aholi farovonligi, ekonometrik tahlil, Asosiy komponentlar tahlili, ijtimoiy-iqtisodiy ko'rsatkichlar.

Kirish

O'zbekistonning sharqiy qismida joylashgan Namangan viloyati muhim ijtimoiy-iqtisodiy ahamiyatga ega bo'lib, qishloq xo'jaligi, sanoat va xizmat ko'rsatish tarmoqlariga asoslangan iqtisodiy tuzilmaga ega. Aholi soni va iqtisodiy salohiyati jihatidan respublikada yetakchi o'rinlardan birini egallaydi. Viloyat aholisining farovonligini oshirish davlat siyosatining ustuvor yo'nalishlaridan biri hisoblanadi. 2015–2024 yillar davomida viloyatda jon boshiga daromad, investitsiya va ta'lim ko'rsatkichlarida o'sish kuzatildi,

qashshoqlik darajasi, ishsizlik va sog'liqni saqlash sohasidagi muammolar sezilarli yaxshilandi, lekin dolzarb masalalardan biri bo'lib qolmoqda.

"O'zbekiston–2030"strategiyasida aholi turmush darajasini yaxshilashga qaratilgan aniq maqsadlar belgilangan. 2030-yilga qadar iqtisodiyot hajmini 2 barobar oshirish, YaIM hajmini 160 milliard dollarga yetkazish, aholi jon boshiga daromadlarni 4 ming dollarga oshirish, xizmatlar sohasini 3 barobar kengaytirish va sanoat hamda qishloq xo'jaligida samaradorlikni oshirish vazifalari belgilangan.

O'zbekistondagi Namangan kabi mintaqalarda iqtisodiy islohotlar va infratuzilma rivojlanishi fonida aholi farovonligini ta'minlash davlat siyosatining ustuvor yo'nalishi bo'lib qolmoqda. Shu bilan birga, bu jarayonlarga ta'sir etuvchi omillarni aniqlash va ularning o'zaro bog'liqliklarini tahlil qilish mintaqaviy rivojlanish strategiyalarini ishlab chiqish uchun muhim ahamiyatga ega.

Tadqiqotda 2015-2024 yillar davri uchun viloyatga xos bo'lgan 12 ta ko'rsatkich, jumladan, jon boshiga daromad, ishsizlik darajasi, YaIM o'sishi, investitsiyalar, qashshoqlik darajasi, savodxonlik darajasi, oliy ma'lumotli aholi ulushi, shifokorlar soni (10 ming aholiga), chaqaloqlar o'limi, umr ko'rish davomiyligi, iqtisodiy faol aholi soni va maishiy xizmatlar ko'rsatilish darajasi.Bu ko'rsatkichlar yordamida Namangan viloyatining ijtimoiy-iqtisodiy ko'rsatkichlarini chuqur o'rganish va viloyatning o'ziga xos sharoitida farovonlik omillarining o'zaro bog'liqliklarini aniqlashda namoyon bo'ladi. Bundan tashqari, tadqiqot 2015-2024 yillar davomida mintaqada amalga oshirilgan iqtisodiy islohotlar natijalarini baholash va kelajakdagi rivojlanish strategiyalarini shakllantirish uchun empirik asos yaratadi. Ushbu tadqiqotda Namangan viloyati aholisining farovonligiga ta'sir etuvchi asosiy omillarni aniqlash va ularning o'zaro bog'liqliklarini tahlil qilish uchun asosiy komponentlar tahlil (AKT) usulidan foydalanildi.

Mavzuga oid adabiyotlar tahlili

Aholi farovonligi, ijtimoiy-iqtisodiy rivojlanishning markaziy ko'rsatkichi sifatida, ko'p yillar davomida tadqiqotchilar e'tiborining markazida bo'lib kelgan. Farovonlik tushunchasi nafaqat moddiy farovonlikni, balki sog'liq, ta'lim, tenglik va ijtimoiy barqarorlik kabi ko'p o'lchovli omillarni o'z ichiga oladi [2]. A.Sen вЂњRivojlanish erkinlik sifatidaвЂќ asarida farovonlik odamlarning qobiliyatlari va erkinliklari nuqtai nazaridan ta'riflab berilgan bo'lib, iqtisodiy o'sish nafaqat uning moddiy natijalari, balki ijtimoiy va insoniy rivojlanishga ta'siri bo'yicha ham baholanishi kerakligi g'oyasiga urg'u berilgan [3]. Farovonlikni o'lchashda yalpi ichki mahsulot (YaIM) kabi an'anaviy iqtisodiy ko'rsatkichlardan tashqari, sog'liq, ta'lim va ijtimoiy tenglik kabi ko'rsatkichlarni ham hisobga olish zarurligini ta'kidladi. Ushbu yondashuvlar aholi farovonligini tahlil qilishda ko'p o'lchovli ma'lumotlarni talab qiladi, bu esa statistik usullarni qo'llashda qiyinchiliklar yuzaga keltiradi. Rivojlanayotgan mamlakatlarda, ayniqsa Markaziy Osiyo mintaqasida, iqtisodiy o'sishning ijtimoiy natijalarga ta'siri notekis bo'lib, qashshoqlik, ishsizlik va sog'liqni saqlash sohasidagi muammolar dolzarb muammolar bo'lib qolmoqda [5]. Farovonlikka ta'sir etuvchi omillarni tahlil qilishda ko'p o'lchovli ma'lumotlar bilan ishlash zarurati statistik usullarni tanlashda muhim rol o'ynaydi. An'anaviy regressiya tahlil ko'pincha o'zgaruvchilar o'rtasidagi multikollinearlik muammosiga duch keladi, bu natijalarning ishonchliligini pasaytiradi [2]. Daromad, investitsiya va ta'lim kabi iqtisodiy va ijtimoiy ko'rsatkichlar o'rtasida yuqori korrelyatsiya mavjud bo'lganda multikollinearlik ayniqsa yaqqol namoyon bo'ladi. Bunday qiyinchiliklarni yengish uchun AKT samarali usul sifatida keng qo'llaniladi [4]. AKT o'zgaruvchilar o'rtasidagi o'zaro

bog'liqlikni kamaytiradi, ma'lumotlarning asosiy tuzilishini aniqlaydi va o'lchovlarni qisqartirish imkonini beradi, bu esa murakkab ma'lumotlar to'plamini tahlil qilishda katta afzallikdir. Markaziy Osiyo kontekstida iqtisodiy islohotlarning ijtimoiy natijalarga ta'siri tahlil qilinib, iqtisodiy o'sish, qashshoqlik va tengsizlik o'rtasidagi munosabatlar o'rganilgan va statistik usullar, xususan AKT ning ahamiyatiga urg'u berilgan [6]. O'zbekiston bo'yicha olib borilgan tadqiqotlarda aholi farovonligiga ta'sir etuvchi omillar odatda keng iqtisodiy ko'rsatkichlar (YaIM, investitsiya) va ijtimoiy ko'rsatkichlar (ta'lim, sog'liq) nuqtai nazaridan o'rganilgan. O'zbekistonda iqtisodiy islohotlarning qashshoqlikni qisqartirishga ta'siri tahlil qilinib, ijtimoiy tengsizlik jiddiy muammo bo'lib qolayotganligiga e'tibor qaratilgan [10]. Shu bilan birga, Namangan viloyati uchun bo'lgani kabi, mintaqaga xos ma'lumotlarga asoslangan chuqur tahlillar kamdan-kam uchraydi.Namangan viloyati statistika agentligi ma'lumotlariga ko'ra, viloyatda jon boshiga daromad, investitsiya va ta'lim ko'rsatkichlarida o'sish kuzatilgan bo'lsa-da, qashshoqlik va sog'liq bilan bog'liq muammolar dolzarb bo'lib qolmoqda. Bu Namangan viloyatida farovonlik omillari o'rtasidagi o'zaro bog'liqliklarni tahlil qilish zaruratini ko'rsatadi. Statistik usullar nuqtai nazaridan, ko'p o'lchovli ma'lumotlarni tahlil qilishda asosiy komponentlar tahlili keng qo'llaniladi[12]. Ushbu usul o'zgaruvchilar sonini qisqartirish va ularning asosiy tuzilishini aniqlash imkonini beradi. Mavjud adabiyotlarni sharhlash shuni ko'rsatadiki, aholi farovonligi o'z tabiatiga ko'ra ko'p o'lchovlidir va uning tahlil uchun murakkab statistik usullarni talab qiladi. AKT usuli iqtisodiy va ijtimoiy ko'rsatkichlarni tahlil qilishda samarali bo'lsa-da, ushbu usulni Namangan kabi o'ziga xos mintaqalarga qo'llash bo'yicha tadqiqotlar hali ham cheklangan. Ushbu tadqiqot AKT yordamida Namangan viloyatining 2015–2024 yillardagi ijtimoiy-iqtisodiy ko'rsatkichlarini tahlil qilish orqali ushbu bo'shliqni to'ldirishga qaratilgan. Tadqiqot nafaqat mintaqadagi farovonlik omillarini aniqlashni, balki ularning vaqt o'tishi bilan dinamikasini tushunishni va mintaqaviy rivojlanish strategiyalari uchun yo'l-yo'riq berishni maqsad qilgan.

Tadqiqot metodologiyasi

Ushbu tadqiqotda Namangan viloyati aholisining farovonligiga ta'sir etuvchi asosiy omillarni aniqlash uchun asosiy komponentlar tahlil usulidan foydalanildi[7]. Bu usul yordamida ko'p o'lchovli ma'lumotlarni, multikollinearlik muammosini va o'lchovlarni qisqartirishlarni AKT yordamida xal qilishga xarakat qildik. Bosh komponentalar usuli butun to'lig'icha umumiy ko'rsatkichlarga asoslanib xulosa chiqaradi. Bosh komponentalarni hisoblash boshlang'ich ma'lumotdan tuzilgan kovariatsion matrisaning hos son va hos vektorlarni hisoblashga keltiriladi[8].

1-jadval

Yillar	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
2015	4,215.3	9.8	6.5	1,245.7	18.5	99.2	14.2	24.5	18.2	72.1	1,456.3	68.5
2016	4,956.8	10.2	7.1	1,356.2	17.8	99.3	14.8	24.8	17.6	72.3	1.478.9	70.2
2017	6,123.5	10.5	8.3	1,892.4	16.9	99.4	15.5	25.2	16.8	72.5	1.512.4	72.8
2018	7,845.6	9.6	5.8	2,345.8	15.7	99.5	16.3	25.8	15.9	73.2	1567.8	75.4
2019	9,234.7	9.3	4.5	3,456.9	16.2	99.6	17.2	26.2	15.4	73.4	1623.5	78.1
2020	10,456.8	11.2	2.9	3,678.5	16.8	99.6	18.1	26.8	14.7	72.0	1645.2	80.3
2021	13.789.4	10.1	8.7	5,234.7	20.5	99.7	19.3	27.2	15.8	72.5	1689.4	83.5
2022	17,234.7	9.5	7.4	5.678.9	19.8	99.7	20.5	27.9	12.4	73.0	1734.6	85.7
2023	21,456.7	7.8	6.2	7,890.1	16.5	99.8	21.8	28.5	12.1	74.1	1789.3	88.2
2024	28,456.9	8.1	6.8	12,456.3	14.9	99.9	23.2	29.3	12.9	74.5	1845.7	905

Tahlil va natijalar

Ushbu tadqiqotda Namangan viloyati aholisining farovonligiga ta'sir etuvchi asosiy omillarni aniqlash uchun asosiy komponentlar tahlili usuli qo'llanildi. Tahlil 2015в–“2024 yillar davri uchun Namangan viloyatiga xos bo'lgan quyidagi 12 ta ko'rsatkich asosida amalga oshirildi: jon boshiga daromad X1 (ming so'm), ishsizlik darajasi X2 (%), YaIM o'sish surati X3 (%), investitsiyalar X4 (mlrd), qashshoqlik darajasi X5(%), savodxonlik darajasi X6 (%), oliy ma'lumotli aholi ulushi X7 (%), shifokorlar soni X8 (10 ming aholiga), chaqaloqlar o'limi X9 (1000 tirik tug'ilganga,), umr ko'rish davomiyligi X10 (yil), iqtisodiy faol aholi soni X11 (ming kishi) va maishiy xizmatlar qoplamasi X12 (%).

1-rasm

	Daromad	Ishsizlik	YaIM	Invest	Qashshoq	Savod	Oliy	Shifo	Ch.o'lim	Umr	Faol	Xizmat
Daromad	1.000	-0.423	0.568	0.983	-0.712	0.720	0.796	0.666	-0.680	0.492	0.729	0.724
Ishsizlik	-0.423	1.000	-0.401	-0.433	0.682	-0.468	-0.499	-0.359	0.405	-0.427	-0.498	-0.464
YaIM	0.568	-0.401	1.000	0.618	-0.565	0.495	0.546	0.457	-0.483	0.344	0.506	0.501
Invest	0.983	-0.433	0.618	1.000	-0.746	0.729	0.807	0.678	-0.695	0.504	0.745	0.739
Qashshoq	-0.712	0.682	-0.565	-0.746	1.000	-0.592	-0.650	-0.529	0.556	-0.570	-0.626	-0.618
Savod	0.720	-0.468	0.495	0.729	-0.592	1.000	0.822	0.690	-0.708	0.511	0.746	0.740
Oliy	0.796	-0.499	0.546	0.807	-0.650	0.822	1.000	0.763	-0.783	0.564	0.822	0.816
Shifo	0.666	-0.359	0.457	0.678	-0.529	0.690	0.763	1.000	-0.861	0.624	0.732	0.726
Ch.o'lim	-0.680	0.405	-0.483	-0.695	0.556	-0.708	-0.783	-0.861	1.000	-0.774	-0.726	-0.720
Umr	0.492	-0.427	0.344	0.504	-0.570	0.511	0.564	0.624	-0.774	1.000	0.532	0.528
Faol	0.729	-0.498	0.506	0.745	-0.626	0.746	0.822	0.732	-0.726	0.532	1.000	0.820
Xizmat	0.724	-0.464	0.501	0.739	-0.618	0.740	0.816	0.726	-0.720	0.528	0.820	1.000

1-jadvalda Namangan viloyatining ijtimoiy va iqtisodiy ko'rsatkichlari keltirilgan[13]. Masalan, jon boshiga daromad 2015 yildagi 4215,3 ming so'mdan 2024 yilda 28456.9 ming so'mgacha oshgan bo'lsa, qashshoqlik darajasi 2021 yildagi eng yuqori ko'rsatkichi 20,5 % dan 2024 yilda 14,9 % gacha pasaygan.Ushbu tendentsiyalar tahlil qilingan davr mobaynida mintaqada iqtisodiy o'sish va ijtimoiy yaxshilanishlar sodir bo'lganligini ko'rsatadi.Ma'lumotlar to'plamida yetishmayotgan qiymatlar yo'q, bu tahlilning ishonchliligini oshiradi.

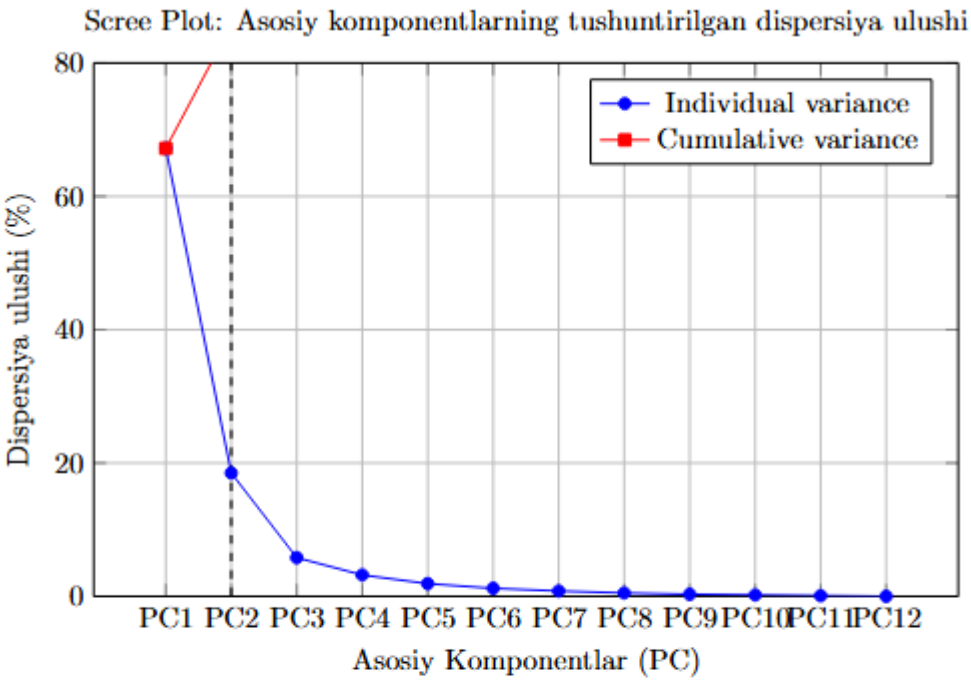
O'zgaruvchilar turli birliklarda (foizlar, ming so'm, milliard so'm) o'lchanganligi sababli, tahlildan oldin ma'lumotlar standartlashtirildi. Standartlashtirish har bir o'zgaruvchining o'rtacha qiymatini 0 ga, standart og'ishini esa 1 ga tenglashtirdi va shu orqali ko'rsatkichlarni solishtirish imkonini ta'minladi. Standartlashtirilgan natijalar quyida keltirilgan(1-rasm).

Komponent	O'z qiymati (λ)	Dispersiya ulushi (%)	Yig'indi ulush (%)
PC1	8.064	67.20	67.20
PC2	2.220	18.50	85.70
PC3	0.696	5.80	91.50
PC4	0.384	3.20	94.70
PC5	0.228	1.90	96.60
PC6	0.150	1.25	97.85
PC7	0.066	0.55	98.40
PC8	0.007	0.06	98.46
PC9	0.004	0.03	98.49
PC10	0.002	0.02	98.51
PC11	0.001	0.01	98.52
PC12	0.000	0.00	98.52

Standartlashtirilgan ma'lumotlar har bir o'zgaruvchining nisbiy o'zgaruvchanligini aks ettiradi. Standartlashtirish barcha o'zgaruvchilarning AKT natijalariga teng hissa qo'shishini ta'minlaydi va o'lchov birliklaridagi farqlar sabab bo'lishi mumkin bo'lgan buzilishlarni bartaraf qiladi.Keyin standartlashtirilgan ma'lumotlar to'plamiga AKT qo'llanildi va aniqlangan asosiy komponentlar asosida har bir yil uchun ballar hisoblab chiqildi. Undan keying bosqichda esa AKT tahlilida har bir asosiy komponentning ma'lumotlar to'plamidagi umumiy dispersiyaga qo'shgan hissasi hisoblab chiqildi. Natijalar quyida keltirilgan(2-rasm).

Asosiy komponentlarning dispersiyaga qo'shgan hissasi PC1 umumiy dispersiyaning 67.2 % tashkil qiladi, bu ushbu komponent Namangan viloyatining ijtimoiy-iqtisodiy ko'rsatkichlaridagi asosiy o'zgaruvchanlikni samarali qamrab olganligini ko'rsatadi. PC2 qo'shimcha 18.5 % dispersiyani tushuntiradi, ya'ni PC1 va PC2 birgalikda ma'lumotlar to'plamidagi umumiy o'zgaruvchanlikning 85.7 % ini izohlaydi.Bu asl o'zgaruvchilarda mavjud bo'lgan ma'lumotlarning katta qismini atigi ikkita asosiy komponent orqali samarali ifodalash mumkinligini ko'rsatadi. PC3 va keyingi komponentlarning hissasi (5.8% va undan kam) nisbatan kichik bo'lib, ular ma'lumotlardagi qo'shimcha, ammo unchalik ahamiyatli bo'lmagan o'zgarishlarni tasvirlaydi.Buni quyidagi grafikdan ham ko'rish mumkin(3-rasm)

Ilova 2. Scree plot grafigi: Asosiy komponentalarning dispersiyaga qo'shgan hissasi



Izoh: Scree plot grafigida tirsak nuqtasi (elbow point) PC2 da joylashgan. Birinchi ikkita asosiy komponent (PC1 va PC2) ma'lumotlardagi umumiy o'zgaruvchanlikning 85.7% ini tushuntiradi. Bu ikki komponentning ma'lumotlarning asosiy tuzilishini ifodalash uchun yetarli ekanligini ko'rsatadi. PC3 dan keyingi komponentlarning dispersiyaga qo'shgan hissasi ahamiyatsiz darajada kichik bo'lib, ularni tahlilga kiritish shart emasligini ko'rsatadi.

Xulosa

Asosiy komponentlar tahlili Namangan viloyati aholisining farovonligiga ta'sir etuvchi asosiy omillarni aniqladi. Iqtisodiy rivojlanish(PC1) Investitsiyalar,jon boshiga daromad,oliy ma'lumot soni kuchli ta'sir ko'rsatdi.Qashshoqlik,chaqaloqlar o'lim 2015вЂ“2024 yillar davomida bo'ylab sezilarli yaxshilanish kuzatildi, bu mintaqaning iqtisodiy rivojlanishi va asosiy ijtimoiy ko'rsatkichlarning yaxshilanganligini ko'rsatadi. Qashshoqlik va Iqtisodiy O'sish (PC2): Yuqori qashshoqlik darajasi ayrim yillarda (masalan, 2021вЂ“2022) YaIM o'sishi bilan birga kuzatildi, sog'liq ko'rsatkichlari, ayniqsa umr ko'rish davomiyligi bilan esa salbiy bog'liqlik kuzatildi. Sog'liq va Maxsus Omillar: PC3 va boshqa kichik komponentlar umr ko'rish davomiyligi va chaqaloqlar o'limi kabi sog'liq bilan bog'liq o'zgaruvchilarni, shuningdek, COVID-19 pandemiyasi kabi o'ziga xos tashqi omillarni o'z ichiga oladi. Iqtisodiy rivojlanishni barqarorlashtirish uchun investitsiyalarni oshirish va ta'lim sifatiini yaxshilashga ko'proq e'tibor qaratilishi kerak.Qashshoqlikni kamaytirish uchun iqtisodiy o'sishning ijtimoiy inklyuzivligini oshirish zarur. Sog'liqni saqlash sohasida chaqaloqlar o'limini kamaytirish va umr ko'rish davomiyligini oshirish, shuningdek, sog'liq xizmatlaridan adolatliroq foydalanishni ta'minlash uchun qo'shimcha choralar ko'rilishi lozim. Ushbu natijalar Namangan viloyati aholisining farovonligini oshirishga qaratilgan strategik rejalashtirish uchun muhun yo'l-yo'riq vazifasini o'taydi.

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Chiziqsiz chegaraviy shartlar bilan bog'langan issiqlik tarqalish tenglamalar sistemasining yechimlari xossalari

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DOI 10.5281/zenodo.18217703

Ushbu maqolada

$$Q_+ = (x,t) : x \in R_+^N, 0 < t < +\infty$$

sohada aniqlangan ko'p komponentli muhitlarda nochiziqli chegaraviy shartlar bilan bog'langan quyidagi:

$$\begin{cases} u_{it} = \nabla(|\nabla u_i|^{m_i-1}\nabla u_i), \ x \in R_+^N, \ t > 0 \\ -|\nabla u_i|^{m_i-1}\dfrac{\partial u_i}{\partial x_1} = u_{i+1}^{q_i}, \ u_{k+1} = u_1, \ x_1 = 0, t > 0 \\ u(x,0) = u_0(x), \ x \in R_+^N, \end{cases} \tag{1}$$

issiqlik tarqalish masalasini qaraymiz. Bu yerda $m_i > 0, q_i > 0$ $u_0(x)$- uzluksiz, manfiy bo'lmagan, chegaralangan funksiyadir. U quyidagi shartlarini qanoatlantiradi,

$$-|\nabla u_{i0}|^{m_i-1}\frac{\partial u_{i0}}{\partial x_1} = u_{(i+1)0}^{q_i}, \ x_1 = 0$$

va ba'zi $x \in R_+^N$ nuqta uchun $sup\ pu_0 \subset B_R(x_0) \cap R_+^N$, $u_0 \neq 0$ bo'ladi. Shu bilan birga, boshlang'ich qiymat $u_0(x)$ kichik soha tashqarisida nolga teng bo'lsa ham, yechimlar chekli vaqt ichida blow-up holatga kelishi mumkin. (1) tenglamani o'ziga xos xususiyati shundaki, unda diffuziya koeffitsiyenti gradiantga bog'liqdir. Bunday turdagi tenglamalar gradiantga bog'liq issiqlik tarqalish jarayonlarini ifodalaydi. Ma'lumki, (1) tenglamalar sistemasi buziluvchi bo'lib, $u_i(x,t) = 0$ yoki $|\nabla u_i| = 0$ bo'lganda klassik ma'noda yechimga ega bo'lmasligi mumkin. Shuning uchun (1) masalaning $|\nabla u_i|^{m_i-1}\nabla u_i \in C(R_+^N \times$

$(0,+\infty)), 0 < u_i \in C(Q), (i = 1,2,...,k)$ sinfda umumlashgan yechimi qaraladi va bu yechim (1) tenglamalar sistemasini integral ayniyat ma'nosida yoki taqsimot ma'nosida qanoatlantiradi.
[1-8] ishlarda nochiziqli chegaraviy shartlarga ega kvazichiziqli, chiziqsiz tenglamalar va ularning sistemalari o'rganilgan. Xususan, [9] ishda Huang, Yin va Wang tomonidan [4] adabiyotdagi masalani ko'p o'lchovli holatga kengaytirganlar:

$$\begin{cases} u_t = \Delta u^m \ x \in R_+^N, \ 0 < t < T \\ -\dfrac{\partial u^m}{\partial x_1} u^p(x,t), \ x_1 = 0, \ 0 < t < T \\ u(x,0) = u_0(x), \ x \in R_+^N, \end{cases} \tag{2}$$

Ular yuqoridagi masala uchun kritik ko'rsatkichlar $p_0 = \frac{1}{2}(m+1)$ va $p_c = m + \frac{1}{N}$ ekanligini aniqlaganlar. Biroq, [4] adabiyotdagi masalaning ko'p o'lchovli holatidagi kritik ko'rsatkichlari hali noma'lum bo'lgan.
[10] adabiyotda quyidagi tez diffuziyali Nyuton tipiga kirmaydigan tenglamalar uchun kritik ko‘rsatkichlarni aniqlashgan, global yechim va chegaralanmagan yechimning hosil bo'lish shartlarini topishgan

$$u_{it} = (|u_{ix}|^{m_i-1}u_{ix})_x, (i = 1,2,...,k), (x,t) \in R_+ \times (0,+\infty) \tag{3}$$

tenglamalar nochiziqli chegaraviy shart orqali o'zaro bog'langan:

$$-|u_{ix}|^{m_i-1}u_{ix}(0,t) = u_{i+1}^{p_i}(0,t), (i = 1,2,...,k), u_{k+1} := u_1, t \in (0,+\infty) \tag{4}$$

$$u_i(x,0) = u_{i0}(x), (i = 1,2,...,k), x \in R_+ \tag{5}$$

Boshlang'ich shartlar nolga teng bo'lmagan, nomanfiy, cheklangan va yetarlicha silliq. Bu yerda $0 < m_i < 1, p_i > 0$. (3) tenglamalar tez diffuziya turiga mansub bo'lgani sababli, (3)вЂ“(5) sistemaning yechimi har qanday $t > 0$ da butun fazoda musbat qiymat qabul qiladi. Ular tez diffuziya xususiyatiga ega bo'lgan Nyuton tipidagi sistema uchun qo'yilgan nochiziqli chegaraviy masalaning kritik ko‘rsatkichini ko'rib chiqdilar. Yuqori va quyi avtomodel yechimlarni qurish yordamida yechimning global mavjudlik va Fujita tipidagi kritik eksponentalarni topishgan.

Ta'rif [11]: $u_i(x,t)$ funksiya Q sohada (1) masalaning yuqori (quyi) yechimi deyiladi, agar $u_i(x,t) = 0$ da Q/D_i, $D_t = x < l(t) \times (0,+\infty)$, $u_{i1}(x,t) \in C_{t,x}^{1,2}(D_t) \cap C(Q), -|\nabla u_i|^{m_i-1}\frac{\partial u_i}{\partial x_1} \in C(Q)$ va quyidagi shartlarni qanoatlantirsa: $u_{it} \geq (\leq) \nabla(|\nabla u_i|^{m_i-1}\nabla u_i)$, $-|\nabla u_i|^{m_i-1}\frac{\partial u_i}{\partial x_i} \geq (\leq) u_{i+1}^{q_i}$,
$u(x,0) \geq (\leq) u_0(x), x \in R$

Natijalar va muhokama

Teorema1. Agar $minN\beta_i - \alpha_i$ shart bajarilsa, u holda (1) masalaning yechimi global bo'ladi.

Isbot: (1) teoremaning isboti yechimlarni taqqoslash usuliga asoslanadi.Buning uchun $Q_+ = (x,t) : x \in R_+^N, 0 < t < +\infty$ sohada quyidagi ko'rinishdagi taqqoslanadigan yechim quramiz.
$\overline{u}_{i+}(x,t) = (\tau+t)^{-\alpha_i} g_i(\eta_i)$ (6)
bu yerda $\eta_i = x(\tau+t)^{\beta_i}, \alpha_i = \dfrac{-\alpha_1 q_i(1+m_i) - m_i}{2m_i}, \beta_i = \dfrac{m_i - \alpha_1 q_i(1-m_i)}{2m_i}$

Shunday bo'lishi uchun $\overline{u}_+(x,t)$ (1) masalaning yuqori yechimi bo'lsin, $g_i(\eta_i)$ funksiyasi quyidagi tengsizlikni qanoatlantirishi kerak:

$$\frac{dg_i}{d\eta_i} = -\,b_i\,\frac{1+m_i}{m_i-1}\left(a_i - b_i\eta_i^{\frac{1+m_i}{m_i}}\right)^{\frac{1}{m_i-1}}\frac{1}{\eta_i^{\frac{1}{m_i}}}. \tag{7}$$

$$\frac{d}{d\eta_i}\left(\eta_i^N\left(a_i - b_i\eta_i^{\frac{1+m_i}{m_i}}\right)^{\frac{m_i}{m_i-1}}\right) =$$

$$\eta_i^{N-1}\left(N\left(a_i - b_i\eta_i^{\frac{1+m_i}{m_i}}\right)^{\frac{m_i}{m_i-1}} - b_i\,\frac{1+m_i}{m_i-1}\,\eta_i^{\frac{1}{m_i}}\left(a_i - b_i\eta_i^{\frac{1+m_i}{m_i}}\right)^{\frac{1}{m_i-1}}\right). \tag{8}$$

$$\overline{g}_i(\eta_i) = (a_i - b_i\eta^{\frac{1+m_i}{m_i}})^{\frac{m_i}{m_i-1}}$$

ohirgi tenglikdan (7) va (8) ni hosil qilinib, avtomodel yechimga qo'yish orqali quyidagilarni hosil qilamiz.

$$\eta_i^{1-N}\frac{d}{d\eta_i}\left[\eta_i^{N-1}\left(-\,b_i\frac{1+m_i}{m_i-1}\right)\eta_i\left(a_i - b_i\eta_i^{\frac{1+m_i}{m_i}}\right)^{\frac{m_i}{m_i-1}}\right] +$$

$$+\alpha_i g_i(\eta_i) + \eta_i\beta_i\left(-\,b_i\frac{1+m_i}{m_i-1}\right)\left(a_i - b_i\eta_i^{\frac{1+m_i}{m_i}}\right)^{\frac{1}{m_i-1}}\frac{1}{\eta_i^{\frac{1}{m_i}}} \le 0,$$

$$\eta_i^{1-N}\left(-\,b_i\frac{1+m_i}{m_i-1}\right)\eta_i^{N-1}\left[N\left(a_i - b_i\eta_i^{\frac{1+m_i}{m_i}}\right)^{\frac{m_i}{m_i-1}} - b_i\frac{1+m_i}{m_i-1}\eta_i^{\frac{1}{m_i}}\left(a_i - b_i\eta_i^{\frac{1+m_i}{m_i}}\right)^{\frac{1}{m_i-1}}\right] +$$

$$+\alpha_i g_i(\eta_i) + \eta_i\beta_i\left(-\,b_i\frac{1+m_i}{m_i-1}\right)\left(a_i - b_i\eta_i^{\frac{1+m_i}{m_i}}\right)^{\frac{1}{m_i-1}}\frac{1}{\eta_i^{\frac{1}{m_i}}} \le 0,$$

$$-\,\beta_i N\left(a_i - b_i\eta_i^{\frac{1+m_i}{m_i}}\right)^{\frac{m_i}{m_i-1}} + \eta_i\beta_i\frac{1+m_i}{m_i-1}\left(a_i - b_i\eta_i^{\frac{1+m_i}{m_i}}\right)^{\frac{1}{m_i-1}}\frac{1}{\eta_i^{\frac{1}{m_i}}} +$$

$$+\alpha_i g_i(\eta_i) - \eta_i\beta_i\frac{1+m_i}{m_i-1}\left(a_i - b_i\eta_i^{\frac{1+m_i}{m_i}}\right)^{\frac{1}{m_i-1}}\frac{1}{\eta_i^{\frac{1}{m_i}}} \le 0,$$

$$-\,\beta_i N g_i(\eta_i) + \alpha_i g_i(\eta_i) \le 0, \qquad g_i(\eta_i)\,(\alpha_i - \beta_i N) \le 0. \tag{9}$$

Shunday qilib, ko'rinib turibdiki $g_i(\eta_i)$ bo'lgani uchun $minN\beta_i - \alpha_i > 0$ bo'ladi.

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Ikki karra nochiziqli manba ega nodivergent ko'rinishdagi parabolik tenglamalar orqali tavsiflanuvchi issiqlik tarqalish tenglamasining yechim xossalari

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DOI 10.5281/zenodo.18217820

Annotatsiya

Mazkur maqolada ikki karra nochiziqli manbaga ega bo'lgan, nodivergent ko'rinishdagi parabolik issiqlik tarqalish tenglamasining umumlashgan yechimlari xossalari o'rganilgan.

Maqolada manba hadlari va o'zgaruvchan zichlikka ega nochiziqli diffuziya tenglamasi qaralib, klassik yechim mavjud bo'lmagan holatlar uchun integral ayniyat maвЂ™nosida umumlashgan yechim tushunchasi qo'llanilgan. O'zgaruvchini almashtirish va avtomodel yechimlar usuli yordamida masala nochiziqli parabolik tenglamadan oddiy differensial tenglamaga keltirilgan. Sekin diffuziya holida yechimlarning global mavjudligi solishtirish prinsipi asosida isbotlangan hamda Fujita tipidagi kritik eksponent aniqlangan. Olingan natijalar issiqlik o'tkazuvchanligi, kimyoviy va biologik jarayonlarni tavsiflovchi nochiziqli modellarni sonli yechish uchun muhim nazariy ahamiyatga ega.

Kalit so'zlar: global yechim, nodivergent tenglama, diffuziya, asimptotik yechim.

Kirish.

Nochiziqli parabolik tenglamalar, xususan, ikki karra nochiziqli va divergent, nodivergent diffuziya tenglamalari so'nggi yillarda intensiv o'rganilayotgan dolzarb yo'nalishlardan biridir. Samarskiy va hammualliflarining kitobida nochiziqli parabolik tenglamalar uchun (blow-up), global mavjudlik va asimptotik xatti-harakat masalalari tizimli ravishda yoritilgan bo'lib, keyingi tadqiqotlar uchun nazariy asos bo'lib xizmat qiladi [1-2].

Aripov va Sadullaeva monografiyasida nochiziqli diffuziya jarayonlarining kompyuter modellashtirish masalalari ko'rib chiqilib, nazariy natijalarni sonli tajribalar bilan tasdiqlash imkoniyatlari ko'rsatilgan. Aripov va Bobokandov tomonidan bajarilgan tadqiqotlarda esa o'zgaruvchan zichlikka ega, ikki karra nochiziqli parabolik tenglamalar uchun Koshi masalasi, umumlashgan yechimlarning mavjudligi va xossalari batafsil tahlil qilingan. Ushbu ishlar nodivergent ko'rinishdagi tenglamalar uchun umumlashgan yechim tushunchasini rivojlantirishga muhim hissa qo'shgan [1-4].

Andreucci va Tedeev [5] tomonidan Fujita tipidagi kritik eksponent masalasi divergent parabolik tenglamalar uchun o'rganilib, yechimlarning global mavjudligi va blow-up hodisasi o'rtasidagi chegaraviy shartlar aniqlangan. Jin va Yin [7] ishlarida esa ikki karra nochiziqli nodivergent ko'rinishdagi parabolik tenglamalar yechimlarining asimptotik xatti-harakati tadqiq etilgan. Zhou va hammualliflar [8] gradient hadli, nodivergent diffuziya tenglamalari uchun chegaraviy masalalarni o'rganib, yechimlarning baholari va sifat xossalarini aniqlagan.

Mahalliy tadqiqotlarda [9-10] Aripov, Bobokandov va boshqalar nodivergent tipdagi diffuziya tenglamalari uchun Koshi masalasi, kuchli va kuchsiz yechimlar, shuningdek, manba hadli ikki karra nochiziqli tenglamalar chuqur o'rganilgan. Ushbu adabiyotlar majmui shuni ko'rsatadiki, nodivergent ko'rinishdagi ikki karra nochiziqli parabolik tenglamalar uchun global yechimlar, ularning baholari va kritik eksponentlarini aniqlash masalasi hanuz dolzarbligini saqlab qolmoqda va mazkur maqoladagi tadqiqotlar ushbu yo'nalishni mantiqan davom ettiradi.

Masalani qo'yilishi $Q = \{(x,t) : x \in R^N,\ t > 0\}$ sohada manba hadi va o'zgaruvchan zichlikni o'z ichiga olgan, nodivergent ko'rinishdagi nochiziqli parabolik diffuziya tenglamasini qaraymiz:

$$|x|^{\alpha_1}\frac{\partial u}{\partial t} = u^{\beta}\frac{\partial}{\partial x}\left(|x|^{\alpha_2}u^{m-1}\left|\frac{\partial u}{\partial x}\right|^{p-2}\frac{\partial u}{\partial x}\right) + |x|^{\alpha_1}u^{q} \tag{1}$$

boshlang'ich shartlar bilan

$$u(x,t)\big|_{t=0} = u_0(x) \geq 0, \quad x \in \mathbb{R} \tag{2}$$

Bu yerda $\beta < 1, m \geq 1, p \geq max\{2, \alpha_2 - \alpha_1\}, q > 1, \alpha_1 < 0, \alpha_2 < 0$ sonli parametrlar bo'lib, $u_0(x)$ uzluksiz, manfiy bo'lmagan, chegaralangan funksiyadir. МаВЂ™lumki, (1) tenglama buziluvchi xarakterga ega bo'lib, $u(x,t) = 0$ yoki $|u_x| = 0$ holatlarda klassik маВЂ™noda yechim mavjud bo'lmasligi mumkin. Shu sababli, (1)-(2) masala uchun yechim integral ayniyat маВЂ™nosida aniqlanadigan umumlashgan yechim sifatida qaraladi. Bunda

$$0 \leq u, \ |x|^{\alpha_2} u^{m-1} |\frac{\partial u}{\partial x}|^{p-2} \frac{\partial u}{\partial x} \in C(R \times (t, +\infty))$$

shart bajarilishi talab etiladi va yechim (1) tenglamani integral tenglik ko'rinishida qanoatlantiradi. Qayd etish joizki, (1)-(2) masalada ishtirok etuvchi parametrlarning turli qiymatlarida ushbu model ko'plab issiqlik almashinuvi, kimyoviy reaksiyalar hamda biologik jarayonlarni tavsiflashda qo'llaniladi. [1-12].

Tadqiqot metodologiyasi

Dastlab, (1) tenglamada quyidagicha belshilash kiritamiz:

$$y = u^{1-\beta} \tag{3}$$

U holda (1), (2) masala quyidagi ko'rinishga keladi:

$$(1-\beta)L(y) = y^{\frac{\beta}{1-\beta}} \left[|x|^{\alpha_1} y_t - \frac{\partial}{\partial x} \left(|x|^{\alpha_2} y^{\frac{m}{1-\beta}-1} \left| \frac{\partial y^{\frac{1}{1-\beta}}}{\partial x} \right|^{p-2} \frac{\partial y}{\partial x} \right) + (1-\beta)|x|^{\alpha_1} y^{\frac{q-\beta}{1-\beta}} \right] = 0 \tag{4}$$

$$y(x,t)\big|_{t=0} = y_0(x) = \left[u_0(x)\right]^{1-\beta}, \qquad x \in \mathbb{R} \tag{5}$$

МаВЂ™lumki, $\beta(1-\beta)$ shart bajarilganda $y = 0$ funksiya (4) tenglamaning trivial yechimi hisoblanadi. Ushbu ishda tenglamaning trivial bo'lmagan yechimlarini aniqlash maqsad qilib olingan. Shu sababli, chiziqli bo'lmagan ajratish usulidan foydalanib, (4), (5) masalaning yechimini quyidagi avtomodel ko'rinishda izlaymiz:

$$y(x,t) = t^{-n_1} g(\eta), \qquad \eta = \frac{|x|}{t^{n_2}} \tag{6}$$

bu yerda $n_1 = \dfrac{1-\beta}{q-1}, n_2 = \dfrac{p+q-(\beta+m+2)}{(q-1)(p-\alpha_2+\alpha_1)}$. (6) ifodani hisobga inobatga olib, (4) tenglama avtomodel o'zgaruvchi orqali quyidagi oddiy differensial tenglamaga keltiriladi:

$$\left(\eta^{\alpha_2} g^{\frac{m+\beta-1}{1-\beta}} \left| \left(g^{\frac{1}{1-\beta}} \right)' \right|^{p-2} g' \right)' + n_2 \eta^{1+\alpha_1} g' + n_1 \eta^{\alpha_1} g + (1-\beta)\eta^{\alpha_1} g^{\frac{q-\beta}{1-\beta}} = 0 \tag{7}$$

Hosil bo'lgan (7) avtomodel tenglama uchun manfiy bo'lmagan, trivial bo'lmagan va fizik mazmunga ega yechimlarni qidiramiz. Bunda yechim quyidagi boshlang'ich shartlarni qanoatlantirishi talab etiladi:

$$g'(0) = g(d) = 0, \qquad d \in \mathbb{R}_+ \tag{8}$$

Olingan natijalar.

Agar $\frac{m+p-2}{1-\beta} - 1 > 0$ shart bajarilsa, ushbu holat sekin diffuziya rejimi sifatida talqin qilinadi. Bu holda (1) tenglama uchun umumlashgan yechimlarning global mavjudligi solishtirish prinsipi asosida isbotlanadi. (4)-(5) masala (1)-(2) masalaga ekvivalent bo'lganligi sababli, keyingi tahlil aynan (4)-(5) masala doirasida olib boriladi. Solishtirish prinsipi natijasida (4)-(5) masalaning yuqori baholovchi yechimi quyidagi avtomodel ko'rinishda aniqlanadi:

$$y_+(x,t) = t^{-n_1}\bar{g}(\eta) \tag{9}$$

$$\bar{g}(\eta) = A(a - \eta^{\gamma_1})_+^{\gamma_2} \tag{10}$$

bu yerda $\gamma_1 = \frac{p-\alpha_2+\alpha_1}{p-1}, \gamma_2 = \frac{(p-1)(1-\beta)}{m-p+\beta+1}, a \geq 0,$

$$A = [\frac{\left|n_2(\frac{1}{1-\beta})^{2-p}\right|^{\frac{1}{p-1}}}{\gamma_1\gamma_2}]^{\gamma_2}, \gamma_1 \neq 0, \gamma_2 \neq 0 \tag{11}$$

1-Teorema. Agar quyidagi shartlardan kamida bittasi bajarilsa $\alpha_1 < -1, \mu_1 > \mu_2$ yoki $\alpha_1 = -1, \mu_1 > 1$, hamda $y(x,t) \leq y_+(x,t), x \in R$ munosabat o'rinli bo'lsa, u halda Q sohada (4), (5) masalaning global umumlashgan yechimi mavjud bo'ladi va u quyidagi bahoni qanoatlantiradi

$$y(x,t) \leq y_+(x,t) = t^{-n_1}\bar{g}(\eta) \tag{12}$$

Bu yerda $\mu_1 = \frac{q-\beta}{1-\beta}, \mu_2 = \frac{m+p-2}{1-\beta} + \frac{p-\alpha_2+\alpha_1}{\alpha_1+1}$

bo'lib, Fujita tipidagi kritik eksponent hisoblanadi.

Isbot. Taqqoslash funksiyasi sifatida (10) tenglama bilan aniqlangan funksiyani tanlaymiz. Ushbu funksiyani (4) tenglamaga qo'yib, quyidagi baholashni olamiz:

$$(1-\beta)Ly = -y_+^{\frac{\beta}{1-\beta}}[(\eta^{\alpha_2}g^{-\frac{m}{1-\beta}-1}\left|(g^{-\frac{1}{1-\beta}})'\right|^{p-2}\bar{g}')' + \tag{13}$$

$$n_2\eta^{1+\alpha_1}\bar{g}' + n_1\eta^{\alpha_1}\bar{g} + (1-\beta)\eta^{\alpha_1}\bar{g}^{\mu_1}] \leq 0$$

Bu yerda $y_+(x,t)$ funksiyaning manfiy bo'lmaganligi hamda $\bar{g}(\eta)$ funksiyaning

$$(\eta^{\alpha_2}g^{-\frac{m}{1-\beta}}\left|(g^{-\frac{1}{1-\beta}})'\right|^{p-2}\bar{g}') + n_2\eta^{1+\alpha_1}\bar{g}' + n_2(\alpha_1+1)\eta^{\alpha_1}\bar{g} = 0$$

tenglamani qanoatlantirishini hisobga olib, (13) tengsizlikni quyidagi ko'rinishga keltiramiz:

$$\eta^{\alpha_1}\bar{g}[n_1 - n_2(\alpha_1+1) + (1-\beta)g^{-\mu_1-1}] \geq 0 \tag{14}$$

η^{α_1} va $\bar{g}$ funksiyalarning manfiy bo'lmaganligi hamda $\beta < 1$ shart bajarilishini inobatga olsak, (14) tengsizlik bajarilishi uchun quyidagi shart yetarli bo'ladi: $n_1 \geq n_2(\alpha_1+1)$

Agar, $\alpha_1 + 1 = 0$ bo'lsa, u holda $0 \leq n_1 = \frac{1}{\mu_1 - 1}$ tengsizlik avtomatik ravishda bajariladi. Olingan tengsizliklar teorema shartlarida keltirilgan munosabatlar bilan mos keladi. Demak, $y(x,t) \leq y_+(x,t)$ bahosi o'rinli bo'ladi. Teorema isbotlandi.

Xulosa

Mazkur maqolada ikki karra nochiziqli manbaga ega bo'lgan nodivergent ko'rinishdagi parabolik issiqlik tarqalish tenglamasi uchun Koshi masalasi o'rganildi. Klassik yechim mavjud bo'lmagan holatlarda umumlashgan yechim tushunchasi kiritilib, masala integral ayniyat маВЂ™nosida qaraldi. O'zgaruvchini mos almashtirish va avtomodel yechimlar usuli yordamida masala soddalashtirilib, tenglamaning sifat xossalarini tahlil qilish imkoniyati yaratildi. Sekin diffuziya holida solishtirish prinsipi asosida yechimlarning global mavjudligi isbotlandi hamda Fujita tipidagi kritik eksponent aniqlanib, yechimlarning global saqlanishi o'rtasidagi chegaraviy shartlar belgilandi. Olingan natijalar nodivergent tipdagi nochiziqli parabolik tenglamalar nazariyasini rivojlantiradi hamda issiqlik almashinuvi, kimyoviy va biologik jarayonlarni modellashtirishda qo'llash uchun nazariy asos bo'lib xizmat qiladi.

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Ellikqal'a tumani misolida 2018–2024–yillar oralig'ida tuproq sho'rlanishi va iqlim omillarining o'zaro bog'liqligini tahlil qilish

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DOI 10.5281/zenodo.18217840

Kalit so'zlar: sho'rlanish, bug'lanish, namlik, NASA POWER, FAO WaPOR, Ellikqal'a tumani, regressiya modeli, GIS tahlil, iqlim determinantlari

Annotatsiya. Mazkur maqolada Qoraqalpog'iston Respublikasi Ellikqal'a tumanida 2018-2024-yillar oralig'ida tuproq sho'rlanishi va iqlim omillari o'rtasidagi o'zaro bog'liqlik empirik ma'lumotlar asosida tahlil qilingan. Tadqiqotda NASA POWER va FAO WaPOR ma'lumotlar bazalaridan olingan oylik meteorologik (harorat, yog'in, bug'lanish, namlik) va gidroklimatik parametrlar, shuningdek, sho'rlanish (EC) qiymatlari ishlatilgan. Tahlil jarayonida ko'p manbali ma'lumotlar GIS muhitida integratsiya qilinib, regressiya modellari (MLR) yordamida iqlimiy determinantlarning sho'rlanish jarayoniga ta'sir darajasi aniqlangan. Natijalarga ko'ra, harorat (T) va bug'lanish (RET) sho'rlanishning asosiy ijobiy determinantlari sifatida, yog'in (PCP) esa manfiy determinant sifatida baholangan. 2018-2024-yillar oralig'ida suv balansining manfiy tendensiyasi (RET $>$ AETI) kuzatilgan bo'lib, bu sho'rlanish jarayonining kuchayganini ko'rsatadi. GIS asosida qurilgan fazoviy xaritalar sho'rlanishning asosan g'arbiy va janubiy massivlarda kuchayganini ko'rsatadi.

Tadqiqot natijalari hududiy meliorativ boshqaruv, suv resurslaridan oqilona foydalanish va qishloq xo'jaligi ekinlarini joylashtirish bo'yicha amaliy qarorlar qabul qilishda ilmiy asos bo'lib xizmat qiladi.

Kirish. O'zbekiston Respublikasi Prezidentining 2022-yil 25-martdagi PQ-179-son qarori agrar sohada barqaror rivojlanishni ta'minlash, suv va yer resurslaridan samarali foydalanish hamda sug'oriladigan yerlarning meliorativ holatini yaxshilashga qaratilgan muhim strategik hujjatdir. Ushbu qarorda ilmiy asoslangan agrotexnologiyalarni joriy etish, sug'orish tizimlarini raqamlashtirish va tuproq sho'rlanishini kamaytirish ustuvor vazifa sifatida belgilangan. Mazkur tadqiqot ana shu yo'nalishlarga mos holda, real kuzatuv ma'lumotlari asosida sho'rlanish va namlik jarayonlarini empirik tahlil qilishga qaratilgan.

Tuproq sho'rlanishi O'zbekistonning sug'oriladigan hududlarida qishloq xo'jaligi barqarorligini cheklovchi asosiy omillardan biri bo'lib, bu muammo Qoraqalpog'iston Respublikasi Ellikqal'a tumanida ayniqsa dolzarbdir. Hududning keskin kontinental iqlimi, kam yog'in va kuchli bug'lanish sharoitida suv balansining buzilishi grunt suvlarining sathi ko'tarilishiga va tuproqning ustki qatlamlarida tuzlar to'planishiga olib keladi. Shu sababli sho'rlanish jarayonining vaqtli dinamikasi va hududiy taqsimotini aniqlash muhim ilmiy-amaliy vazifa hisoblanadi.

So'nggi yillarda ushbu jarayonlarni modellashtirish bo'yicha xalqaro tadqiqotlar olib borilgan bo'lsa-da [1], ularning aksariyati nazariy yondashuvlarga asoslangan. Ushbu ish esa Ellikqal'a tumani bo'yicha 2018-2024-yillar real kuzatuv ma'lumotlariga tayanganligi bilan farqlanadi. Tadqiqotda tuproqning elektr o'tkazuvchanligi (EC) va asosiy meteorologik omillar o'rtasidagi bog'liqlik ko'p omilli regressiya modeli orqali baholanib, natijalar GIS asosida fazoviy tahlil qilinadi. Tadqiqot natijalari suv-meliorativ boshqaruvni takomillashtirish uchun ilmiy asos bo'lib xizmat qiladi hamda kelgusida [6-8] adabiyotlarda keltirilgan matematik va sun'iy intellekt asosidagi modellar bilan kengaytirilishi rejalashtirilgan.

Ma'lumotlar va tadqiqot usullari. Tadqiqot obyekti sifatida Qoraqalpog'iston Respublikasining Ellikqal'a tumani tanlandi. Ushbu hudud Orolbo'yi havzasida joylashgan bo'lib, tuproq sho'rlanishi va namlik tanqisligi eng keskin namoyon bo'ladigan mintaqalardan biridir. Keskin kontinental iqlim sharoitida yillik yog'in miqdorining bug'lanishdan ancha kam bo'lishi suv balansining buzilishiga olib keladi va sho'rlanish jarayonlarini kuchaytiradi. Natijada grunt suvlari sathining ko'tarilishi tuproqning ustki qatlamlarida tuzlar to'planishini jadallashtiradi. Ellikqal'a tumani sug'oriladigan dehqonchilikka ixtisoslashgan bo'lib, asosan paxta, g'alla va yem-xashak ekinlari yetishtiriladi.

So'nggi yillarda sho'rlanish darajasining oshishi, sug'orish infratuzilmasining eskirishi va suv resurslarining qisqarishi hududda qishloq xo'jaligi barqarorligiga salbiy ta'sir ko'rsatmoqda. Shu sababli sho'rlanish jarayonini ilmiy asosda monitoring qilish, uning iqlim omillari bilan bog'liqligini aniqlash va xavfli hududlarni fazoviy baholash dolzarb vazifa hisoblanadi. Mazkur tadqiqot O'zbekiston Respublikasi Prezidentining 2022-yil 25-martdagi PQ-179-son qarorida belgilangan suv resurslaridan samarali foydalanish, yerlarning meliorativ holatini yaxshilash va raqamli monitoring tizimlarini joriy etish bo'yicha ustuvor vazifalarga mos holda amalga oshirildi.

Tadqiqot 2018-2024-yillar oralig'ida to'plangan meteorologik va tuproq sho'rlanishi ma'lumotlariga asoslandi. Meteorologik parametrlar - o'rtacha harorat, yog'in miqdori, nisbiy namlik, bug'lanish va shamol tezligi - NASA POWER, Copernicus ERA5 va UzHydromet ochiq ma'lumotlar bazalaridan olindi[9]. Barcha ma'lumotlar oylik vaqt qatorlariga keltirilib, o'zaro moslashtirildi hamda yetishmayotgan qiymatlar interpolatsiya usullari yordamida to'ldirildi.

Tuproq sho'rlanishi darajasi tuproqning elektr o'tkazuvchanligi (EC) ko'rsatkichi orqali baholandi. EC qiymatlari 2018 va 2024-yillarda o'tkazilgan dala va laboratoriya tadqiqotlari asosida aniqlanib, 0-30 sm chuqurlik oralig'i uchun standartlashtirildi. Qo'shimcha aniqlikni ta'minlash maqsadida SoilGrids va FAO ma'lumotlar bazalaridan olingan global sho'rlanish ko'rsatkichlari ham integratsiya qilindi.

Fazoviy tahlilni amalga oshirish uchun relyef, yer qoplami va sug'orish tarmoqlariga

oid GIS qatlamlari ochiq geoma'lumotlar bazalaridan olinib, yagona koordinata tizimida qayta ishlanib, Ellikqal'a tumani chegaralari doirasida moslashtiriladi.

Ma'lumotlarni qayta ishlash va tahlil bosqichlari. Tahlil ishlari ketma-ket bosqichlarda amalga oshirildi. Dastlab 2018-2024-yillar oralig'idagi asosiy meteorologik parametrlarning (harorat, yog'in va bug'lanish) vaqt bo'yicha o'zgarishi tahlil qilinib, chiziqli trendlar yordamida baholandi. Natijalar o'rganilgan davrda o'rtacha haroratning oshishi, yog'in miqdorining kamayishi hamda bug'lanish intensivligining kuchayganini ko'rsatdi, bu esa namlik tanqisligini yuzaga keltirgan.

Keyingi bosqichda tuproq sho'rlanishini ifodalovchi elektr o'tkazuvchanlik (EC) ko'rsatkichi bilan asosiy meteorologik omillar o'rtasidagi bog'liqlik Pearson va Spearman korrelyatsiya koeffitsientlari yordamida aniqlandi. Tahlil natijalariga ko'ra, harorat, bug'lanish va ET/P nisbati sho'rlanish jarayoniga eng sezgir determinantlar sifatida ajratildi.

Sho'rlanishning iqlimiy omillar bilan miqdoriy bog'liqligini baholash uchun ko'p omilli chiziqli regressiya (Multiple Linear Regression, MLR) modeli qo'llanildi va u quyidagi umumiy ko'rinishda ifodalanadi:

$$EC = \alpha_0 + \alpha_1 T + \alpha_2 P + \alpha_3 ET + \alpha_4 + \varepsilon$$

bu yerda EC - tuproqning elektr o'tkazuvchanligi, T - o'rtacha harorat, P - yog'in miqdori, ET - bug'lanish, RH - nisbiy namlik, $\alpha_0, ..., \alpha_4$ - regressiya koeffitsientlari, ϵ - tasodifiy xatolik. Model aniqligi R^2, RMSE, va MAE ko'rsatkichlari yordamida baholandi. Ushbu model sho'rlanishning asosiy iqlimiy determinantlariga sezgirligini aniqlash va bashorat asoslarini shakllantirishga xizmat qildi.

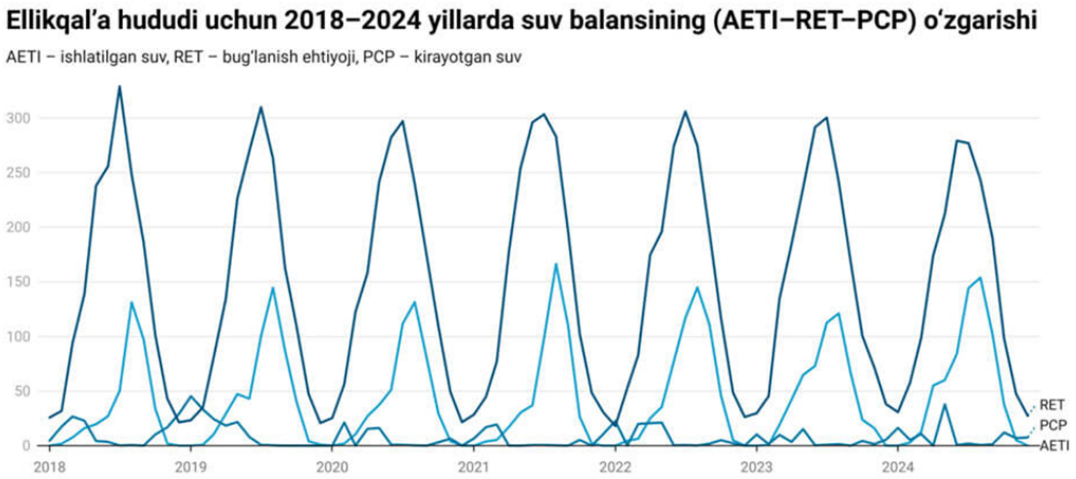
Fazoviy tahlil uchun ArcGIS va QGIS dasturlari qo'llanilib, 2018 va 2024-yillarga oid EC qiymatlari asosida fazoviy interpolatsiya amalga oshirildi. Interpolatsiya usullari sifatida Inverse Distance Weighting (IDW) va Ordinary Kriging ishlatildi. Har ikki yil uchun EC xaritalari qurilib, o'zgarish farqi quyidagi formula bilan ifodalandi:

$$\Delta EC = EC_{2024} - EC_{2018}$$

Fazoviy tahlil natijasida sho'rlanish darajasi o'zgarayotgan hududlar aniqlanib, ularning tabiiy va sug'orish sharoitlari bilan bog'liqligi baholandi. Natijalar sho'rlanish jarayonining hudud bo'yicha notekis taqsimlanganini ko'rsatdi. GIS asosida yaratilgan xaritalar suv-meliorativ va agrotexnik qarorlarni asoslashda amaliy ahamiyatga ega bo'lib, taklif etilgan yondashuv kelgusida qo'shimcha ko'rsatkichlar va regressiya modellarini integratsiya qilish uchun metodik asos bo'lib xizmat qiladi.

Natijalar va muhokama. Ellikqal'a tumani uchun 2018-2024-yillar oralig'ida oylik suv balansining asosiy komponentlari - AETI (haqiqiy bug'lanish va intersepsiya), RET (potensial bug'lanish ehtiyoji) va PCP (yog'in miqdori) - tahlil qilindi. Tahlil natijalari RET qiymati har yili iyun-avgust oylarida maksimal (300-320 mm/oy) darajaga yetadi, bu esa hududda suv tanqisligining kuchayishini bildiradi. AETI esa RETdan ancha past qiymatlarda (100-140 mm/oy) o'zgaradi, bu tuproq namligi cheklanganligini va namlik defitsiti mavjudligini 1-rasmda 2018-2024 yillarda Ellikqal'a tumanida suv balansi o'zgarishining dinamikasi ko'rsatilgan.

Grafikdan ko'rinadiki, har yili yoz oylarida RET $>$ AETI $\gg$ PCP tengsizlik kuzatiladi, bu iqlimiy suv defitsiti ($\Delta ET \approx 200\,\text{mm/oy}$) mavjudligini bildiradi. Mazkur



1-rasm. 2018-2024-yillarda AETI, RET va PCP ning oylik o'zgarishi

nomutanosiblik, ayniqsa 2021 va 2023-yillarda kuchayib, bug'lanishning oshishi sho'rlanish jarayonini tezlashtirgan.

Sho'rlanish darajasining hozirgi holati (2024-yil holatiga ko'ra). Qoraqalpog'iston Respublikasi Sho'rlangan yerlar bo'yicha 2024-yilgi ma'lumotlar Ellikqal'a tumani uchun sho'rlanish darajasining yuqoriligini ko'rsatmoqda. Tuman bo'yicha jami 29 052 gektar sug'oriladigan maydonning 86.5 % qismi turli darajada sho'rlangan.

1-jadval. 2024-yil holatiga ko'ra Ellikqal'a tumanida sho'rlangan yerlarning toifalar bo'yicha taqsimoti

Sho'rlanish toifasi	**Maydon (ga)**	**Ulushi (%)**
Kuchsiz sho'rlangan	11 520.3	37.7
O'rtacha sho'rlangan	10 349.1	33.5
Kuchli sho'rlangan	2 024.4	6.5
Juda kuchli sho'rlangan	1 111.8	3.8
Jami sho'rlangan yerlar	25 139.9	86.5

Fazoviy tahlil va GIS natijalari. 2018 va 2024-yillarga oid tuproq elektr o'tkazuvchanligi (EC) qiymatlari asosida fazoviy interpolatsiya IDW va Ordinary Kriging usullari yordamida amalga oshirildi[5]. Natijada sho'rlanish darajasining har ikki yil uchun hududiy taqsimoti xaritalar ko'rinishida aniqlandi.

2018-yilgi EC xaritasi tuman markaziy va shimoliy hududlarida sho'rlanish nisbatan pastligini, janubiy va g'arbiy qismlarda esa o'rtacha sho'rlanish zonalarini ko'rsatdi. 2024-yilga kelib, sho'rlanish darajasi tuman bo'ylab oshgani, ayniqsa Amudaryo o'zaniga yaqin pasttekisliklarda kuchli sho'rlanish zonalari kengaygani aniqlandi.

$\Delta EC = EC_{2024} - EC_{2018}$ xaritasi sho'rlanish o'zgarishining fazoviy dinamikasini aks ettirdi. Natijalarga ko'ra, sho'rlanish ortgan maydonlar asosan janubiy va g'arbiy

hududlarda joylashib, umumiy maydonning 64 % ini tashkil etdi. Sho'rlanish kamaygan hududlar esa markaziy irrigatsiya tarmoqlari yaxshilangan zonalarda kuzatilib, 36 % ni tashkil qildi. Bu holat bug'lanishning ortishi bilan birgalikda meliorativ tizimlar samaradorligi yetarli emasligini ko'rsatadi.

Regressiya modelining natijalari. Sho'rlanish va iqlimiy omillar o'rtasidagi miqdoriy bog'liqlikni baholash uchun ko'p omilli chiziqli regressiya (MLR) modeli qurildi, bunda EC bog'liq o'zgaruvchi, harorat, yog'in, bug'lanish va nisbiy namlik esa tushuntiruvchi omillar sifatida kiritildi.

$$EC = \alpha_0 + \alpha_1 T + \alpha_2 P + \alpha_3 ET + \alpha_4 + \varepsilon$$

Natijada quyidagi asosiy koeffitsientlar va aniqlik ko'rsatkichlari olindi:

Таблица 3: 2-jadval. Regressiya koeffitsientlari

Parametr	**Belgilanish**	**Koeffitsient**	**Ta'sir yo'nalishi**
Harorat (°C)	a_1	+0.42	Musbat — harorat oshganda EC ortadi
Yog'in (mm)	a_2	−0.31	Manfiy — yog'in ortganda EC kamayadi
Bug'lanish (mm)	a_3	+0.56	Kuchli ijobiy — bug'lanish sho'rlanishni kuchaytiradi
Nisbiy namlik (%)	a_4	−0.18	Zaif manfiy — namlik oshsa EC pasayadi

Modelning aniqlik ko'rsatkichlari quyidagicha:

$$R^2 = 0.82, RMSE = 0.13, MAE = 0.09,$$

bu esa model 80 % dan ortiq dispersiyani tushuntira olishini bildiradi. Bundan ko'rinadiki, Ellikqal'a tumanida sho'rlanish jarayoni asosan bug'lanish (ET) va harorat (T) bilan bog'liq, yog'in va nisbiy namlik esa sho'rlanishni pasaytiruvchi omil sifatida ishtirok etadi.

Muhokama va amaliy talqin. Olingan natijalar xalqaro tadqiqotlar bilan mos keladi[11]. Jumladan, yuqori harorat va past namlik sharoitida bug'lanishning kuchayishi sho'rlanish jarayonini tezlashtirishi avvalgi ishlarida ham qayd etilgan. PHREEQC asosidagi modellashtirish natijalari ham eritmalar harakati sho'rlanishga bevosita ta'sir ko'rsatishini tasdiqlaydi. Ellikqal'a tumanida aniqlangan $AETI < RET$ holati suv balansining manfiy ekanini ko'rsatib, sho'rlanish jarayonining faollashishini izohlaydi. Bu sharoit hududda meliorativ boshqaruv choralarini kuchaytirish zarurligini ko'rsatadi. Olingan natijalar asosida quyidagi amaliy yo'nalishlar tavsiya etiladi: sug'orishda suvni tejovchi texnologiyalarni keng joriy etish, drenaj tizimlarini takomillashtirish va raqamli monitoring bilan integratsiya qilish, shuningdek, GIS asosidagi xavf zonalari xaritalaridan agrotexnik qarorlar qabul qilishda foydalanish. Kelgusida sho'rlanish monitoringini sun'iy intellekt va masofaviy zondlash ma'lumotlari bilan kengaytirish maqsadga muvofiq.

Xulosa. 2018-2024-yillar oralig'ida olib borilgan tahlillar Ellikqal'a tumanida tuproq sho'rlanishi dinamikasining barqaror o'sish tendensiyasini ko'rsatdi. Sho'rlanishning ayniqsa g'arbiy va janubiy hududlarda kuchaygani aniqlanib, bu holat hududiy suv balansidagi nomutanosiblik bilan bog'liq ekanligi qayd etildi.

Natijalarga ko'ra, bug'lanish intensivligi (RET) va havo harorati (T) sho'rlanish jarayoniga eng katta ta'sir ko'rsatuvchi asosiy iqlimiy omillar hisoblanadi. GIS asosidagi fazoviy tahlil sho'rlanish xavfi yuqori bo'lgan zonalarni aniqlash imkonini berib, meliorativ tadbirlarni hududiy darajada rejalashtirish uchun ilmiy asos yaratdi.

Taklif etilgan yondashuv kelgusida tuproq namligi, vegetatsiya indekslari va sun'iy intellekt asosidagi modellar bilan kengaytirilishi mumkin. Ushbu natijalar Ellikqal'a tumani va o'xshash iqlim sharoitidagi hududlarda suv resurslaridan oqilona foydalanish, sug'orish jarayonlarini optimallashtirish va sho'rlanish xavfini kamaytirishga xizmat qiladi.

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Irratsional ifodalar qatnashgan tenglamalar sistemasi va uni yechishda GeoGebra dasturidagi tahlili

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DOI 10.5281/zenodo.18217855

Annotatsiya.

Ushbu maqolada biz irratsional ifodalar qatnashgan tenglamalar sistemasini va uni yechish tahlilini ko'rib chiqamiz. Shu bilan birga irratsional ifodalar qatnashgan tenglamalar sistemasini yechishda funksialar grafiklari va ularning kesishish nuqtalaridan yechim topish orqali analitik hamga grafik usulda yechim topishning qulayliklarini tahlil qilamiz.

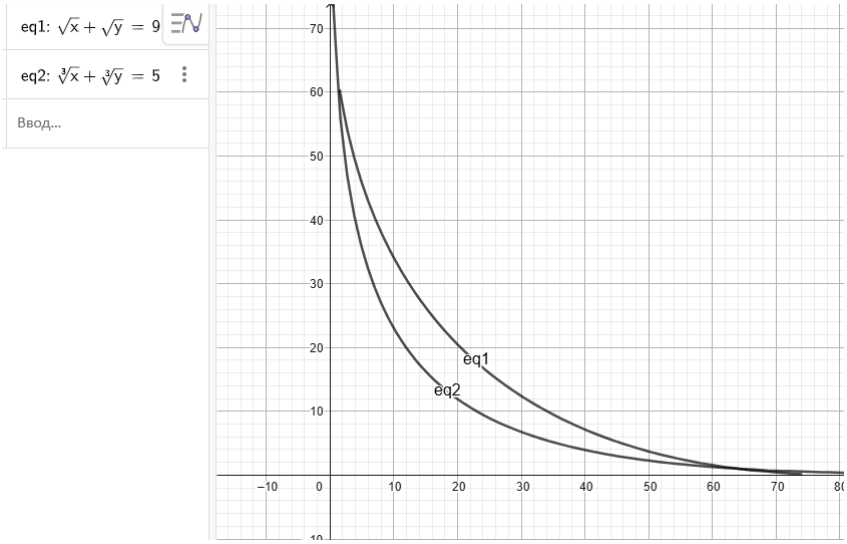
Kalit so'zlar: irratsional ifoda, tenglamalar sistemasi, funksialar grafiklarini chizish, Geogebra dasturi.

Kirish

Biz quyida irratsional ifoda qatnashgan noodatiy tenglamalar sistemasini va uni yechishning ikki hil ya'ni analitik hamda grafik usullarini ko'rib chiqamiz. Shuning ushbu tenglamalar sistemasini yechishda bir nechta masalan o'zgaruvchi kiritish, to'la kvadratga keltirish usullaridan foydalanamiz. **Misol.** Quyidagi tenglamalar sistemasining yechimini toping:

$$W: \begin{cases} \sqrt{x} + \sqrt{y} = 9, \\ \sqrt[3]{x} + \sqrt[3]{y} = 5. \end{cases}$$

Yechish. Avvalo, tenglamalar sistemasidagi har bir tenglamani funksiya sifatida qabul qilib, GeoGebra dasturida grafiklarini chizamiz.



Grafikdan ko'rinadiki, tenglamalar sistemasi ikkita yechimga ega.

Endi ushbu sistemani analitik usulda yechamiz. Quyidagi belgilashlarni kiritamiz:

$$\sqrt[6]{x} = t, \qquad \sqrt[6]{y} = z.$$

Shunda sistema quyidagi ko'rinishga keladi:

$$W : \begin{cases} t^3 + z^3 = 9, \\ t^2 + z^2 = 5. \end{cases}$$

To'la kub va to'la kvadrat formulalaridan foydalanamiz:

$$\begin{cases} (t+z)^3 - 3tz(t+z) = 9, \\ (t+z)^2 - 2tz = 5. \end{cases}$$

Endi quyidagi belgilashlarni kiritamiz:

$$t + z = A, \qquad tz = B.$$

Natijada sistema:

$$\begin{cases} A^3 - 3AB = 9, \\ A^2 - 2B = 5. \end{cases}$$

Ikkinchi tenglamadan:

$$B = \frac{A^2 - 5}{2}.$$

Bu ifodani birinchi tenglamaga qo'yib, quyidagi tenglamaga kelamiz:

$$(A-3)(A^2 + 3A - 6) = 0.$$

Endi

$$(A-3) = 0.$$

bo'lgan holni tahlil qilamiz.Bundan esa,

$$A = 3, B = 2.$$

ekanligi kelib chiqadi. Bularni yuqoridagi belgilashlarga qo'yib tenglamalarning yechimlarini topsak, u holda

$$(1, 64); (64, 1).$$

ga teng bo'lgan ikkita yechimga kelamiz. Yuqoridagi chizmadan ko'rinib turibdiki, bu topilganlari tenglamalar sistemasining haqiqiy yechimlari va haqiqiy sonlar to'plamida bu ikkita yechimdan boshqa yechimlari mavjud emas. Shuning uchun

$$(A^2 + 3A - 6) = 0.$$

bo'lgan holning tahlilini tekshirish shart emas.

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Statistika tahlilida Python dasturidan foydalanish

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DOI 10.5281/zenodo.18217870

Qisqacha annotatsiya: statistika, sonli o'lchov, markaziy tendensiya, arifmetik o'rtacha, mediana, moda, tarqoqlik(dispersiya), asimmetriya, cho'zilganligi, musbat asimmetriya, manfiy asimmetriya, asimmetriya koeffitsiyenti, Python

Statistikaning sonli o'lchovlari nima?

Sonli o'lchovlar — bu ma'lumotlar to'plamini bitta yoki bir nechta son orqali tavsiflovchi ko'rsatkichlar.

Ular asosan 3 guruhga bo'linadi:

- **Markaziy tendensiya o'lchovlari**
- **Tarqoqlik (dispersiya) o'lchovlari**
- **Shakl o'lchovlari** (asimmetriya, cho'qqililik)

Markaziy tendensiya o'lchovlari

1. Arifmetik o'rtacha. Arifmetik o'rtacha ma'lumotlarning yig'indisini ularning hajmiga nisbati

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Bu ma'lumotlarning "o'rtacha qiymati" hisoblanib, har bir ma'lumot shu qiymatni bir xilda qabul qiladi deb hisoblash mumkin bo'lgan kattalik.

Masalan, 2024-2025 o'quv yili uchun o'tish ballarni:

62.3, 58.9, 75.4, 67.6, 68.5, 111.0, 125.6, 102.5, 176.2, 148.3

deb olsak, bu ma'lumotlar uchun o'rtacha $\bar{x} = 99.63$ tashkil qiladi.

Bu ma'lumotlarni Python dasturi yordamida quyidagi kodlar yordamida hisoblash mumkin:

import statistics

data = [62.3, 58.9, 75.4, 67.6, 68.5, 111.0, 125.6, 102.5, 176.2, 148.3]

print(statistics.mean(data))

2. Mediana. Ma'lumotlarni o'sish yoki kamayish tartibida yozilganda uning o'rtadagi qiymati. Agar ma'lumotlar toq sonda bo'lsa uning medianasi o'rtadagi qiymati bo'ladi. Agar ma'lumotlar juft sonda bo'lsa, ularning o'rtasidagi ikki qiymati yig'indisini yarmi bo'ladi.

Masalan, quyidagi tanlanma olingan: 3, 5, 4, 4, 5, 3, 5, 4, 5, 4, 4 – bu tanlanma uchun medianani aniqlashda avval o'sish tartibida yozib olinadi. 3, 3, 4, 4, 4, 4, 4, 5, 5, 5, 5 – bu ifodani *variatsion qator* deyiladi. Tanlanma 11 ta bo'lgani uchun o'rtadagi element 6-element bo'lib, u 4 ga teng. Demak, mediana 4 ga teng ekan.

Python dasturida tanlanmani variatsion qatorida yozish quyidagicha

data = [3, 5, 4, 4, 5, 3, 5, 4, 5, 4, 4]

variatsion_ qator = sorted(data)

print(variatsion_ qator).

Medianani esa

print(statistics.median(data))

kod orqali yoziladi.

3. Moda. Tanlanmadagi eng ko'p qatnashgan varianta(tanlanmani har bir elementi) moda deyiladi. Quyidagi tanlanma olingan: 3, 5, 4, 4, 5, 3, 5, 4, 5, 4, 4 – bu tanlanma uchun moda 4. Python dasturida quyidagicha aniqlanadi:

print(statistics.mode(data))

kodi orqali yoziladi.

4. Tarqoqlik (dispersiya) o'lchovlari.

Dispersiya - ma'lumotlarning o'rtacha qiymat (matematik kutilish) atrofida anchalik tarqoq yoki zich joylashganini o'lchaydi, ya'ni o'zgaruvchanlik darajasini ko'rsatadi.

Oddiyroq qilib aytganda, ma'lumotlar guruhidagi sonlar o'rtacha son atrofida qanchalik tarqoqlik bilan joylashganini anglatadi.

Formulasi

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

bu yerda, n-tanlanma hajmi. Dispersiya Python dasturida quyidagi kod orqali aniqlanadi

print(statistics.var(data))

tanlanma dispersiya uchun ddof=1, ya'ni

np.var(data, ddof=1)

Bosh to'plam dispersiyasi

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

bu yerda n-bosh to'plam hajmi.

Python yordamida quyidagi kod orqali hisoblanadi:

np.var(data)

Asimmetriya (skewness)– bu ma'lumotlar taqsimotining simmetrik emasligini, ya'ni qiymatlarning o'rtacha atrofida qaysi tomonga ko'proq "cho'zilganini" ko'rsatuvchi muhim ko'rsatkichdir.

Agar chap va o'ng tomonlar taqsimotlar teng (*simmetrik taqsimot*) ya'ni o'rtacha, moda, mediana teng $\bar{x} = M_0 = M_e$ bo'lsa, asimmetriya $\approx$ bo'ladi. O'ngga og'ish bo'lsa, $\bar{x} > M_e > M_0$ *musbat asimmetriya* bo'ladi. Chapga og'ish bo'lsa (ya'ni $\bar{x} < M_e < M_0$), u holda *manfiy asimmetriya* bo'ladi.

Formulasi

$$\gamma_1 = \frac{E[(X-\mu)^3]}{\sigma^3}$$

bu yerda X-tasodifiy miqdor, μ, E-matematik kutilma(o'rtacha qiymat), σ -standart og'ish

$\gamma_1 > 0 \rightarrow$ o'ngga og'gan

$\gamma_1 < 0 \rightarrow$ chapga og'gan

$\gamma_1 = 0 \rightarrow$ simmetrik.

Quyidagi $4, 3, 3, 4, 4, 5, 4, 4, 3, 4, 5, 4$ tanlanma uchun $\bar{x} \approx 3.9, M_0 = 4, M_e = 4$, dispersiyasi $s^2 \approx 0.447$ va asimmetriya koeffitsiyenti $\gamma_1 \approx 0.11$ ga teng. Mana shu kattaliklarni Python yordamida hisoblash quyidagicha:

import numpy as np

from scipy import stats

x = np.array([4, 3, 3, 4, 4, 5, 4, 4, 3, 4, 5, 4])

var_ qator = np.sort(x)

mean = np.mean(x)

mode = stats.mode(x, keepdims=True).mode[0]

median = np.median(x)

variance = np.var(x, ddof=1)

skewness = stats.skew(x)

print("Variatsion qator: var_ qator)

print("O'rtacha qiymat: mean)

print("Moda: mode)

print("Mediana: median)

print("Dispersiya: variance)

print("Asimmetriya: skewness)

Shu kodlarni python dasturiga kiritish orqali quyidagi natijalarni olishimiz mumkin.

Variatsion qator: [3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 5, 5]

O'rtacha qiymat: 3.916 , Moda: 4 , Mediana: 4.0 , Dispersiya: 0.44 Asimmetriya: 0.075

Asimmetriya koeffitsiyentini *Fisher-Pearson* formulasi yordamida hisoblash mumkin. Formulasi

$$\gamma_1 = \frac{n}{(n-1)(n-2)} \frac{\sum (x_i - \bar{x})^3}{s^3}$$

Bu formula yordamida hisoblab $\gamma_1 \approx 0.086$?$_1$?$0,086$ natijani olish mumkin.

Bu natijani Python yordamida quyidagi kod orqali olish mumkin:

import numpy as np

from scipy.stats import skew

x = np.array([4,3,3,4,4,5,4,4,3,4,5,4])

print(skew(x, bias=False))

E'tibor beradigan bo'lsak, asimmetriya koeffitsiyenti 2 xil chiqmoqda, sababi 2 xil usul yordamida topilganidir. 1-usul standart ya'ni Bias=True, oddiy formula yordamida topilgan unda natija 0,075 hosil bo'ladi. 2-usul Fisher-Pearson(Bias=False) tuzatilgan asimmetriyada esa 0,086 natija kelib chiqadi. Bu natijalarni ikkalasi ham to'g'ri hisoblanadi. Nazariy statistika, imtihon, ilmiy ishda Fisher-Pearson ishlatiladi, Tezkor tahlillarda standart formula ishlatiladi.

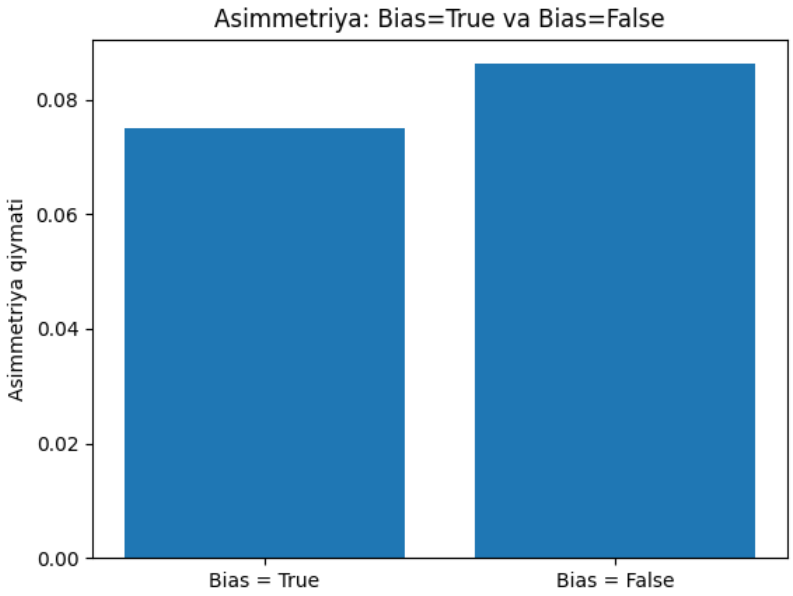
Python yordamida quyidagi kodlar orqali aniqlash mumkin:

from scipy.stats import skew

```
import numpy as np
x = np.array([4,3,3,4,4,5,4,4,3,4,5,4])
skewness = skew(x, bias=False)
print("Asimmetriya: skewness)
```

Asimmetriya koeffitsiyentini natijalari farqini grafikda ham ko'rish mumkin Bu grafikni Python yordamida quyidagi kod orqali chiqarish mumkin.

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import skew
x = np.array([4, 3, 3, 4, 4, 5, 4, 4, 3, 4, 5, 4])
skew_bias_true = skew(x, bias=True)
skew_bias_false = skew(x, bias=False)
labels = ['Bias = True', 'Bias = False']
values = [skew_bias_true, skew_bias_false]
plt.figure()
plt.bar(labels, values)
plt.ylabel("Asimmetriya qiymati")
plt.title("Asimmetriya: Bias=True va Bias=False")
plt.show()
```



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Противопоточная разностная схема для нелинейных одномерных симметрических гиперболических систем с нелокальными характеристическими скоростями

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DOI 10.5281/zenodo.18218024

Абстракт.

В этой работе дадим доказательство дискретной устойчивости по Ляпунову противопоточной разностной схемы для смешанной задачи для одномерной нелинейной симметрической гиперболической системы, записанная в инвариантах Римана, с нелокальными характеристическими скоростями и диссипативными граничными условиями.

Ключевые слова: гиперболическое система, инвариантов Римана, противопоточная разностная схема, нелокальная характеристическая скорость, устойчивость, явная разностная схема.

1 Введение

Построение дискретных функций Ляпунова является популярный и эффективный инструмент для доказательства экспоненциальной устойчивости разностных схем для гиперболических систем с диссипативными граничными условиями. Он позволяет свести устойчивость разностной схемы к отрицательности приращения специально сконструированной квадратичной функции. Для систем с нелокальными характеристическими скоростями появляются дополнительные сложности: вклад нелокальности обычно входит в оценки «остатков» и требует более строгого необходимого условия устойчивости CFL (Куранта-Фридрихса-Леви) или добавления искусственной вязкости. Кроме того, смена знака характеристических скоростей заставляет работать с локальным upwind-правилом и учитывать ситуации, где минимальные по модулю характеристические скорости λ_{min} может быть близка к нулю — это ключевые технические места в современных работах [1].

Классическая основа: формальные критерии диссипативности на границах, конструкция непрерывных (весовых) функционалов Ляпунова для 1-D симметрических

гиперболических систем и задачи управления/стабилизации через граничный контроль. Обзор методов и теорем даётся в монографиях и обзорных главах [1].

Последние работы показывают как сконструировать дискретные аналоги функционалов Ляпунова для схем конечных разностей или конечных объёмов (особенно противопоточных), и на этой основе доказывают априорные оценки экспоненциальной устойчивости численного решения. Эти статьи дают практически конструктивные формулы для дискретной энергии и оценок остатков (включая случаи с нелокальностью)[2].

В теории уравнений в частных производных всё активнее изучаются гиперболические системы с нелокальными характеристическими скоростями: хорошо известны вопросы корректности постановки, эффекты зависимости характеристических скоростей от интегралов по пространству, влияние на волновую структуру и устойчивость; см. обзорные работы по нелокальным законам баланса. Для разностных схем это переводится в проблемы большой операторной нормы ядра (N) и дополнительного слагаемого в оценках остатков [3].

Есть работы, где прямо рассматривают численную стабилизацию через обратной связи с границей и строят дискретные функционалы Ляпунова для неоднородных/неравномерных (в пространстве) систем — полезно для практики реализации диссипативных границ [4].

2.Постановка задачи

Рассмотрим систему в форме инвариантов Римана (диагонализованная форма) на отрезке $x \in (0, L),\ t > 0$:

$$\partial_t w_k(x,t) + \lambda_k(w(x,t),\, \mathbf{N}[w(\cdot,t)])\, \partial_x w_k(x,t) = 0, \qquad k = 1, \dots, m,$$

где $w = (w_1, \dots, w_m)^{\cdot}$ — переменные (инварианты Римана). Оператор $\mathbf{N}[\cdot]$ — нелокальный оператор; скорости λ_k зависят локально от $w(x,t)$ и нелокально через $\mathbf{N}[w]$.

Граничные условия (диссипативные) задаются смешанно: формально при $x = 0$ и $x = L$ задаются линейные/невырожденные соотношения вида

$$w_{\text{in}}(0,t) = B_0\, w_{\text{out}}(0,t), \qquad w_{\text{in}}(L,t) = B_L\, w_{\text{out}}(L,t),$$

и матрицы B_0, B_L диссипативны.

Построить дискретную функцию Ляпунова V^n для численного решения w_i^n (на шаге n по времени, узел i по пространству) такой, что для некоторого $\gamma > 0$

$$V^{n+1} \le (1 - \gamma \Delta t)\, V^n \quad \Longrightarrow \quad V^n \le V^0 e^{-\gamma t^n},$$

то есть дискретное решение экспоненциально убывает во времени и устойчиво в смысле Ляпунова.

2.1. Предположения (условия теоремы)

1) (Регулярность и липшицевость) Для всех k функция $\lambda_k(w, \zeta)$ гладкая и локально липшицева по аргументам: существуют константы L_w, L_ζ такие, что

$$\left|\lambda_k(w^1, \zeta^1) - \lambda_k(w^2, \zeta^2)\right| \le L_w\, |w^1 - w^2| + L_\zeta\, |\zeta^1 - \zeta^2|.$$

2) (Ограниченность характеристических скоростей) Существуют числа $\lambda_{\min} > 0$ и

$\lambda_{\max} > 0$ такие, что по модулю характеристические скорости ограничены:

$$0 < \lambda_{\min} \le |\lambda_k(w, \zeta)| \le \lambda_{\max} < \infty,$$

и (важно) знак каждой характеристики постоянен по рассматриваемому классу решений (то есть характеристика k всегда движется в одну сторону — направление не меняется). Это типичная техническая гипотеза, необходимая для корректной противоточной дискретизации без смены направления потока.

3) (Нелокальность ограничена) Оператор $\mathbf{N}$ линейно непрерывен в нормах L^2 (или ℓ^2 в дискретном случае):существует C_N с

$$|\mathbf{N}[w]| \le C_N \, |w|_{L^2}.$$

4) (Диссипативные граничные условия) Матрицы B_0, B_L такие, что при подстановке в энергетическую форму граничные вклады дают отрицательный поток энергии. Формально существует $\delta > 0$ такое, что граничные формы удовлетворяют оценки диссипативности.

5) (CFL) Для схемы используется условие CFL

$$\max_k \frac{\Delta t}{\Delta x} \, \lambda_{\max} \le \nu < 1.$$

Эти гипотезы — стандартны для подобного рода оценки; при необходимости их можно ослаблять, но это требует дополнительной технической работы.

2.2. Противопоточная разностная схема

Разобьём отрезок $[0, L]$ равномерной сеткой

$$x_i = i\Delta x, \qquad i = 0, \dots, N, \qquad L = N\Delta x, \qquad t^n = n\Delta t.$$

Для каждой характеристики k в ячейке i используем классический противопоточный вариант:

$$w_{k,i}^{n+1} = w_{k,i}^n - \frac{\Delta t}{\Delta x} \left(F_{k,i+1/2}^n - F_{k,i-1/2}^n \right),$$

где поток определяется по противопоточному правилу, учитывая знак скорости $\lambda_{k,i+1/2}^n$:

$$F_{k,i+1/2}^n = \lambda_{k,i+1/2}^n \begin{cases} w_{k,i}^n, & \text{если } \lambda_{k,i+1/2}^n \ge 0, \\ w_{k,i+1}^n, & \text{если } \lambda_{k,i+1/2}^n < 0. \end{cases}$$

и $\lambda_{k,i+1/2}^n$ — аппроксимация характеристической скорости на интерфейсе (например, $\lambda_k(w_i^n, \mathbf{N}_h[w^n])$ или среднее $\frac{1}{2}\big(\lambda_k(w_i^n, \cdot) + \lambda_k(w_{i+1}^n, \cdot)\big)$).Оператор $\mathbf{N}_h$ — дискретная версия $\mathbf{N}$ (сохраняет непрерывность/ограниченность).

На границах $i = 0$ и $i = N$ входящие компоненты w_{in} задаются диссипативной схемой соответствующей граничной матрицей B_0 и B_L (в дискретном виде). Это реализуется так: при вычислении потоков на граничных интерфейсах используются $w_{\text{in}}(0, t^n) = B_0 \, w_{\text{out}}(0, t^n)$ и аналогично для $x = L$.

2.3. Дискретная функция Ляпунова (энергия) — построение

Определим дискретную энергию в виде взвешенной ℓ^2-нормы:

$$V^n = \sum_{i=0}^{N} \sum_{k=1}^{m} \alpha_k \, e^{-2\sigma x_i} \left(w_{k,i}^n\right)^2 \, \Delta x,$$

где $\alpha_k > 0$ — постоянные веса по компонентам (подбираются ниже),$\sigma > 0$ — положительный параметр экспоненциального веса (позволяет управлять вкладом граничных членов). Экспоненциальный вес — стандартный способ: он переводит граничные потоки в отрицательный вклад в энергию при подходящем выборе α_k, σ.

Цель — доказать, что при корректном выборе α_k, σ и CFL выполняется

$$V^{n+1} - V^n \le -\gamma \Delta t \, V^n,$$

с некоторым $\gamma > 0$ (зависит от данных).

2.4. Оценка приращения энергии по одному шагу времени

Сделаем основную алгебраическую операцию:рассмотрим изменение энергии на шаге $n \to n+1$. Обозначим для краткости вес $w_i := w_{k,i}^n$, $\widetilde{w}_i := w_{k,i}^{n+1}$, $\alpha = \alpha_k$, $\ell_i := \lambda_{k,i+1/2}^n$ — локальные скорости интерфейсов (при необходимости индекс k и n вернём).

Тогда, для фиксированного k,

$$\Delta V_k = \sum_i \alpha e^{-2\sigma x_i} (\widetilde{w}_i^2 - w_i^2) \Delta x = \sum_i \alpha e^{-2\sigma x_i} (\widetilde{w}_i - w_i)(\widetilde{w}_i + w_i) \Delta x.$$

Используя схему $\widetilde{w}_i - w_i = -\frac{\Delta t}{\Delta x} \left(F_{i+1/2} - F_{i-1/2}\right)$, получаем

$$\Delta V_k = -\frac{\Delta t}{\Delta x} \sum_i \alpha e^{-2\sigma x_i} \left(F_{i+1/2} - F_{i-1/2}\right) (\widetilde{w}_i + w_i) \Delta x.$$

Перегруппируем сумму по потокам (сдвиг индексов) — получим внутренние и граничные вклады:

$$\Delta V_k = -\frac{\Delta t}{\Delta x} \sum_i \alpha \Big[e^{-2\sigma x_i} F_{i+1/2} (\widetilde{w}_i + w_i) - e^{-2\sigma x_{i+1}} F_{i-1/2} (\widetilde{w}_{i+1} + w_{i+1}) \Big] \Delta x.$$

Запишем аккуратно, соберём вклад по каждому интерфейсу $i + 1/2$ и получаем:

$$\Delta V_k = -\frac{\Delta t}{\Delta x} \sum_{i=0}^{N-1} \alpha \Phi_{i+1/2} + R_k.$$

где

$$\Phi_{i+1/2} = F_{i+1/2} \Big(e^{-2\sigma x_i} (\widetilde{w}_i + w_i) - e^{-2\sigma x_{i+1}} (\widetilde{w}_{i+1} + w_{i+1}) \Big),$$

а R_k — остаток, возникающий из-за вариации весов $e^{-2\sigma x_i}$ и возможной неточности в том, что $\widetilde{w}$ и w различаются. Этот остаток можно оценить по модулю как $C\Delta t$ умноженное на энергию V^n.

Теперь используем противопоточную структуру: на интерфейсе $i + 1/2$

$$F_{i+1/2} = \ell_{i+1/2} \, w_{i+1/2}^{\text{up}},$$

где $w^{\mathrm{up}}_{i+1/2}$ — соответствующее противопоточное значение: либо w_i, либо w_{i+1} в зависимости от знака $\ell_{i+1/2}$. Подставляя в Ф и приводя с учётом знака ℓ, получаем выражение, которое распадается на разность квадратов.

Стандартное действие: для $\ell > 0$ получаем вклад вида

$$\ell_{i+1/2}\Big[e^{-2\sigma x_i}(\widetilde{w}_i + w_i)w_i - e^{-2\sigma x_{i+1}}(\widetilde{w}_{i+1} + w_{i+1})w_i\Big],$$

и аналогично для $\ell < 0$. После некоторой алгебры (используя, что $\widetilde{w}$ близко к w и заменяя среднее $\widetilde{w} + w$ на $2w$ с контролируемой ошибкой $O(\Delta t)$) получаем основной ведущий вклад:

$$\Delta V_k \approx -2\Delta t \sum_i \alpha e^{-2\sigma x_{i+1/2}} \ell_{i+1/2} (w^{\mathrm{up}}_{i+1/2})^2 \Delta x + (\text{границы}) + R_k,$$

где $x_{i+1/2} \equiv \frac{1}{2}(x_i + x_{i+1})$. Ключевое: ведущий член — сумма положительных величин, умноженных на $-\ell$ и вес $e^{-2\sigma x}$, что даёт убывание при правильном выборе знака/веса.

2.5. Граничные вклады и диссипативность

Суммирование даёт граничные слагаемые на $x = 0$ и $x = L$. Для фиксированной k общий граничный вклад имеет вид

$$B_k = -\frac{\Delta t}{\Delta x}\alpha\Big[e^{-2\sigma x_N} F_{N+1/2}(\widetilde{w}_N + w_N) - e^{-2\sigma x_0} F_{1/2}(\widetilde{w}_0 + w_0)\Big].$$

Подставляя граничные соотношения (входящие компоненты через матрицы B_0, B_L) и суммируя по всем k, приходим к матричной граничной форме. Требуем, чтобы итоговый граничный вклад был $\le -\kappa\Delta t\,|w|^2_{\text{гран}}$ с $\kappa > 0$. Это — условие «диссипативности» граничных условий. Конкретно: если на границе линейное соотношение между входящими и выходящими компонентами имеет вид $w_{\text{in}} = B\,w_{\text{out}}$ и матрица B удовлетворяет $|B| < 1$ в соответствующей скалярной форме, то граница даёт отрицательный вклад в энергию (взвешенный поток). Это стандартное условие в теории симметрических гиперболических систем (см. работы по стабилизации и границам).

Таким образом суммарный вклад от всех интерфейсов и границ может быть оценён как

$$\Delta V \le -2\Delta t \sum_{i,k} \alpha_k e^{-2\sigma x_{i+1/2}} |\ell_{k,i+1/2}| \left(w^{\mathrm{up}}_{k,i+1/2}\right)^2 \Delta x - \kappa\Delta t\,|w|^2_{\text{гран}} + \sum_k R_k.$$

2.6. Оценка остаточных членов (влияние нелокальности и нелинейности)

Остатки R_k происходят из трёх источников:

- замены $\widetilde{w} + w$ на $2w$ (ошибка $O(\Delta t)$);
- дискретизации нелокального оператора $\mathbf{N}_h$ и изменений λ между соседними интерфейсами;
- вариации весов $e^{-2\sigma x_i}$ при суммировании.

Нужно показать, что эти остатки линейно ограничены энергией:

$$|R_k| \le C_k \Delta t\, V^n,$$

где C_k зависит от липшицевости λ_k и свойств $\mathbf{N}$, а суммарно $\sum_k C_k = C$.

Обоснование: по липшицевости λ_k

$$\left|\lambda^n_{k,i+1/2} - \lambda^n_{k,i-1/2}\right| \le L_w \left|w^n_{i+1} - w^n_{i-1}\right| + L_\zeta \left|\mathbf{N}_h[w^n]_{i+1/2} - \mathbf{N}_h[w^n]_{i-1/2}\right|.$$

Разности по пространству аппроксимируются через градиенты, которые оцениваются через ℓ^2-норму (через неравенство обратного типа в сетках и через сопряжённость стандартных разностных операторов).Благодаря ограниченности $\mathbf{N}_h$ и непрерывности оператора, получаем линейную оценку в V^n.

Аналогично, при переходе от $\widetilde{w} + w$ к $2w$ возникает слагаемое порядка Δt умноженное на $\max|\lambda|$ и на $\sum(w^2)$ — опять $O(\Delta t)V^n$.

Итого:

$$\sum_k |R_k| \le C\Delta t\, V^n.$$

Заметим: если нелокальность сильная (большой C_N или большие липшицевские константы), то C может быть большим; в таком случае требуется более строгий CFL (меньшее Δt), чтобы обеспечить убывание энергии. Это естественно: сильная нелинейность/нелокальность требует шагов меньше для стабильности.

2.7. Сведение к неравенству убывания

Соединив оценки, получаем (сумма по k):

$$\Delta V \le -2\Delta t \sum_{i,k} \alpha_k e^{-2\sigma x_{i+1/2}} \left|\ell_{k,i+1/2}\right| \left(w^{\text{up}}_{k,i+1/2}\right)^2 \Delta x - \kappa\Delta t\, |w|^2_{\text{границы}} + C\Delta t\, V^n.$$

Нижняя граница на $|\ell_{k,i+1/2}| \ge \lambda_{\min}$ и отношение между суммами (площадь ячеек Δx и сумма по интерфейсам) дают, что первый суммарный член контролирует всю энергию V^n снизу: существует $\mu > 0$ (зависит от выборов α_k, σ и $\lambda_{\min}$), такое что

$$2\sum_{i,k} \alpha_k e^{-2\sigma x_{i+1/2}} \left|\ell_{k,i+1/2}\right| \left(w^{\text{up}}_{k,i+1/2}\right)^2 \Delta x \ge \mu V^n.$$

(Интуитивно: противоточная часть собирает квадратные значения w по всем узлам с весами, сравнимыми с весами в V^n.)

Тогда

$$\Delta V \le -\Delta t\, \mu V^n - \kappa\Delta t\, |w|^2_{\text{границы}} + C\Delta t\, V^n = -\Delta t(\mu - C)V^n - \kappa\Delta t\, |w|^2_{\text{границы}}.$$

Если выбрать параметры схемы / веса так, чтобы $\mu > C$ (в частности, при фиксированных данных это обеспечивается малым Δt через CFL — в наш термин μ зависит слабее от Δt, а C линейно зависит от параметров), можно потребовать Δt настолько мал, что C небольшие по сравнению с μ, то положим $\gamma := \mu - C > 0$ и имеем

$$V^{n+1} - V^n \le -\gamma\Delta t\, V^n,$$

что эквивалентно

$$V^{n+1} \le (1 - \gamma\Delta t)\, V^n.$$

Применим несложный дискретный результат: при $\Delta t > 0$,

$$V^n \le (1 - \gamma \Delta t)^n V^0 \le e^{-\gamma n \Delta t} V^0 = e^{-\gamma t^n}\, V^0.$$

То есть получаем экспоненциальную по времени (в дискретном смысле) убывающую оценку энергии — дискретную устойчивость по Ляпунову.

Теорема (дискретная устойчивость по Ляпунову). Пусть выполнены предположения 1–5.Тогда существует выбор положительных весов α_k и параметра $\sigma > 0$, а также число $\nu \in (0, 1)$ такое, чтопри условии CFL $\max_k \dfrac{\Delta t}{\Delta x}\, \lambda_{\max} \le \nu$ дискретная upwind-схема для указанной системы удовлетворяет оценке

$$V^{n+1} \le (1 - \gamma \Delta t)\, V^n \qquad \text{с некоторым } \gamma > 0,$$

следовательно

$$V^n \le V^0 e^{-\gamma t^n}.$$

В частности, численное решение устойчиво по Ляпунову и экспоненциально убывает при $t \to \infty$.

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Информационное моделирование перспективное планирования наукоемких производств

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DOI 10.5281/zenodo.18218894

АННОТАЦИЯ

В статье автор констатируют что, проблемы создания высоких технологий и исползования накопленного научно-технического, производсвенного, интеллектуалного и кадрового потенциала, будуший актульными всегда приобрели оссобую значимость в настояшшее время, в постсоциалических странах. Кроме этого приводит множества проектных решеный является рационалным, но толко с точки зрения опредленного критерия.

Автор приводит пять шагов технология сопосновительного многофакторного анализа.

ANNOTATSIYA

Maqolada muallif postsotsialistik mamlakatlarda doimo dolzarb bo‘lib kelgan yuqori texnologiyalarni yaratish, to‘plangan ilmiy-texnikaviy, ishlab chiqarish, intellektual va kadrlar salohiyatidan foydalanish muammolari hozirgi davrda alohida ahamiyat kasb etganini ta’kidlaydi. Bundan tashqari, u oqilona bo’lgan ko’plab dizayn qarorlariga olib keladi, lekin faqat ma’lum bir mezon nuqtai nazaridan.

Muallif qiyosiy ko’p o’lchovli tahlil texnologiyasining besh bosqichini taqdim etadi.

ABSTRACT

In the article the author states that the problems of creating high technologies and using the accumulated scientific and technical, production, intellectual and personnel potential, the future of current always acquired special significance at the present time, in post-socialist countries. In addition, it gives many project solutions that are rational, but only from the point of view of a certain criterion. The author gives five steps of the technology of substantiating multifactorial analysis.

Проблемы создания высоких технологий использования накопленного научно-технического производственного к кадрового потенциала будучи актуальными всегда приобрели особую значимость в настояшее время когда постсоциалистических странах стоит перед историческим выбором стратегии своего развития на многие десятилетия вперед.

Практически эта стратегия должна обеспечивать перевоз экономики на траекторию устойчивого роста а для ее реализации необходима выработать эффективную научно-промышленную политику – основу единой государственной концепции долгосрочного социально экономического развития.

Ядром новой научно промкшленной политику должна стать система мер, обеспечивающих прогрессивные структурно – технологические сдвиги в продвижении

самого передового технологического уклада. Вытеснение старых укладов новым стимулирует экономический рост а их сохранение неизбежно приводит к замедлению технологических сдвигов и темпов роста экономики снижению конкурентоспособности товаров на внутреннем и мировом рынках.

Усовершенствование методологии и технологии перспективного планирования и финансирования крупных наукоемких производств исследование проблем концептуального характера особенно важны в настоящее переходное время, когда интенсивно происходят радикальные структурные изменение промышленного потенциала появляются новые формы финансовой и производственной организации и кооперации, к управлению приходят новые кадры не всегда в достаточной мере владеющие современными методами управления, планирования и прогнозирования.

Повышение уровня значимости информационно – технологического обеспечения формирования и управления реализацией программ научно-технического развития требует разработки специальной информационной технологии и оснащения ее многофункциональным инструментарием техника-экономического обоснования и механизмом управления программными мероприятиями создание которых возможно только на базе современных вычислительных платформ.

Способом исследования, определяющим направления развития и комплекс наиболее эффективных мероприятий служит диалоговый расчёт и последующий сопоставительный многофакторный анализ вариантов проектных решений. Воздействуя управляющими параметрами на процесс выработки решения ЛПР получает различные варианты программно-плановых решений, отличающихся техника-экономическими показателями, степенью использования локальных и общественных ресурсов, степенью выполнения заказов на продукцию (в том числе научную) и т.п.

Каждое из этих проектных решений является рациональным но только с точки зрения определённого критерия. Выбор лучшего варианта программно-планового решения, удовлетворяющего требованиям ЛПР и целям планирования, предоставляется самому ЛПР, поскольку комплекс методов и моделей может не производить сопоставительный анализ вариантов и поиск из них наиболее подходящего на практике.

Технология сопоставительного много факторного анализа в этом случае осуществляется за несколько шагов.

1. Задавая ресурсные ограничения критерии и другие управлявшие параметры расчетов. ЛПР с помощью информационной технологии генерирует серию вариантов программно-плановых решений. Количество решения должно быть обозримым чтобы ЛПР мог их анализировать

2. На основании имеющихся опыта и знаний целей расчетов, учёта информационных факторов ЛПР определяет вариант проектного решения, наиболее приемлемый с его точки зрения.

3. Для достижения целей планирования улучшения проектных решений в желаемом направления ЛПР, изменяя соответствующие параметры базового расчета формирует задание на новый расчет. Его результаты подвергаются аналитическому исследованию, сопоставляющему технико-экономические показателей нового расчета с базавоми. В исследовании указываются показатели базового расчета., условия, при которых был проведен новый расчет-показатель, принятый в качестве критерия расчета., ресурсные показателей, принятые в качестве ограничений расчета и их количественные значения., техника экономические показателе по которым производится сопоставительный анализ и величаний их изменит относительно базового

варианта. В качестве проектных показателей для сравнительного многофакторного анализа могут быть приняты: объем выпуска продукции, объем финансирования ИНОКР, объем инвестиций трудоемкость работ и другие критерии.

4. На основании анализа ЛПР решает вопрос о направлениях улучшения проектного решения. Дальнейшего действие ЛПР могут быть следующие: закончит расчета улучшая базовой вариант., продолжить расчеты, взяв за основу новый базовый вариант.

5. ЛПР получает новый вариант проектного решения но основе многофакторного анализа, а также своих знаний и опыта.

Задача рассмотрения всей совокупности альтернатив программно-плановых проектов, оценки их реализуемости и эффективности являются наиболее важной и трудоемкой для ЛПР. Степень детализации проектов увеличивается постепенно, по мере принятия решений на промежуточных этапах планирования. По этому набор моделей информационной технологии и методов их решения должен обеспечивать ЛПР оперативность проведение расчетом как на агрегированных, так и детализированных исходных данных. При этом программное обеспечение моделей должно строиться по принципу их информационной независимости, что является необходимым условием производительности и надежности программного обеспечения технологии.

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Статистический анализ и прогнозирование динамики заготовления молока в Республике Узбекистан

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DOI 10.5281/zenodo.18219225

Аннотация: Производство молока и молочной продукции является одной из актуальных задач по выполнению продовольственной программы республики. Решение этой задачи зависит от расширения зелёных площадей, на которых выращиваются кормовые культуры для скота. В частности, требуется увеличение площадей под посевы газонных трав и кукурузы для заготовки кормов, сена и силоса. Таким

образом, увеличение зелёных насаждений на территории областей Республики, во-первых, способствует улучшению экологического состояния и чистоты атмосферы, а во-вторых, позволяет обеспечить выполнение продовольственной программы по молоку и молочной продукции.

В статье методом статистического анализа временных рядов изучена статистическая закономерность рядов динамики ??$_?$? — заготовки молока в Республике Узбекистан (по материалам ЦСУ РУз за 2011–2023 годы). Построены точечные и интервальные оценки среднего объёма заготовки молока в Республике Узбекистан с 95%-й доверительной вероятностью, определены явные виды трендов и выполнено прогнозирование среднего объёма заготовки молока на последующие годы. С помощью статистического критерия Дарбина–Уотсона установлено, что средний объём заготовки молока в Республике Узбекистан обладает автокорреляционной зависимостью. Следовательно, объём заготовки молока в Республике Узбекистан в текущем году зависит от объёмов заготовки молока в предыдущие годы.

Keywords: динамический ряд, тренд, автокорреляция.

Введение.

Почти в каждой области встречаются явления, которые важно изучать в их развитии и изменении во времени. Можно, например, стремиться предсказать будущее на основании знания прошлого, управлять процессом, описать характерные особенности ряда на основании ограниченного количества информации. При обработке временных рядов опираются на разработанные математической статистикой, настоящей время статистика располагает разнообразными методами анализа временных рядов от самых элементарных до весьма сложных([1,2,3,4]).

Материалы и методы. В настоящей работе, проведена обработка и анализ объем заготовление молоко в Республике Узбекистан за период наблюдений 2011-2023 годы, как дискретный стационарный временный ряд. Изучение объем заготовление молоко в Республике Узбекистан как дискретный стационарный динамический ряд и прогнозирование ее объем на следующие годы на основании опытных данных, играют важную роль при определении экономический эффективности фермерского, дехканского хозяйств. В общем случае временный ряд y_t, tCT состоит из четырех составляющих: тренд; колебания относительно тренда; эффект сезонности; случайная компонента В данной работе использована методы обработки и анализа временных рядов, такие, как методы определения тренда, проверки нормальности и случайности, а также проверка автокорреляции, метод скользящей средней, метод конечных разностей, метод наименьших квадратов, критерия Дарбина – Уотсона и другие методы. Используя методы статистического анализа временных рядов построены точечные и интервальные оценки для средней объем заготовление молоко в Республике Узбекистан, определены явные виды трендов и прогнозирована средней объем заготовление молоко для последующих лет, проверено различные статистические гипотезы. Изучению и анализу динамических рядов посвящены работы: Андерсона[1], Кендала[2], Тихомирова[3], Сулайманова[4], Файзиева[5] и другие.

Результаты и их обсуждение. Предположим, что средней объем заготовление молоко Республике Узбекистан за период наблюдений, 2011-2023 годы образует

стационарный дискретный временный ряд. Используя выше изложенных методы статистического анализа временных рядов, построим точечные и интервальные оценки для средней объем заготовление молоко республике, определим явные вид тренда и прогнозируем заготовление объем молоко для последующих лет, проверим различные статистические гипотезы. На рисунки-1, с помощи опытных данных (таблица-1, стольба-3) геометрически изображены y_t - динамика средней объем заготовление молоко в Республике Узбекистан виде а) точечная график, б) гистограмма, с) круговая диаграмма, д) диаграмма:

Геометрическое изображение наблюденных данных, система координат дают основание в первом приближении, предполагать гипотезу что, трендовая часть процесса (общее направление развития процесса) имеет линейную зависимость вида $y(t) = a_1 t + a_0$. Где неизвестные параметры определяются методом наименьших квадратов т.е. на основании опытных данных, решая следующую систему нормальных уравнений:

$$\begin{cases} a_0 T + a_1 \sum t = \sum y_t, \\ a_0 \sum t + a_1 \sum t^2 = \sum y_t t \end{cases} \tag{1}$$

Решая система уравнение (1) и используя, вычисления по таблице- 1, имеем:

$$\sum y_t = 125951{,}5 \text{ ming tonna},$$

$$a_0 = \frac{1}{T} \sum y_t = \frac{125951{,}5}{13} = 9688{,}58 \text{ ming tonna},$$

$$a_1 = \frac{1}{\sum t^2} \sum y_t t = \frac{77362{,}6}{182} = 425{,}07.$$

Отсюда, находим уравнение линейного тренда (тенденция) средней объем заготовление молоко Республике Узбекистан [1,2,3,4,5]:

$$y(t) = 425,07t + 9688,58 \tag{2}$$

Подставляя в уравнение (2) значение t =1 находим ожидаемые средней объем заготовление молоко Республике Узбекистан в 024 году, будет в среднем y(1)=10113,65 тысяча тонна.

С помощью статистических критериев ([1]-[5]) установлено, что в уравнении (2) $y(t) = a_1 t + a_0$ основная гипотеза $H_0 : a_1 = 0$ отвергается, принимается альтернативная гипотеза $H_1 : a_1 \neq 0$ с уровень значимости ?=0,05. Следовательно, средней объем заготовление молоко Республике Узбекистан имеет линейный тренд. Многих задачах наблюдения, выборка наблюдения является статистически независимо, то во временных рядах они, как правило, зависимы, и характер этой зависимости может определяться положением наблюдений в последовательности. Автокорреляцией


SERIES1
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8431.6
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9598.2
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10976.9
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2011
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а)


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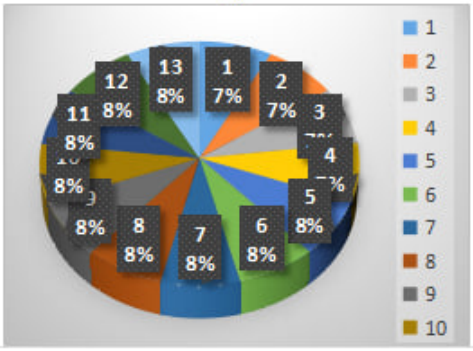
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Годы наблюдений
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13

с) д)

Рис.1.

Таблица 4: К расчёту данных для определения тренда временного ряда

№	Годы наблюдений	$y(t)$, тыс. т	t	t^2	$y(t)\,t$	$y(t)\,t^2$
1	2011	6766,2	-6	36	-40597,2	243583,2
2	2012	7310,9	-5	25	-36554,5	182772,5
3	2013	7885,5	-4	16	-31542,0	126168,0
4	2014	8431,6	-3	9	-25294,8	75884,4
5	2015	9027,8	-2	4	-18055,6	36111,2
6	2016	9598,2	-1	1	-9598,2	9598,2
7	2017	10047,9	0	0	0	0
8	2018	10466,4	1	1	10466,4	10466,4
9	2019	10714,3	2	4	21428,6	42857,2
10	2020	10976,9	3	9	32930,7	98792,1
11	2021	11274,2	4	16	45096,8	180387,2
12	2022	11627,2	5	25	58136,0	290680,0
13	2023	11824,4	6	36	70946,4	425678,4
Сумма		125951,5	0	182	77362,6	1722978,8

Таблица 5: Составим таблица для расчета конечные разности.

№ п/п	Годы наблюд	y(t) тысяча тонна	Y(t)^2	$\Delta Y(t)$	ΔY(t)^2	?^2Y(t)	Δ^2Y(t)^2	Δ ^3Y(t)	Δ ^3Y(t)^2
1	2011	6766,2	45781462,44						
2	2012	7310,9	53449258,81	544,7	296698,09				
3	2013	7885,5	62181110,25	574,6	330165,16	29,9	894,01		
4	2014	8431,6	71091878,56	546,1	298225,21	-28,5	812,25	-58,4	3410,56
5	2015	9027,8	81501172,84	596,2	355454,44	50,1	2510,01	78,6	6177,96
6	2016	9598,2	92125443,24	570,4	325356,16	-25,8	665,64	-75,9	5760,81
7	2017	10047,9	100960294,4	449,7	202230,09	-120,7	14568,49	-94,9	9006,01
8	2018	10466,4	109545529	418,5	175142,25	-31,2	973,44	89,5	8010,25
9	2019	10714,3	114796224,5	247,9	61454,41	-170,6	29104,36	-139,4	19432,36
10	2020	10976,9	120492333,6	262,6	68958,76	14,7	216,09	185,3	34336,09
11	2021	11274,2	127107585,6	297,3	88387,29	34,7	1204,09	20	400
12	2022	11627,2	135191779,8	353	124609	55,7	3102,49	21	441
13	2023	11824,4	139816435,4	197,2	38887,84	-155,8	24273,64	-211,5	44732,25
Сумма		125951,5	1254040508	5058,2	2365568,7	-347,5	78324,51	-185,7	131707,29

представляет собой корреляционную зависимость между последующими и предшествующими членами временного ряда. Проверяем наличие автокорреляции средние урожайности молока в республике т.е.

$$Y_t = \rho Y_{t-1} + \varepsilon_t, \quad \text{где} \quad \rho = \mathrm{Cov}(Y_t, Y_{t+1}) = M\big[(Y_t - \bar{y}_t)(Y_{t+1} - \bar{y}_t)\big].$$

Для дальнейшей исследование нам необходима вычислить следующие конечные разности по наблюденным опытным данным. Обозначим,

$$\Delta Y_t = Y_{t+1} - Y_t, \quad \Delta^2 Y_t = \Delta Y_{t+1} - \Delta Y_t, \quad \Delta^3 Y_t = \Delta^2 Y_{t+1} - \Delta^2 Y_t.$$

конечные разности (таблица -2).

По таблице- 2 вычислим

$$V_k = \frac{\sum_{t=k}^{T} \left(\Delta^k y_t\right)^2}{(T-k)\, C_{2k}^k} \tag{3}$$

коэффициенты вариации разностей и установим, что $V_1?V_2?V_3$. Следовательно, конечные разности первого порядка элиминируют линейную тенденцию. Используя таблицу-3, формулы из литературы [1,2,3,4,5] определяются значения коэффициентов автокорреляции R_L при L=1,2,3,4,5 (где: L, временной сдвиг, т.е. промежуток

Таблица 6: Таблица для расчёта автокорреляции

№ п/п	Годы	$y(t)$, тыс. т	$y(t)\cdot y(t+1)$	$y(t)\cdot y(t+2)$	$y(t)\cdot y(t+3)$	$y(t)\cdot y(t+4)$	$y(t)\cdot y(t+5)$
1	2011	6766,2					
2	2012	7310,9	49467011,58				
3	2013	7885,5	57650101,95	53354870,1			
4	2014	8431,6	66487381,8	61642584,44	57049891,92		
5	2015	9027,8	76118798,48	71188716,9	66001343,02	61083900,36	
6	2016	9598,2	86650629,96	80928183,12	75686606,1	70171480,38	64943340,84
Сумма		125951,5	1160058775	1062564651	963451920,6	862488762,7	759171149,3

времени отставания одного явления от другого, связанного с ним):

$$R_L = \frac{\sum_{t=1}^{N-L} Y_t Y_{t+L} - \frac{1}{N-L}\sum_{t=1}^{N-L} Y_t \sum_{t=L+1}^{N} Y_t}{\sqrt{\left[\sum_{t=1}^{N-L} Y_t^2 - \frac{1}{N-L}\left(\sum_{t=1}^{N-L} Y_t\right)^2\right]\left[\sum_{t=L+1}^{N} Y_t^2 - \frac{1}{N-L}\left(\sum_{t=L+1}^{N} Y_t\right)^2\right]}} \tag{4}$$

Отличие (4) значения R_L от нуля, даёт основание полагать, что между средней объем заготовление молоко Республике Узбекистан имеется существенная автокорреляционная зависимость.

С другой стороны проверим гипотезу, существования автокорреляционная зависимость между средней объем заготовление молоко Республике Узбекистан с помощью критерия Дарбина – Уотсона :

$$\sum_{t=1}^{T-1}(Y_{t+1} - Y_t)^2 / \sum_{t=1}^{T} Y_t^2. \tag{5}$$

Вычисляется по формуле (5) $d = 0,005$ сравнивается с $d = 1,22$ табличным значением [1],[2],[5]. Поскольку $d = 0,005 < d = 1,22$. Следовательно, критерия Дарбина – Уотсона 95% гарантией доказывает, что средней объем заготовление молоко Республике Узбекистан имеет автокорреляционную зависимость $Y_t = \rho Y_{t-1} + \varepsilon_t$. Следовательно, средней объем заготовление молоко Республике Узбекистан в этом году, зависит от объема прошлых лет. Проверка статистическая гипотза о нормальности, (y_t) - средний объем заготовление молоко Республике Узбекистан ([1]-[5]):

$$H_0: \ P(\bar{y}_t < x) = \Phi_{a,\sigma}(x), \qquad H_1: \ P(\bar{y}_t < x) \neq \Phi_{a,\sigma}(x)$$

принимается с уровень значимости $\alpha = 0,05$ (см.таблица-4).

Тогда с помощью следующие формулы построим интервальные оценка для средней объем заготовление молоко Республика Узбекистан:

$$\bar{Y}_{T+i} - t(T-2;\alpha)\,\bar{\sigma}_y \ \leq \ a_0 + a_1(T+i) \ \leq \ \bar{Y}_{T+i} + t(T-2;\alpha)\,\bar{\sigma}_y \tag{6}$$

Таблица 7: Оценка основных параметров динамического ряда.

Выборочные характеристики	Оценки выборочных характеристик
Средний объём заготовления молока в Республике Узбекистан, $\bar{y}_t$ (тыс. т)	9688,57
Дисперсия	2812476,01
Среднеквадратическое отклонение $\bar{\sigma}_t$	1677,04
Коэффициент вариации $v(\%)$	17,31%
Асимметрия A_ζ	-0,46
Эксцесс E_{K_ζ}	-1,08
Ошибка среднего значения $\bar{y}_T$, m_y	$m_y = \frac{\sigma_y}{\sqrt{n}} = \frac{1677,04}{3,61} = 464,55$
Предельная ошибка m'_y	$m'_y = t\, m_y = 2,18 \cdot 464,55 = 1012,72$
Интервальная оценка (95%) $\bar{y}_T \pm t m_y$ для среднего объёма заготовления молока в Республике Узбекистан	$\bar{y}_T \pm t m_y = 9688,57 \pm 1012,72$ (8675,85; 10701,29) тыс. т
Проверка статистической гипотезы $H_0: P(\bar{y}_t < x) = \Phi_{a,\sigma}(x)$, $H_1: P(\bar{y}_t < x) \neq \Phi_{a,\sigma}(x)$	94%: гипотеза H_0 принимается

где

$$\bar{\sigma}_y = \bar{\sigma} \left[\frac{\frac{1}{T} + \left(\frac{T-1}{2} + i\right)^2}{\sum_{i=1}^{T} (t - \bar{t})^2} \right]^{0.5}$$

Значение $t_{.} = t(T-2;\alpha)$ определяется по таблице Стьюдента (см. [5]-стр.181). По формуле (6) построим интервальная оценка для среднего объема заготовление молоко Республике Узбекистан с вероятностью 0.94 (94-95% гарантией) (8675,85 ; 10701,29) тысяча тонна.

На основании выборочных данных, используя пакет программы x7.2019 и Excel ЭВМ [4,5,6], вычислим числовые характеристики y_t-для динамического ряда заготовление молоко Республике Узбекистан (таблица - 4).

Выводы

На основании выше изложенных статистических анализов, динамика $\bar{y}_t$ - заготовление молоко в Республике Узбекистан как дискретный временный ряда с надежностью $\gamma = 0,94$ (таблица-4) можно сделать следующие выводы:

1. построены точечные оценка для среднего, ежегодного $\bar{y}_t$?-значение заготовление молока республике Узбекистан $\bar{y}_t$?= 9688,57 тысяча тонна ;

2. 94% гарантией интервальные статистические оценки для среднего объема заготовление молоко в Республике Узбекистан (8675,85 ; 10701,29) тысяча тонна;

3.определены явные вид тренда т.е. математический модель указывающие основное направление изучаемого случайного процесса и установлена её линейность y(t)=425,07 t+9688,58 ;

4. критерием Дарбина – Уотсона установлены, что, средние объм заготовление-молоко Республике Узбекистан имеет автокорреляционную зависимость

$$Y_t = \rho Y_{t-1} + \varepsilon_t, \quad \text{где} \quad \rho = \mathrm{Cov}(Y_t, Y_{t+1}) = M\big[(Y_t - \bar{y}_t)(Y_{t+1} - \bar{y}_t)\big].$$

т.е. объм заготовление молоко Республике Узбекистан в этом году, зависит от объема заготовленного молока прошлых и последующей лет;

5. целом, динамический ряд, средней объем заготовление молоко в Республике Узбекистан образует нестационарный временный ряд.

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Моделирование течения с помощью уравнения вязкой мелкой воды с пористыми средами

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DOI 10.5281/zenodo.18219281

Введение

Уравнения мелкой воды представляют собой фундаментальную модель в гидродинамике [1,2], предназначенную для описания течений жидкостей в системах, где вертикальный масштаб значительно меньше горизонтального. Они широко применяются для моделирования океанических волн, речных потоков, наводнений и атмосферных явлений. Традиционные уравнения мелкой воды, известные как уравнения Сен-Венана, выводятся путем интегрирования полных уравнений Навье-Стокса по глубине, пренебрегая вертикальными ускорениями. Это приближение справедливо для случаев с малым отношением глубины к длине волны.

Введение вязкости добавляет диссипативные эффекты, связанные с внутренним трением жидкости, что особенно важно для ламинарных или слабо турбулентных потоков [3]. Вязкие мелкие воды учитывают диффузию импульса, предотвращая неустойчивости в численных расчетах и обеспечивая более точное описание реальных течений, таких как сток в каналах или фильтрация в грунтах.

Пористая структура усложняет модель [4,5], моделируя взаимодействие жидкости с твердой матрицей, например, в почвах, песчаных фильтрах или биологических тканях. Пористость влияет на проницаемость среды, снижая эффективную скорость течения и добавляя сопротивление. Это актуально для гидрогеологии (моделирование грунтовых вод), экологии (транспорт загрязнителей) и инженерии (проектирование дренажных систем).

Современные приложения расширяют сферу использования модели: анализ наводнений в пористых зонах, оптимизация водных сооружений, моделирование кровотока в органах человека и биологических тканях. Течения в этих областях характеризуются низкими числами Рейнольдса, что делает вязкостные эффекты критическими, а также требуют учета переменных уровней пористости. Решение таких задач зачастую связано с использованием численных методов для нелинейных уравнений.

Математическая модель

Математическая модель основана на модифицированных уравнениях Сен-Венана с добавлением вязких и пористых терминов. Рассмотрим двумерный случай в плоскости (x, y). Основные переменные: $H(x, y, t)$ — глубина жидкости, $u(x, y, t)$, $v(x, y, t)$ — компоненты горизонтальной скорости, f — пористость $(0 < f \leq 1)$, K — проницаемость среды.

$$\frac{\partial H}{\partial t} + \frac{\partial (Hu)}{\partial x} + \frac{\partial (Hv)}{\partial y} = 0. \tag{1}$$

$$\frac{\partial (Hu)}{\partial t} + \frac{\partial (Hu^2)}{\partial x} + \frac{\partial (Huv)}{\partial y} = -gH\,\frac{\partial (z_0 + p)}{\partial x} + 2\frac{\partial}{\partial x}\left(\nu H \frac{\partial u}{\partial x}\right) + \\ + \frac{\partial}{\partial y}\left(\nu H\Big(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\Big)\right) - \frac{(\tau_{xz})_0}{\rho} - \frac{f}{K}u + q_x. \tag{2}$$

$$\frac{\partial (Hv)}{\partial t} + \frac{\partial (Huv)}{\partial x} + \frac{\partial (Hv^2)}{\partial y} = -gH\,\frac{\partial (z_0 + H)}{\partial y} + \frac{\partial}{\partial x}\left(\nu H\Big(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\Big)\right) + \\ + \frac{2}{3}\frac{\partial}{\partial y}\left(\nu H \frac{\partial v}{\partial y}\right) - \frac{(\tau_{yz})_0}{\rho} - \frac{f}{K}v + q_y. \tag{3}$$

где g — гравитационная постоянная, $z_0(x, y)$ — рельеф дна, ν — кинематическая вязкость, $(\tau_{xz})_0, (\tau_{yz})_0$ связаны с шероховатостью дна:

$$(\tau_{xz})_0 = \rho\, C_f\, u\sqrt{u^2 + v^2}, \qquad (\tau_{yz})_0 = \rho\, C_f\, v\sqrt{u^2 + v^2},$$

и C_f — трение жидкости о дно канала.

Безразмеризация

Водим безразмерные переменные

$$x = Lx', \quad y = Ly', \quad u = Uu', \quad v = Uv', \quad Re = \frac{UL}{\nu}, \quad Fr = \frac{U^2}{gH_0}.$$

Здесь L — характерная ширина канала, U — характерная скорость, Re — число Рейнольдса, Fr — число Фруда, H_0 — характерная глубина потока. Для удобства у штриховых переменных штрихи опускаем.

Напишем уравнения (1)-(3) в стационарном безразмерном виде.

$$\frac{\partial (Hu)}{\partial x} + \frac{\partial (Hv)}{\partial y} = 0, \tag{1}$$

$$\frac{\partial(Hu^2)}{\partial x}+\frac{\partial(Huv)}{\partial y}=-\frac{1}{Fr}H\,\frac{\partial(z_0+H)}{\partial x}+\frac{2}{Re}\frac{\partial}{\partial x}\left(\nu H\frac{\partial u}{\partial x}\right)+$$

$$+\frac{1}{Re}\frac{\partial}{\partial y}\left(\nu H\Big(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\Big)\right)-\frac{(\tau_{xz})_0}{\rho}-\frac{f}{Da}u-\frac{\sin\alpha}{Fr},\qquad(2)$$

$$\frac{\partial(Huv)}{\partial x}+\frac{\partial(Hv^2)}{\partial y}=-\frac{1}{Fr}H\,\frac{\partial(z_0+H)}{\partial y}+\frac{1}{Re}\frac{\partial}{\partial x}\left(\nu H\Big(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\Big)\right)+$$

$$+\frac{2}{Re}\frac{\partial}{\partial y}\left(\nu H\frac{\partial v}{\partial y}\right)-\frac{(\tau_{yz})_0}{\rho}-\frac{f}{Da}v-\frac{\cos\alpha}{Fr}.\qquad(3)$$

Численный метод решения

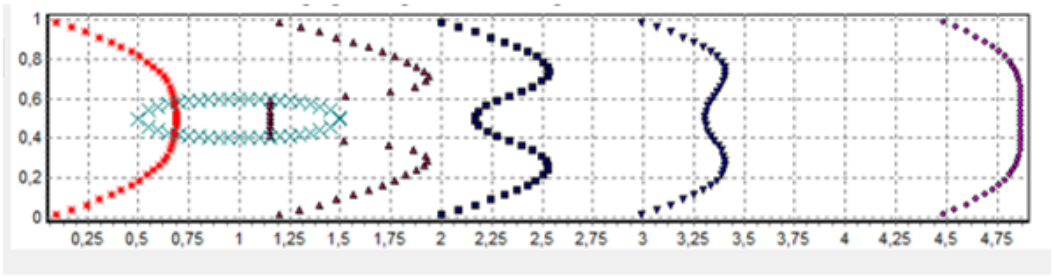
Для решения системы уравнений (1)-(3) применяем SIMPLE подобный алгоритм изложенное в работе [6,7], и использованных во многих работах [8-14]. Пространственная дискретизация осуществляется с использованием по степенным законом изложенной в работе [6],

Система нелинейных алгебраических уравнений решается итерационно методом релаксации, с использованием предобусловливания для ускорения сходимости. Граничные условия реализуются через фиктивные узлы: на непроницаемых стенках — условие no-slip ($u=0, v=0$), на входных границах — заданные потоки или скорости, на выходных — свободные условия с нулевым градиентом.

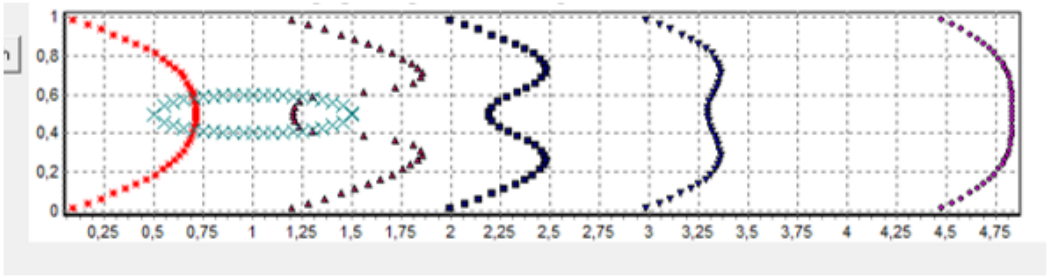
Численные результаты

Рассмотрим некоторые задачи. Исследуем течение с пористым препятствием эллиптического типа с полуосями $a=0,5; b=0,1$ и центром $(1;0,5)$. Входные параметры расчёта: $Re=100; Fr=0,05; f=0,7; sin\alpha=0,0005; C_f=1/6$.

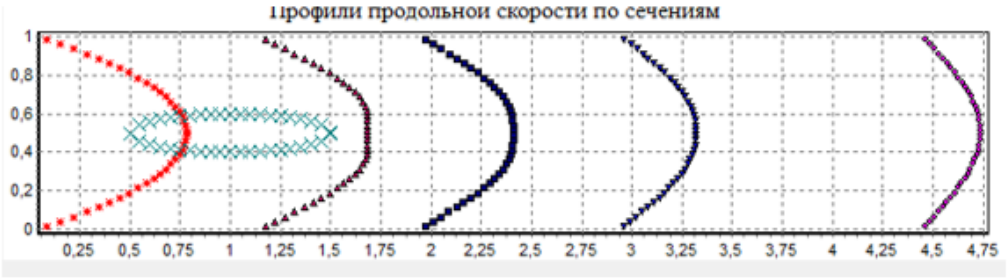
На рис.1 приведены профили поперечной скорости при различных числах Дарси, показывающие существенной влияние его на формирование скоростей.



a) Da=0,01



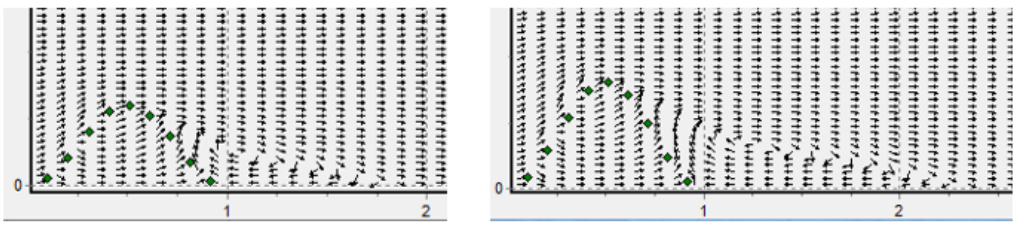
b) Da=10



c) Da=1000

Рисунок 1. Профиль продольной скорости по сечениям:
$x = 0,05; x = 1,15; x = 1,95; x = 2,95;\ = 4,45.$

Рассмотрим теперь влияние на течение пористой структуры, расположенной на одной из берегов канала. Входные параметры расчёта: $Re = 100; Da = 1; Fr = 0,05; f = 0,7; sin\alpha = 0,0005; C_f = 1/6.$



а) б)

Рисунок 2. Направление поле скоростей

На рис.2 представлен направление поле скоростей около пористой структуры где

течение претерпевает существенное изменение. При заданных параметрах наблюдается вторичные течения за пористым слоям (ромбики на рис.2 представляет границу пористого слоя).

Выводы

Модель уравнения вязкой мелкой воды с пористой структурой объединяет гидродинамику и фильтрацию, позволяя анализировать сложные течения в неоднородных средах. Она способствует пониманию экологических и инженерных проблем, таких как управление водными ресурсами и предотвращение катастроф.

Будущие направления включают трехмерные расширения, интеграцию турбулентных моделей. Однако остаются вызовы в калибровке параметров и численном разрешении. Эта модель подчеркивает роль междисциплинарных подходов в механике жидкостей для устойчивого развития.

Эта модель подчеркивает важность междисциплинарных подходов в механике жидкостей, интегрирующих гидродинамику, фильтрацию, гидрогеологию и экологию для обеспечения более устойчивого и эффективного управления водными ресурсами.

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Применение метода перемещаемого узла к задаче оценки стоимости европейских опционов

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DOI 10.5281/zenodo.18219326

Аннотация

В работе рассматривается задача определения цены европейского опциона в рамках модели Блэка–Шоулза [1,2,3]. Исходное уравнение параболического типа приводится к уравнению теплопроводности [3]. Для аппроксимации используется метод перемещаемого узла (далее МПУ) [4,5]. Построены два приближённо-аналитических решения:
— прямое применение метода к уравнению Блэка–Шоулза [6];
— применение метода после приведения уравнения Блэка–Шоулза к уравнению теплопроводности [5,6].

Полученные решения сравниваются с точным решением уравнения Блэка–Шоулза на трёх типовых задачах определения цены опциона вышеуказанными приближённо-аналитическими решениями и проведён сравнительный анализ полученных решений с точным. В качестве критерия точности используется среднеквадратичное отклонение от точного решения [4,6].

Ключевые слова: уравнение Блэка–Шоулза, уравнение теплопроводности, метод перемещаемого узла, европейский опцион, численные методы.

Введение

Модель Блэка–Шоулза является одной из фундаментальных моделей финансовой математики и широко используется для оценки стоимости европейских опционов [1,2,3]. В рамках данной модели цена опциона определяется решением параболического дифференциального уравнения второго порядка. Несмотря на наличие точного

аналитического решения, численные и приближённые методы остаются актуальными, особенно при рассмотрении более сложных моделей или при необходимости быстрого вычисления.

Целью настоящей работы является исследование эффективности метода перемещаемого узла при оценке европейских колл-опционов и сравнение полученных результатов с точным решением Блэка–Шоулза с использованием предложенных в [4,5,6] приближённых аналитических методов.

Постановка задачи

Первое представление. Задача 1.

Классическое уравнение Блека–Шоулза имеет следующий вид [1]:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0. \tag{1}$$

Здесь $V(S,t)$ — цена опциона, S — цена актива, K — цена исполнения, $t \in [0,T]$ — время до исполнения, r — безрисковая процентная ставка, σ — волатильность.

Уравнение (1) рассматривается в области $S \in [0,L]$, $t \in [0,T]$ с начальными и граничными условиями:

$$\begin{aligned} V(S,T) &= \max\{S-K,\,0\},\\ V(0,t) &= 0,\\ V(L,t) &= L - K\exp\bigl(-r(T-t)\bigr). \end{aligned} \tag{2}$$

Здесь T — время до экспирации опциона (в годах), K — цена исполнения опциона. Уравнение (1) и условия (2) будем называть задачей 1.

Для решения данной задачи предлагается приближённо-аналитический метод, который позволяет получить приближённо-аналитическое решение уравнения (1) с начальными и граничными условиями (2) [6].

Второе представление. Задача 2.

При замене переменных

$$x = \ln\frac{S}{K}, \qquad \tau = T - t,$$

$$V(S,t) = Ke^{\alpha x+\beta\tau}\,u(x,\tau),$$

где

$$\alpha = \frac{1}{2} - \frac{r}{\sigma^2}, \qquad \beta = -\left(\frac{r}{\sigma^2} - \frac{1}{2}\right)^2 - \frac{2r}{\sigma^2},$$

уравнение (1) преобразуется к уравнению теплопроводности [3]:

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial x^2}. \tag{3}$$

Уравнение (3) рассматривается в области

$$W < x < E, \qquad 0 < \tau < T,$$

где

$$W = \ln \frac{S_{\min}}{K}, \qquad E = \ln \frac{S_{\max}}{K}.$$

С начальными и граничными условиями

$$\begin{aligned} u(x,0) &= u^0(x) = e^{-\alpha x} \max\bigl(e^x - 1,\, 0\bigr), \\ u(W,\tau) &= u_W = 0, \qquad u(E,\tau) = u_E = 0. \end{aligned} \tag{4}$$

Данная постановка и её приближённые решения рассматриваются в [5,6].

Получение приближённо-аналитических решений

Для получения приближённо-аналитических решений задачи используем метод перемещаемых узлов. Способ получения приближённо-аналитического решения достаточно прост, однако представляет более грубое приближение решения задачи.

Первый способ. Приближённое решение задачи 1 с помощью МПУ

Рассмотрим задачу 1 и выберем произвольную точку (S,t) внутри области, где ищутся решения. Аппроксимируем уравнение (1) с перемещаемым узлом, что даёт систему приближённых уравнений [4,6]. Тогда взамен уравнения (1) получим приближённое уравнение:

$$\frac{C - V_T}{t - T} + \frac{1}{2}\sigma^2 S^2 \cdot \frac{2}{L - 0}\left(\frac{V_p - C}{L - S} - \frac{C - V_l}{S - 0}\right) + rS\frac{V_p - V_l}{L - 0} - rC = 0 \tag{5}$$

Здесь C — приближённое решение. Используя граничные и начальные условия, имеем:

$$V_T = V(S,T), \quad V_l = V(0,t) = 0, \quad V_p = V(L,t). \tag{6}$$

Решая алгебраическое уравнение (5) с учётом граничных и начального условий, получим приближённо-аналитическое решение задачи 1.

Второй способ. Приближённое решение задачи 2 с помощью МПУ

Обозначим через $U(x,\tau)$ приближённое аналитическое решение задачи 2, полученное с помощью метода перемещаемого узла, где использованы граничные и начальное условия. Используется неявная схема МПУ для уравнения теплопроводности (3) с начальными и граничными условиями (4), что приводит к явным формулам приближённого решения [5,6]:

$$\frac{U(x,\tau) - U^0(x)}{\tau} = \frac{2}{E - W}\left(\frac{U_E(\tau) - U(x,\tau)}{E - x} - \frac{U(x,\tau) - U_W(\tau)}{x - W}\right). \tag{7}$$

Решая уравнение (7), получим приближённо-аналитическое решение задачи 2:

$$U(x,\tau) = \frac{(E-x)(x-W)}{2\tau + (E-x)(x-W)}\, U^0(x) + \frac{2\tau\big[U_E(\tau)(x-W) + U_W(\tau)(E-x)\big]}{2\tau + (E-x)(x-W)}. \qquad (8)$$

где

$$u(x,0) = u^0(x) = e^{-\alpha x}\max\big(e^x - 1, 0\big),$$
$$u(W,\tau) = u_W = 0, \qquad u(E,\tau) = u_E = 0.$$

4. Точные решения для сравнения

Для задачи 1 используется классическое решение Блэка–Шоулза с функциями нормального распределения [1]:

$$V(S,t) = S \cdot N_1 - K \cdot \exp(-r(T-t)) \cdot N_2, \qquad (9)$$

где

$$N_1 = \int_{-\infty}^{d_1} n_z\, dz, \quad N_2 = \int_{-\infty}^{d_2} n_z\, dz, \quad n_z = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right),$$
$$d_1 = \frac{\ln(S/K) + 0.5\sigma^2(T-t)\left(\frac{2r}{\sigma^2} + 1\right)}{\sigma\sqrt{T-t}}, \quad d_2 = \frac{\ln(S/K) + 0.5\sigma^2(T-t)\left(\frac{2r}{\sigma^2} - 1\right)}{\sigma\sqrt{T-t}},$$

где N_1, N_2 — функции нормального распределения.

Для задачи 2 точное решение выражается через свёртку с гауссовым ядром [3,5]:

$$u(x,\tau) = \frac{1}{\sqrt{4\pi\tau}} \int_0^{+\infty} e^{-\alpha\xi}(e^{\xi} - 1) e^{-\frac{(x-\xi)^2}{4\tau}}\, d\xi, \qquad (10)$$

которое приводится к виду

$$u(x,\tau) = e^{-\alpha x + \alpha^2\tau} F(d_1) - e^{-(\alpha+1)x + (\alpha+1)^2\tau} F(d_2),$$
$$d_1 = \frac{x + (2\alpha+1)\tau}{\sqrt{2\tau}} + \frac{\sqrt{\tau}}{2}, \quad d_2 = \frac{x + (2\alpha+1)\tau}{\sqrt{2\tau}} - \frac{\sqrt{\tau}}{2},$$

где F — функция нормального распределения.

5. Численные эксперименты и результаты

В Таблице (1) приведены параметры трёх типовых задач для оценки точности методов [4,6].

Таблица 1. Параметры тестовых задач

№ задачи	K	r	σ	T
1	1.0	0.05	0.2	1
2	0.6	0.20	0.2	1
3	1.2	0.03	0.25	0.5

В таблице (2) представлены результаты сравнений приближённо-аналитических решений с точными. Для сравнения используется среднеквадратичное отклонение (СКО), которое считается как норма L_2 разности точного и приближённого решений:

$$\text{СКО} = \left(\frac{1}{|\Omega|^2} \iint_{\Omega} \left(V_{\text{точн.}}(S,t) - V_{\text{приб.}}(S,t) \right)^2 \, dS \, dt \right)^{\frac{1}{2}}, \tag{11}$$

где

- $V_{\text{точн.}}(S,t)$ — точное решение уравнения Блэка–Шоулза,
- $V_{\text{приб.}}(S,t)$ — приближённое решение, полученное методом перемещаемого узла,
- $\Omega = [S_{\min}, S_{\max}] \times [0, T_1]$ — область сравнения, $T_1 < T$ (используется $T_1 = 0.99T$ для исключения вырожденной точки $t = T$),
- площадь области: $|\Omega| = (S_{\max} - S_{\min})T_1$.

Таблица 2. Среднеквадратичное отклонение (СКО) для двух способов решения

Задача	СКО при решении 1 способом	СКО при решении 2 способом
1	0.127	0.084
2	0.059	0.074
3	0.152	0.091

6. Заключение

В Таблице 2 представлены значения среднеквадратичного отклонения между точным решением уравнения Блэка–Шоулза и приближёнными решениями, полученными методом перемещаемого узла двумя различными способами. Анализ полученных результатов показывает, что в большинстве рассмотренных случаев второй способ обеспечивает более высокую точность аппроксимации.

Для первой и третьей тестовых задач значения среднеквадратичной ошибки при использовании второго способа оказываются существенно меньшими по сравнению с первым способом. Это указывает на более точное воспроизведение поведения решения во всей расчётной области и более корректный учёт граничных условий.

В то же время для второй задачи наблюдается меньшая погрешность при использовании первого способа. Данный эффект обусловлен особенностями параметров задачи, в частности повышенным значением процентной ставки и меньшим значением страйка, что усиливает влияние начального условия на общее поведение решения. Это подтверждает, что эффективность метода перемещаемого узла зависит от конкретных параметров задачи.

В целом полученные результаты демонстрируют, что оба рассмотренных способа обладают сопоставимой точностью и могут рассматриваться как эффективные приближённые методы решения задачи оценки европейского опциона. При этом выбор конкретного способа целесообразно осуществлять с учётом параметров модели и характера решаемой задачи.

Анализ результатов показывает, что во всех рассмотренных случаях методы МПУ демонстрируют сопоставимую точность, с преимуществом второго способа в большинстве задач. Такие выводы согласуются с ранее опубликованными исследованиями [4,5,6]. В дальнейшем целесообразно рассмотреть оптимизацию параметров метода и расширение применимости на более сложные финансовые инструменты.

Метод перемещаемого узла, применённый в данной работе для задачи оценки европейских опционов, также успешно используется в инженерных задачах, таких как расчёт потоков жидкости в трубах с различным сечением и других технологических процессах [9]. Это подтверждает универсальность и широкий потенциал метода.

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Трансформация проектного менеджмента в эпоху Industry 4.0: от контроля исполнения к управлению ценностью

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DOI 10.5281/zenodo.18224538

В условиях ускоряющихся технологических сдвигов и роста инновационной активности проектный менеджмент приобретает статус ключевого организационного механизма, обеспечивающего реализацию стратегических изменений. Современный этап эволюции управления проектами характеризуется переходом от преимущественно регламентированных «каскадных» практик к гибридным, адаптивным и технологически насыщенным подходам, что связано с распространением концепции Четвёртой промышленной революции (Industry 4.0). Она задаёт новую логику создания ценности за счёт интеграции цифровых технологий в производственные и управленческие контуры: промышленного Интернета вещей (IIoT), аналитики больших данных, роботизации и решений на базе искусственного интеллекта. В результате проекты всё чаще выступают не только способом достижения сроков и бюджета, но и инструментом цифровой трансформации - модернизации процессов, бизнес-моделей и цепочек поставок.

Зарубежные исследования подчёркивают, что цифровизация проектной деятельности формирует «социотехническую» трансформацию: эффект достигается не только внедрением технологий, но и переобучением команд, изменением ролей, механизмов принятия решений и культуры управления. В этом контексте ИИ и автоматизация усиливают данные-ориентированное управление проектами: расширяются практики предиктивной аналитики рисков и сроков, интеллектуального планирования ресурсов, автоматизированной отчётности и поддержки управленческих решений, включая применение генеративного ИИ для работы с требованиями, коммуникациями и проектной документацией. Одновременно усиливается тренд fit-for-purpose delivery - выбор методологии «под задачу» вместо универсального доминирования одного подхода; по данным PMI , результативность проектных команд определяется не столько типом методологии, сколько её соответствием контексту и зрелости организации. Дополнительно отмечается рост значимости системного управления портфелем (PPM) как «единого источника правды» для синхронизации инициатив цифровой трансформации, распределения ресурсов и контроля создаваемой ценности на уровне всей организации.

Эмпирические и обзорные работы по пересечению проектного менеджмента и Industry 4.0 фиксируют следующие устойчивые эффекты:

- повышение производительности и прозрачности операций за счёт мониторинга в реальном времени и сквозной цифровой трассируемости;
- снижение транзакционных издержек и потерь времени благодаря автоматизации рутинных процессов управления;

- рост качества управленческих решений вследствие использования аналитики и ИИ;
- более рациональное использование ресурсов и повышение устойчивости проектов, особенно в инфраструктурных и промышленных программах.



Рис.1. Концептуальная схема влияния Industry 4.0, инноваций, Agile и Lean подходов на результаты проектного управления
Источник: Составлено автором

Вместе с тем, массовое внедрение искусственного интеллекта, аналитики данных и автоматизации формирует новую технологическую среду, в которой традиционные модели проектного управления оказываются недостаточно адаптивными. В этих условиях особую роль приобретают инновации в проектном управлении, включая цифровые инструменты, гибридные методологии, а также Agile и Lean подходы, обеспечивающие гибкость и ориентацию на создание ценности. Их интеграция формирует механизм быстрой проектной реакции, основанный на итеративном принятии решений, приоритизации задач и непрерывном улучшении процессов. Представленная концептуальная схема отражает логику влияния Industry 4.0 и инноваций на результаты проектного менеджмента, демонстрируя взаимосвязь технологических драйверов, управленческих механизмов и достигаемых эффектов.

Табл.1.

Структура принятия управленческих решений и проектной адаптации в контексте Industry 4.0

Этап управленческого решения	Подходы и инструменты	Проектная реакция	Эффект
1. Формирование информации	IoT, Big Data, Lean visual management, Kanban	Оперативное выявление отклонений по срокам, затратам и качеству	Повышение прозрачности и производительности
2. Анализ и интерпретация	AI-аналитика, Lean value stream mapping, Agile metrics	Быстрая диагностика причин отклонений и узких мест проекта	Рост качества управленческих решений
3. Принятие решения	Автоматизация, Lean стандартизация, Agile backlog prioritization	Немедленная корректировка приоритетов, объёма работ и последовательности задач	Снижение транзакционных издержек и временных потерь
4. Реализация и адаптация	Гибридные методы, Agile итерации, Kaizen	Гибкая перенастройка плана проекта и перераспределение ресурсов	Рациональное использование ресурсов и устойчивость проектов

Источник: Составлено автором

Анализ представленной концептуальной схемы позволяет уточнить логику формирования управленческих решений и раскрыть механизмы проектной реакции на изменения среды на различных этапах управленческого цикла. В представленной ниже таблице формализована логика принятия управленческих решений в проектном управлении в условиях Industry 4.0, учитывающая роль инноваций, Lean и Agile подходов; каждый этап управленческого процесса сопоставлен с соответствующими механизмами проектной реакции и результирующими эффектами.

В совокупности рассмотренные в таблице механизмы принятия управленческих решений и проектной реакции демонстрируют качественное изменение роли проектного менеджмента в условиях цифровой трансформации. Усиление инновационной и адаптивной составляющей управления, а также интеграция Lean и Agile подходов в управленческий цикл приводят к переосмыслению традиционных функций проектного контроля. В совокупности это смещает фокус проектного менеджмента от контроля исполнения к управлению ценностью и организационными способностями - цифровыми компетенциями, межфункциональным взаимодействием и адаптивностью, что соответствует международной повестке развития «проектной профессии» в условиях цифровой экономики.

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К построение математической модели двухмерных задач фильтрации структурированных флюидов в двухслойном несвязанном пласте

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DOI 10.5281/zenodo.18224644

Аннотация. В работе рассмотрена задача математического моделирования двухмерных, аномально структурированных флюидов, в несвязанной двухслойной пластовой среде. Решения задачи построена с использованием метода итерации, метода фиктивных областей, метода переменных направлений, метода прямых и метода потоковой прогонки. Для подвижных границ предложен, использовать метод «челночных» итераций. На тестовых данных проведена апробации алгоритмов расчета.

Ключевые слова. Аномальные, структурные, флюид, моделирование, пористые, слоистые, пласт, изолированные, границы, подвижные, метод, переменный, фиктивные, потоковое, алгоритмы, схемы.

Введение. Известно что, все основные запасы полезных ископаемых, в частности флюидов: нефти, газа, газоконденсата, минеральные и термальные воды потребляемый в народном хозяйстве, находятся в подземных пористых средах. По своему построение эти среды является многослойными, и их можно квалифицировать, как многопластовые среды имеющие границы пластов как проницаемыми, так и непроницаемыми кровлями и подошвами. Если эти пласты гидро динамически связанные, то при фильтрации флюидов происходить обмен между пластами, и их называют

перетоками из одного пласта в другую. Перетоки зависит от структуры среды а также от величины горизонтальных и вертикальных характеристик пласта. При непроницаемом случаи , эти пласты изолированный друг от друга и переток в них не происходить, из одного слоя к другому. В таких средах взаимообмен происходить только через скважины, если она вскрыта по высоте каждого пласта. Однако при этом будет неизвестно, какая величина отбора выходить из каждого пласта в единой дебит общей скважины. Для многопластовых пористых сред разработан различные модели фильтрации и они исследован, для ньютоновских флюидов в [1-5]. К исследование процесса фильтрации, неньютоновских и неструктурированных флюидов в многослойных пластах работах посвящены работы [4-6].

Задача фильтрации аномально структурированных флюидов для изолированных двухслойных сред рассмотрен в [7], а для трехслойных сред в [8]. Процесс вытеснение различных флюидов в двухслойных изолированных пластах изучена в [9,10]. По исследование задачи нелинейной фильтрации флюидов в неизолированных многопластовых структурах при различных моделей фильтрации существует широкий набор исследований [11,12], где исследован динамика изменения пластовых давлений и величины коэффициентов перетоков между пластами.

В работе [13] исследован эволюция поверхности раздела при вытеснения вязких жидкостей. Эти исследование проведена в основном для одномерных математических моделей который является частным случаем многомерных задач.

Постановка задачи. Рассмотрим гидродинамически несвязанную двухпластовую среду, где нижний область $D_1(x,y)$ насыщена структурными флюидами. Верхний пласт – область $D_2(x,y)$ насыщена флюидами, имеющей тоже области аномальных подвижностей, где движения подчиняется варианту криволинейного закона фильтрации. Пластовые характеристики у этих пластов по мощности одинаковые и постоянные, что предполагает рассмотреть двухмерные уравнение. Ввиду того что эти пласты гидродинамически не связанные, то изменение физических и механических характеристик пласта, могут происходит в результате разработки их через совместно пробуренных связывающих скважин. Такую физическую задачи математически моделируется так: надо найти непрерывную функции $u(x,y,t)$ и $V(x,y,t)$ определенную в области $D_1(x,y)$ и $D_2(x,y)$ а также подвижные границы возмущенных зон $S_i(x,y,t),\ R_i(x,y,t)$ из следующей краевой задачей:

$$\frac{\partial}{\partial x}\left(A_i\big(|\nabla u|,\beta_i\big)\,\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_i\big(|\nabla u|,\beta_i\big)\,\frac{\partial u}{\partial y}\right)=M_i\,\frac{\partial u}{\partial t}+F_1(x,y,t),(x,y)\in D_1,\ t>0. \tag{1}$$

$$\frac{\partial}{\partial x}\left(B_i\big(|\nabla V|,\beta_i\big)\,\frac{\partial V}{\partial x}\right)+\frac{\partial}{\partial y}\left(B_i\big(|\nabla V|,\beta_i\big)\,\frac{\partial V}{\partial y}\right)=M_2\,\frac{\partial V}{\partial t},\qquad (x,y)\in D_2,\ t>0. \tag{2}$$

Initial conditions

$$u(x,y,0)=u_0(x,y),\qquad V(x,y,0)=V_0(x,y), \tag{3}$$

$$S_i(x,y,0)=S_0,\qquad R_i(x,y,0)=R_0. \tag{4}$$

Conditions on unknown moving boundaries

$$A_1\big(|\nabla u|,\beta_1\big)\ \frac{\partial u}{\partial n}\bigg|_{n\in S_1-0} = A_2\big(|\nabla u|,\beta_2\big)\ \frac{\partial u}{\partial n}\bigg|_{n\in S_1+0},\ u(x,y,t)|_{n\in S_1-0} = u(x,y,t)|_{n\in S_1+0}\,. \tag{5}$$

$$A_2\big(|\nabla u|,\beta_2\big)\ \frac{\partial u}{\partial n}\bigg|_{n\in S_2-0} = A_3\big(|\nabla u|,\beta_3\big)\ \frac{\partial u}{\partial n}\bigg|_{n\in S_2+0},\ u(x,y,t)|_{n\in S_2-0} = u(x,y,t)|_{n\in S_2+0}\,. \tag{6}$$

$$B_1\big(|\nabla V|,\beta_1\big)\ \frac{\partial V}{\partial n}\bigg|_{n\in R_1-0} = B_2\big(|\nabla V|,\beta_2\big)\ \frac{\partial V}{\partial n}\bigg|_{n\in R_1+0},\ V(x,y,t)|_{n\in R_1-0} = V(x,y,t)|_{n\in R_1+0}\,. \tag{7}$$

$$B_2\big(|\nabla V|,\beta_2\big)\ \frac{\partial V}{\partial n}\bigg|_{n\in R_2-0} = B_3\big(|\nabla V|,\beta_3\big)\ \frac{\partial V}{\partial n}\bigg|_{n\in R_2+0},\ V(x,y,t)|_{n\in R_2-0} = V(x,y,t)|_{n\in R_2+0}\,. \tag{8}$$

Условия на совместной скважине с контуром в γ_r.

$$\int_{\gamma_r}\left(\frac{H_1K_1}{\mu_1}\,\rho_1\,\frac{\partial u}{\partial n} + \frac{H_2K_2}{\mu_2}\,\rho_2\,\frac{\partial V}{\partial n}\right)\,d\sigma = Q(t), \tag{9}$$

где

$$Q(t) = Q_1(t) + Q_2(t). \tag{10}$$

Считаем естественные границы областей D_1 и D_2 замкнутыми, т. е. отсутствует внешняя подпитка. Тогда выполняются условия

$$b_1A\big(|\nabla u|,\beta\big)\ \frac{\partial u}{\partial n}\bigg|_{(x,y)\in\Gamma_1} = 0, \tag{11}$$

$$b_2B\big(|\nabla V|,\beta\big)\ \frac{\partial V}{\partial n}\bigg|_{(x,y)\in\Gamma_2} = 0. \tag{12}$$

В задаче (1)–(12) естественные параметры пласта и флюида $k_1(x,y)$, $k_2(x,y)$, $m_1(x,y)$, $m_2(x,y)$, $\mu_1(x,y)$, $\mu_2(x,y)$, ρ_1, ρ_2, H_1, H_2, $v_1(x,y)$, $v_2(x,y)$ входят в функции A_i и B_i и отражают пластовые и физические характеристики флюидов, как это показано в работах [3,4,13]. Функции A_i и B_i в зависимости от принадлежности к различным зонам структурированного флюида принимают различные выражения.

$$A\big(|\nabla u|,\beta\big) = \begin{cases} \dfrac{k_1(x,y)\,\beta_1}{\nu_1(x,y)\,h_1}, & (x,y)\in\big((x_1,y_1),\,S_1(x,y,t)\big),\\[2ex] \dfrac{k_1(x,y)\,\beta_2}{\nu_1(x,y)\,h_1}\,\dfrac{|\nabla u|-\beta_2\gamma_0}{|\nabla u|}, & (x,y)\in\big(S_1(x,y,t),\,S_2(x,y,t)\big),\\[2ex] \dfrac{k_1(x,y)\,\beta_3}{\mu_1(x,y)\,h_1}, & (x,y)\in\big(S_2(x,y,t),\,\Gamma_1(x,y)\big). \end{cases}$$

$$B\big(|\nabla V|,\beta\big)=\begin{cases}\dfrac{k_2(x,y)\,\bar\beta_1}{\nu_2(x,y)\,h_2}, & (x,y)\in\big((x_0,y_0),\,R_1(x,y,t)\big),\\[2ex] \dfrac{k_2(x,y)\,\bar\beta_2}{\nu_2(x,y)\,h_2}\,\dfrac{|\nabla V|-\bar\beta_2^{-1}\gamma_0}{|\nabla V|}, & (x,y)\in\big(R_1(x,y,t),\,R_2(x,y,t)\big),\\[2ex] \dfrac{k_2(x,y)\,\bar\beta_3}{\nu_2(x,y)\,h_2}, & (x,y)\in\big(R_2(x,y,t),\,\Gamma_2(x,y)\big).\end{cases}$$

Если возникает необходимость, то могут быть использованы другие виды аппроксимации в зонах аномальных подвижностей и ползучести. Для нелинейной задачи (1)–(12) аналитическое решение построить невозможно, поэтому решение ищется приближёнными методами.

Методы. Ввиду того, что границы области фильтрации $\Gamma_1(x,y)$ и $\Gamma_2(x,y)$ являются произвольными естественными контурами и описываются непрерывной функцией в виде уравнения поверхности, при дискретизации области сеточные узлы могут не совпадать с естественными линиями границы. В этом случае возникает задача учёта границы области, которая может располагаться между двумя узловыми точками.

Тогда начальные значения прогонных коэффициентов или потоков задаются с использованием интерполяционных полиномов, которые необходимо строить по нескольким приграничным точкам. Это, в свою очередь, требует разработки обучающих алгоритмов для распознавания и отслеживания естественной границы. Однако можно избегать от этой процедуры если использовать метод фиктивных областей [14,15]. Согласно методу, исходные произвольные области Γ_1 и Γ_2 дополняются до прямоугольных областей $\overline{D}_1$ и $\overline{D}_2$ путём введения фиктивных функций относительно каждого физического и геометрического параметра задачи (1)–(12).

При этом эти функции принимают вид:

$$K_{1\varepsilon_1}=\begin{cases}K_1(x,y), & (x,y)\in D_1,\\ \varepsilon_1K_1(x,y), & (x,y)\in(\overline{D}_1\setminus D_1),\end{cases}\qquad K_{2\varepsilon_2}=\begin{cases}K_2(x,y), & (x,y)\in D_2,\\ \varepsilon_2K_2(x,y), & (x,y)\in(\overline{D}_2\setminus D_2),\end{cases}$$

$$M_{1\varepsilon_1}=\begin{cases}M_1, & (x,y)\in D_1,\\ \varepsilon_1M_1, & (x,y)\in(\overline{D}_1\setminus D_1),\end{cases}\qquad M_{2\varepsilon_2}=\begin{cases}M_2, & (x,y)\in D_2,\\ \varepsilon_2M_2, & (x,y)\in(\overline{D}_2\setminus D_2).\end{cases}$$

Здесь $\overline{D}_1$ и $\overline{D}_2$ — Где $\overline{D}_1$ и $\overline{D}_2$ прямоугольные области, $\varepsilon_{1,2}\in(0;1)$ — бесконечно малый параметр, регулирующие исходные и фиктивные области. Ввод такого параметра равносильно предположение, что функцию $u(x,y,t)$ и $V(x,y,t)$ представить как сумма двух функций с регулирующим параметром

$$\bar u(x,y,t)=u(x,y,t)+\varepsilon_1\,\tilde u(x,y,t),\qquad \bar V(x,y,t)=V(x,y,t)+\varepsilon_2\,\tilde V(x,y,t).$$

После этого задача (1)–(12) записывается относительно функций $\bar u(x,y,t)$ и $\bar V(x,y,t)$ (мы его не приведем). В дальнейшем не будем различать $\bar u$ и u, а также $\bar V$ и V, считая, что исходная область является прямоугольник состоящей как из фиктивный так и из естественный подобласти $\overline{D}_1$ и $\overline{D}_2$.

При разработке и построение вычислительного алгоритма придерживаем исходную обозначение задачи (1)–(12), подразумевая что там есть исходный и фиктивный область.

Вводим функции потоков:

$$W_{x,i}(x,y,t) = A_i\big(|\nabla u|,\beta\big)\,\frac{\partial u}{\partial x}, \qquad W_{y,i}(x,y,t) = A_i\big(|\nabla u|,\beta\big)\,\frac{\partial u}{\partial y}, \tag{13}$$

$$\omega_{x,i}(x,y,t) = B_i\big(|\nabla V|,\bar{\beta}\big)\,\frac{\partial V}{\partial x}, \qquad \omega_{y,i}(x,y,t) = B_i\big(|\nabla V|,\bar{\beta}\big)\,\frac{\partial V}{\partial y}. \tag{14}$$

С учётом их уравнение (1)–(3) примет вид

$$\frac{\partial W_{x,i}}{\partial x} + \frac{\partial W_{y,i}}{\partial y} = M_1\,\frac{\partial u}{\partial t} + F_1(x,y,t), \qquad (x,y)\in D_1,\ t>0, \tag{15}$$

$$\frac{\partial \omega_{x,i}}{\partial x} + \frac{\partial \omega_{y,i}}{\partial y} = M_2\,\frac{\partial V}{\partial t}, \qquad (x,y)\in D_2,\ t>0. \tag{16}$$

$$u(x,y,0) = u_0(x,y), \qquad V(x,y,0) = V_0(x,y), \tag{17}$$

условия (5)–(8) запишется так

$$W_{n,1}(x,y,t)\big|_{(x,y)\in S_1-0} = W_{n,1}(x,y,t)\big|_{(x,y)\in S_1+0}\,, \tag{18}$$

$$W_{n,2}(x,y,t)\big|_{(x,y)\in S_2-0} = W_{n,2}(x,y,t)\big|_{(x,y)\in S_2+0}\,. \tag{19}$$

$$\omega_{n,1}(x,y,t)\big|_{(x,y)\in R_1-0} = \omega_{n,1}(x,y,t)\big|_{(x,y)\in R_1+0}\,, \tag{20}$$

$$\omega_{n,2}(x,y,t)\big|_{(x,y)\in R_2-0} = \omega_{n,2}(x,y,t)\big|_{(x,y)\in R_2+0}\,. \tag{21}$$

При этом (9) записывается так

$$\oint_{\gamma_r}\big(W_n+\omega_n\big)\,d\sigma = Q. \tag{22}$$

$$W_n\big|_{n\in\overline{D}_1} = 0, \qquad \omega_n\big|_{n\in\overline{D}_2} = 0. \tag{23}$$

где W_n, ω_n — радиальные потоки на γ_r, W_n, ω_n — граничные потоки с учётом параметров контуров области.

Решаем задачи (15)–(23) с применением метода расщепления по переменному t и метода переменных направлений [16,17] по пространственным переменным [18–20], получаем систему разностных уравнений в потоковых сетках [21,22].

Эту процедуру запишем относительно задачи (15). При этом индекс i, определяющий принадлежность уравнений к областям фильтрации структурированных флюидов, считаем само собой разумеющимся, и далее индексы (i, j, k) используются как дискретизация области по переменным (x, y, t), а именно:

$$x_i = ih_1, \qquad y_j = jh_2, \qquad t_k = k\tau.$$

Таким образом имеем:

$$\frac{1}{h_1}\left[(W_x)^{k+\frac{1}{2}}_{i+\frac{1}{2},j} - (W_x)^{k+\frac{1}{2}}_{i-\frac{1}{2},j}\right] + \frac{1}{h_2}\left[(W_y)^{k}_{i,j+\frac{1}{2}} - (W_y)^{k}_{i,j-\frac{1}{2}}\right] = \frac{M^{(s-1)}}{0{,}5\tau}\left(u^{k+\frac{1}{2}}_{i,j} - u^{k}_{i,j}\right) + (f_1)^{k+\frac{1}{2}}_{i,j}, \tag{24}$$

$$(W_x)^{k+\frac{1}{2}}_{i-\frac{1}{2},j} = X^{(s-1)}_{i-\frac{1}{2},j}\,\frac{1}{h_1}\left(u^{k+\frac{1}{2}}_{i,j} - u^{k+\frac{1}{2}}_{i-1,j}\right), \tag{25}$$

$$\frac{1}{h_1}\left[(W_x)^{k+\frac{1}{2}}_{i+\frac{1}{2},j} - (W_x)^{k+\frac{1}{2}}_{i-\frac{1}{2},j}\right] + \frac{1}{h_2}\left[(W_y)^{k+1}_{i,j+\frac{1}{2}} - (W_y)^{k+1}_{i,j-\frac{1}{2}}\right] = \frac{M^{(s-1)}}{0{,}5\tau}\left(u^{k+1}_{i,j} - u^{k+\frac{1}{2}}_{i,j}\right) + (f_1)^{k+1}_{i,j}, \tag{26}$$

$$(W_x)^{k+1}_{i,j-\frac{1}{2}} = X^{(s-1)}_{i,j-\frac{1}{2}}\,\frac{1}{h_2}\left(u^{k+1}_{i,j} - u^{k+1}_{i,j-1}\right). \tag{27}$$

где

$$(W_x)^{k+\frac{1}{2}}_{i+\frac{1}{2},j} = (W_x)^{(s)}\left(A_{i+\frac{1}{2},j},\, y_j,\, t_{k+\frac{1}{2}},\, u^{k+\frac{1}{2}}_{i+\frac{1}{2},j}\right), \qquad (W_y)^{k}_{i,j+\frac{1}{2}} = (W_y)^{(s)}\left(A_i,\, y_{j+\frac{1}{2}},\, t_k,\, u^{k}_{i,j+\frac{1}{2}}\right),$$

$$A^{(s-1)}_{i-\frac{1}{2},j} = A\left(\left|\nabla u^{(s-1),k+\frac{1}{2}}_{i-\frac{1}{2},j}\right|, \beta_{i-\frac{1}{2},j}\right).$$

Такие же формулы аналогично можно записать и для $j + \frac{1}{2}$. При этом граничные условия (17)–(23) тоже будут дискретными (их не приводим).

Аналогично строится система разностных уравнений и для (16). Из-за громоздкости их здесь не приводим. Полученные системы, а также система уравнений (24)–(27) с соответствующими начальными и краевыми условиями решается потоковым вариантом метода прогонки [16,17].

Использование потоков (13) и (14) позволяет не выделять особые точки при итерационных вычислениях. Так как функции $A(|\nabla u|, \beta)$ и $B(|\nabla V|, \beta)$ сильно меняются, но поток за счет колебания величины производных остается конечным, что обеспечивает устойчивости вычислений.

Ход вычислений численных алгоритмов по прямой и по обратном направлениям аналогично задачи [13,18–25]. Положение продвижение подвижных границ в структурированных задачах определяются аналогично как в задачах с границей разделом. При этом рекомендуется использовать метод челночных итераций [21,22]. Если оба пласта эксплуатируется единой скважиной (условия (22)), когда отсутствует другие внутренние скважины по каждому пласту, определения величины дебитов, отдельно

первого и второго пласта, требует использования отдельной методики. Она предполагает учет запаса флюида по каждому пласту с целью определения сроки разработки.

Можно, искусственно задать, нулевые дебиты извлекаемый из каждого пласта с обеспеченном планового отбора $Q(t)$ на период эксплуатации месторождений. Относительно дебита можно использовать линейных регрессионных формул, при этом задавая коэффициенты регрессии приближенно.

Обсуждение результатов. Приведена частичные результаты полученные после апробации с использованием тестовых данных при следующих значений параметров: $u_0 = v_0 = 1, \mu_1 = \mu_2 = 0{,}01, h_1 = h_2 = 0{,}01214, f_r = q = 0{,}1121, \beta = 10^{-4}$, $m = 0{,}1, k = 0{,}21, M_1 = M_2 = 1, \Delta\tau = 0{,}00012$. Результаты приведены таблице 1.

Таблица 1. Изменение поля давлений в области D_1.

$y \setminus x$	0,0	0,2	0,4	0,5	0,6	0,8	1,0
0,0	0,9999	0,9998	0,9976	0,9923	0,9976	0,9998	0,9999
0,2	0,9964	0,9902	0,9891	0,9822	0,9891	0,9902	0,9964
0,4	0,9926	0,9891	0,9702	0,9697	0,9702	0,9891	0,9926
0,5	0,9923	0,9822	0,9697	0,9501	0,9697	0,9822	0,9923
0,6	0,9926	0,9881	0,9702	0,9697	0,9702	0,9891	0,9926
0,8	0,9964	0,9902	0,9891	0,9822	0,9891	0,9902	0,9964
1,0	0,9999	0,9998	0,9976	0,9923	0,9976	0,9998	0,9999

Если область D_2 — круг и в точках $(0{,}25; 0{,}25)$, $(0{,}8; 0{,}3)$, $(0{,}5; 0{,}35)$, $(0{,}8; 0{,}5)$, $(0{,}25; 0{,}5)$, $(0{,}5; 0{,}65)$, $(0{,}8; 0{,}75)$, $(0{,}9; 0{,}75)$ имеет источники, то имеем результаты, которые приведены в таблице 2 (здесь тоже приведена давления для одного пласта и поскольку совместные источники одинаковые, то картина в обоих областях тоже аналогично).

Таблица 2. Поля давления с учетом фиктивных областей.

$y \setminus x$	0,00	0,1	0,25	0,35	0,5	0,75	0,8	0,9	1,0
0,0	0,0000	0,0000	0,0000	0,0000	0,9897	0,0000	0,0000	0,0000	0,0000
0,10	0,0000	0,0000	0,9873	0,9884	0,9893	0,9884	0,9873	0,0000	0,0000
0,25	0,0000	0,9857	0,9740	0,9865	0,9870	0,9865	0,9740	0,9857	0,0000
0,35	0,0000	0,9861	0,9853	0,9862	0,9750	0,9862	0,9855	0,9861	0,0000
0,5	0,9965	0,9757	0,9733	0,9855	0,9861	0,9855	0,9733	0,9857	0,9965
0,65	0,0000	0,9818	0,9855	0,9862	0,9750	0,9862	0,9855	0,9861	0,0000
0,75	0,0000	0,9857	0,9741	0,9865	0,9870	0,9865	0,9746	0,9857	0,0000
0,90	0,0000	0,0000	0,9872	0,9881	0,9895	0,9884	0,9873	0,0000	0,0000
1,00	0,0000	0,0000	0,0000	0,0000	0,9897	0,0000	0,0000	0,0000	0,0000

Таблица 3. Результаты расчета при пяти скважин для разных t

$y \setminus x$	t=0,026			t=0,50		
	0,25	0,50	0,75	0,25	0,50	0,75
0,25	0,9702	0,9998	0,9702	0,9015	0,9905	0,9015
0,50	0,9998	0,9700	0,9998	0,9905	0,8988	0,9905
0,75	0,9702	0,9998	0,9702	0,9015	0,9905	0,9015

Заключение

Приведенные частичные результаты расчета, позволяют сделать вывод, что в зависимости от расположений источников, давления обоих пластах изменяется однообразно, так как коэффициенты и параметры взято одинаковыми и постоянными и поэтому при эксплуатации пласта, единой скважиной расположенный в центре области, линии изобар почти распространяется как круг. Если в области расположены различные источники, а исходная область круг, то значения давлении будет различными. Здесь можно наблюдать влияние скважин и интерференции давления. Там, где в таблице имеется угловые ячейки нули, то здесь имеется фиктивные области. Предложенные математические модели и численные алгоритмы позволяет провести вычислительный эксперимент с целью определение прогнозных параметров при эксплуатации месторождений аналогичного типа.

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Математическое моделирование нелинейных дробных осцилляторов и их приложения в биофизике

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DOI 10.5281/zenodo.18224663

Аннотация. Приводится информация о новых достижениях в области математического моделирования нелинейных дробных осцилляторов и их приложений в задачах биофизики. Дробные осцилляторы описывают колебательные системы, в которых их текущие состояния зависят от предыдущих, т.е. от предыстории. В статье сначала даются предварительные сведения, вводятся понятия дробной производной и дробного осциллятора. Далее дается математическое описание широкого класса нелинейных дробных осцилляторов, а также в частном случае, приводится математическое описание дробных осцилляторов Ван дер Поля-Эйри, Зимана, ФитцХью-

Нагумо. Все эти дробные осцилляторы имеют приложения в задачах биофизики: механической работы человеческого сердца, ЭКГ, распространение нервного импульса в клеточной мембране. Методика исследования дробных осцилляторов основана на применении численных методов: нелокальной явной конечно-разностной схемы и метода Адамса-Башфорта-Мултона из семейства методов предиктор-корректор. Приводятся примеры применения численных алгоритмов к решению соответствующих математических моделей дробных осцилляторов, строятся осциллограммы и фазовые траектории. Даются некоторые аспекты качественного анализа: бифуркационный анализ, амплитудно-частотный анализ для вынужденных колебаний. В заключении дается анонс задач, которые необходимо решить для развития данной тематики исследований.

Ключевые слова: математическое моделирование, дробные осцилляторы, эффекты наследственности, дробные производные, осциллограммы, фазовые траектории, предельный цикл.

Abstract. This article presents information on new advances in the mathematical modeling of nonlinear fractional oscillators and their applications to biophysical problems. Fractional oscillators describe oscillatory systems in which their current states depend on previous ones, i.e., on the underlying history. This article begins with preliminary information and introduces the concepts of fractional derivatives and fractional oscillators. It then provides a mathematical description of a broad class of nonlinear fractional oscillators, as well as a specific mathematical description of the van der Pol-Airy, Zeeman, and FitzHugh-Nagumo fractional oscillators. All of these fractional oscillators have applications in biophysical problems, including the mechanical work of the human heart, electrocardiograms, and the propagation of nerve impulses in cell membranes. The methodology for studying fractional oscillators is based on the application of numerical methods: a nonlocal explicit finite-difference scheme and the Adams-Bashforth-Multon method from the predictor-corrector family of methods. Examples of applying numerical algorithms to solving the corresponding mathematical models of fractional oscillators are provided, and oscillograms and phase trajectories are constructed. Some aspects of qualitative analysis are discussed: bifurcation analysis and amplitude-frequency analysis for forced oscillations. The conclusion provides an overview of the problems that need to be solved to advance this research topic.

Keywords: mathematical modeling, fractional oscillators, heredity effects, fractional derivatives, oscillograms, phase trajectories, limit cycle.

Annotatsiya. Ushbu maqolada matematik modellashtirish sohasida nochiziqli kasrli osillyatorlar va ularning biofizika masalalaridagi qo‘llanilishiga oid yangi yutuqlar haqida ma’lumot keltiriladi. Kasrli osillyatorlar — bu tebranishli tizimlar bo‘lib, ularning joriy holatlari avvalgi holatlarga, ya’ni tizimning o‘tmishdagi holatlariga bog‘liq bo‘ladi. Maqolada avvalo dastlabki ma’lumotlar berilib, kasrli hosila va kasrli osillyator tushunchalari kiritiladi.

Keyin nochiziqli kasrli osillyatorlarning keng sinfining matematik tavsifi, shuningdek xususiy hollarda Van der Pol–Eyri, Ziman, FitzHugh–Nagumo kasrli osillyatorlarining matematik modellari bayon etiladi.

Ushbu kasrli osillyatorlarning barchasi biofizika masalalarida jumladan: inson yuragining mexanik ishi, EKG, nerv impulsining hujayra membranasi bo‘ylab tarqalishi

kabi jarayonlarni tavsiflashda qo'llaniladi. Kasrli osillyatorlarni tadqiq etish metodikasi sonli usullarni qo'llashga asoslanadi: nolokal aniq chekli-ayirmali sxema hamda prediktor–korrektor usullar oilasiga mansub Adams–Bashfort–Multon usuli.

Kasrli osillyatorlarning mos matematik modellarini yechishda sonli algoritmlarni qo'llash misollari keltirilib, osillogrammalar va fazaviy trayektoriyalar quriladi. Sifat tahlilining ayrim jihatlari — bifurkatsion tahlil, majburiy tebranishlar uchun amplituda–chastota tahlili ko'rib chiqiladi. Xulosada ushbu tadqiqot yo'nalishini rivojlantirish uchun hal etilishi lozim bo'lgan masalalar e'lon qilinadi.

Kalit so'zlar: matematik modellashtirish, kasrli osillyatorlar, irsiyat effektlari, kasr hosilalar, osillogrammalar, fazaviy trayektoriyalar, chegaraviy sikl.

Введение. Исследование нелинейных колебательных систем является важной прикладной задачей в различных областях наук. Это связано с тем, что колебания характеризуют повторяющиеся процессы или движения вблизи положения равновесия, свойственные природе и технике: от качания маятника и пульсации сердца до звуковых и световых волн.

Обычно колебания описываются с помощью производных целочисленного порядка. Однако в случае, когда колебательная система обладает памятью, которая проявляется в том, что ее текущее состояние зависит от предыдущих, нам необходимо использовать интегро-дифференциальные уравнения [1]. Частный случай интегро-дифференциальных уравнений при специальном выборе ядра подынтегрального выражения приводит нас к математическому аппарату дробных производных [2]. Поэтому нелинейные колебательные системы, которые учитывают эффекты памяти называют дробными осцилляторами.

В настоящей работе приведен некоторый обзор новых результатов, полученных учениками научной школы «Применение дробного исчисления к математическому моделированию динамических процессов» в рамках изучения нелинейного дробного осциллятора, который описывает релаксационные колебания (автоколебания). Автоколебания имеют важное практическое значение при исследовании биологических систем, например, описывают механическую работу сердца или возбуждение нейрона в клеточной мембране. В работе с помощью специально разработанного программного обеспечения на языке программирования Python при водятся примеры, которые подтверждают существование автоколебаний. Наличие эффектов памяти в биологических системах и методика их описания может привести нас к разработке метода идентификации заболеваний сердца или нервной системы. Это является конечной целью наших исследований.

Предварительные сведения. Введем некоторые понятия и определения. **Определение 1.** Дробной производной порядка $\alpha \in (1,2)$ для функции $x(t) \in C^2[0,T]$ в смысле Герасимова-Капуто будем называть оператор:

$$\partial_{0t}^{\alpha} x(t) = \frac{1}{\Gamma(2-\alpha)} \int_0^t \frac{\ddot{x}(\tau)}{(t-\tau)^{\alpha-1}}\, d\tau. \tag{1}$$

где $(\cdot)$ - гамма-функция Эйлера, $T > 0$ - конечное время, $\ddot{x}(t) = \frac{d^2x}{dt^2}$.

Определение 2. Дробной производной порядка $\beta \in (0,1)$ для функции $x(t) \in C^1[0,T]$ в смысле Герасимова-Капуто будем называть оператор:

$$\partial_{0t}^{\beta} x(t) = \frac{1}{\Gamma(1-\beta)} \int_0^t \frac{\dot{x}(\tau)}{(t-\tau)^{\beta}}\, d\tau \tag{2}$$

где $\dot{x}(t) = \frac{dx}{dt}$.

Определение 3. Дробной производной порядка $\alpha(t) \in (1,2)$ и $\alpha(t) \in C[0,T]$ для функции $x(t) \in C^2[0,T]$ типа Герасимова-Капуто будем называть оператор:

$$\partial_{0t}^{\beta} x(t) = \frac{1}{\Gamma(1-\beta)} \int_0^t \frac{\dot{x}(\tau)}{(t-\tau)^{\beta}}\, d\tau \tag{2}$$

Определение 4. Дробной производной порядка $\beta(t) \in (0,1)$ и $\beta(t) \in C[0,T]$. Для функции $x(t) \in C^1[0,T]$ типа Герасимова-Капуто будем называть оператор:

$$\partial_{0t}^{\beta(t)} x(t) = \frac{1}{\Gamma(1-\beta(t))} \int_0^t \frac{\dot{x}(\tau)}{(t-\tau)^{\beta(t)}}\, d\tau \tag{4}$$

Определение 5. Осциллятор — это система, которая совершает периодические или квазипериодические колебания вокруг точки равновесия.

Определение 6. Если осциллятор описывается с помощью дробных производных (1)-(4), то тогда его будем называть дробным осциллятором. Если осциллятор описывается с помощью целочисленных производных, то тогда осциллятор будем называть классическим.

Пример математического описания классического осциллятора может иметь вид:

$$\ddot{x}(t) + \lambda(x(t),t)\dot{x}(t) + \omega(t)x(t) = f(x(t),t), \tag{5}$$
$$x(0) = k_1, \qquad \dot{x}(0) = k_2.$$

Осциллятор (5) представляет собой задачу Коши для описания нелинейных колебаний. Первое слагаемое в левой части уравнения (5) здесь отвечает за силу инерции, второе – за силу трения, с причем нелинейным коэффициентом $\lambda(x(t),t)$, третье слагаемое – это возвращающая сила с частотой собственных колебаний $\omega(t)$. В правой части уравнения (5) находится сила внешнего воздействия, причем она может содержать нелинейные слагаемые от решения $x(t)$, которые указывают на не изохронность колебаний.

Обобщением классического нелинейного осциллятора (5) является дробный нелинейный осциллятор.

Постановка задачи и методика решения. Рассмотрим следующую задачу Коши:

$$\partial_{0t}^{\alpha(t)} x(t) + \lambda(x(t),t)\partial_{0t}^{\beta(t)} x(t) + \omega(t)x(t) = f(x(t),t), \quad x(0) = k_1,\ \dot{x}(0) = k_2 \tag{6}$$

Задача Коши (6) представляет собой дробный нелинейный осциллятор, который является объектом нашего исследования. В силу того, что этот осциллятор описывает

широкий класс дробных нелинейных осцилляторов мы остановимся на следующих его частных случаях.

Случай 1. Дробный осциллятор Ван дер Поля-Эйри [3].

$\lambda(x(t),t) = \lambda_0(ax^2(t) - b)$, λ_0, a, b положительные константы; $\omega(t) = ct$, c-положительная константа; $f(x(t),t) = \delta cos(\varphi t)$, $\delta\varphi$- амплитуда и частота внешнего воздействия, $\alpha(t) = \alpha_0$, $\beta(t) = \beta_0$ - положительные константы. Случай 2. Дробный осциллятор ФитцХью-Нагумо [4].

$\lambda(x(t),t) = \lambda_0(x^2(t) + p)$, λ_0, p-положительные константы; $\omega(t) = q$, q-положительная константа $f(x(t),t) = a + bz(t) - gx^3(t)$, a, b-заданные константы, $1 - \frac{2b}{3} < a < 1$, $0 < b < 1$, $b < \lambda_0^2$; $\alpha(t), \beta(t), z(t)$-заданные функции из класса $C[0,T]$

Случай 3. Дробный осциллятор Зимана [5]. Дробный осциллятор Зимана можно записать в виде задачи (6) со следующими параметрами:

$\lambda(x(t),t) = \frac{3x^2(t)-1}{\epsilon}$; $\omega(t) = \frac{1}{\epsilon}$; $f(x(t),t) = 0; \epsilon >$ 0-малый параметр, $\alpha(t) = \alpha_0$, $\beta(t) = \beta_0$

Все три случая обладают общим свойством, они описывают автоколебания, которые часто возникают в биологических системах. Например, они могут описывать колебания человеческого сердца, например, сокращение мышечного волокна под действием электрохимического потенциала (Случай 3) или активацию и деактивацию нейрона в клеточной мембране в зависимости от интенсивности раздражителя (Случай 2).

Для каждого из трех случаев строилась нелокальная явная конечно-разностная схема первого порядка точности. Шаг сетки выбирался достаточно малым, для сохранения устойчивости схемы. Кроме того, для некоторые случаев (Случай 1 и 2) строился также метод Адамса-Башфорта-Мултона из семейства предиктор-корректор, который показал более высокую точность, чем нелокальная явная конечно-разностная схема.

Результаты исследования. На рис.1 приведены результаты исследований (Случаи 1, 2, 3).

Здесь мы видим, что Случаи 1-3 описывают автоколебательные режимы, фазовые траектории выходят на предельный цикл. Осциллограммы это тоже косвенно подтверждают. Важный вопрос пока остается открытым является ли этот предельный цикл устойчивым? С помощью компьютерный экспериментов было показано, что не всегда. Однако этот факт должен подтверждаться строгим математическим доказательством.

Заключение. В работе приведены примеры трех случаев нелинейного дробного осциллятора (6). С помощью численного анализа и программного обеспечения написанного на языке программирования Python показано наличие автоколебательных режимов. Вопрос устойчивости таких режимов пока остается открытым. Однако с помощью разработанного программного обеспечения в настоящее время проводится качественный анализ нелинейных дробных осцилляторов (Случаи 1-3): исследуются бифуркации, хаотические и регулярные режимы, который поможет выявить колебательные режимы, которые соответствуют отклонениям от нормы в работе сердца или нервной системы.

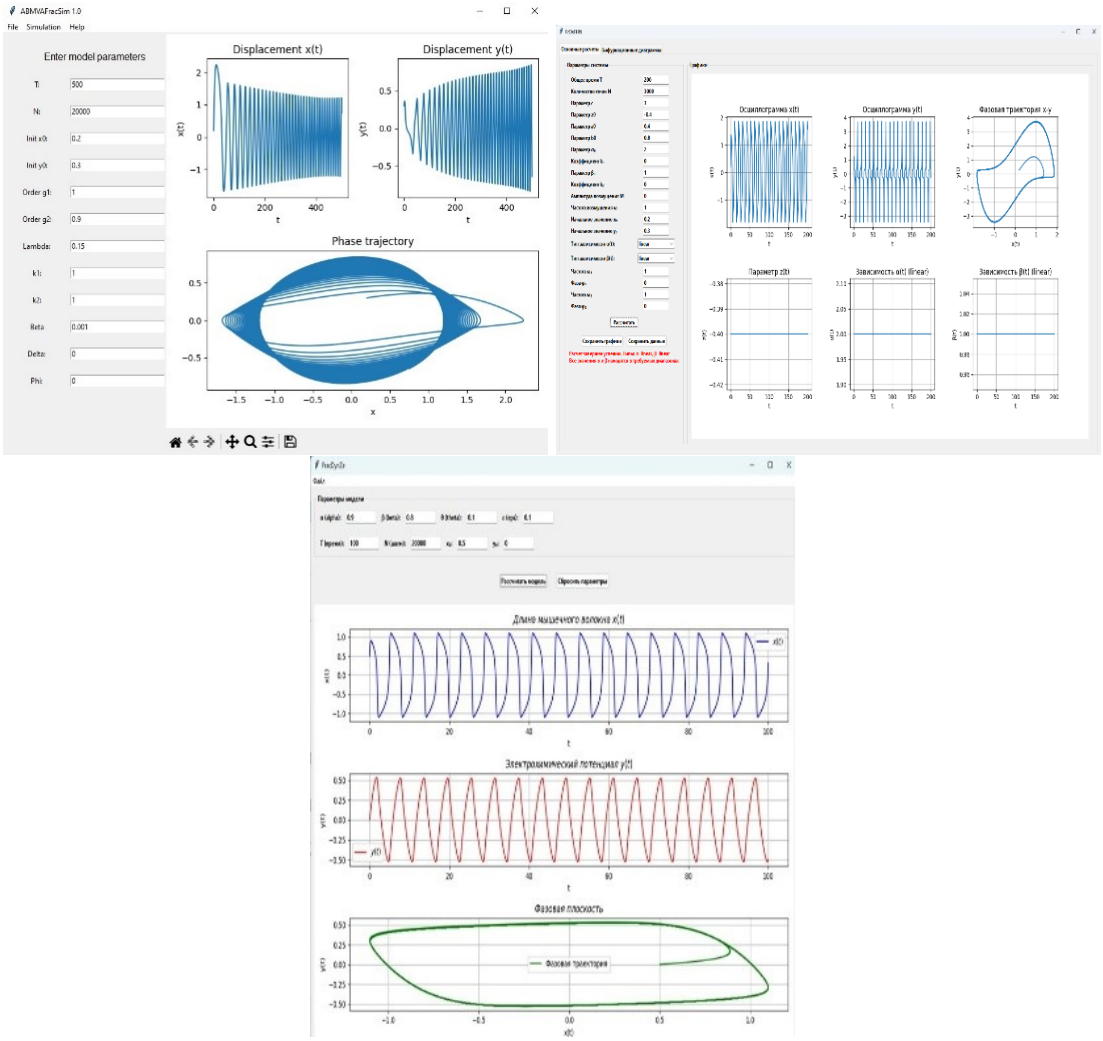

Рис. 3: Результаты исследования: а) Случай 1; б) Случай 2; в)Случай 3

Работа выполнена в рамках рамочного соглашения о сотрудничестве между ИКИР ДВО РАН и Национальным университетом Узбекистана имени Мирзо Улугбека (г. Ташкент, Узбекистан) №1118 от 28.04.2022 (0468/01/22 НТМИ) (2022-2027 гг.) по физико-математическим наукам.

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Многомерные статистические методы в анализе цифрового развития туристического сектора

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DOI 10.5281/zenodo.18224693

Аннотация. В статье исследуются особенности цифрового развития туристического сектора на основе применения многомерных статистических методов. Целью исследования является выявление ключевых латентных факторов цифрового туризма, построение композитного индекса цифрового развития туристической отрасли и типологизация стран по уровню и структуре цифровизации. Эмпирическую базу составляют международные статистические данные, отражающие состояние цифровой инфраструктуры, человеческого капитала, инновационной активности, внешнеэкономических аспектов ИКТ и туристической инфраструктуры. В работе последовательно применяются корреляционный анализ, факторный анализ, метод главных компонент (PCA) и кластерный анализ. На основе PCA сформирован композитный индекс цифрового туризма, позволяющий провести межстрановой сравнительный анализ и построить рейтинг стран. Кластерный анализ выявил четыре устойчивые группы стран, различающиеся по уровню цифровой зрелости туристического сектора. Отдельное внимание уделено позиции Республики Узбекистан и формированию рекомендаций по приоритетным направлениям цифровой трансформации туризма. Полученные результаты могут быть использованы при разработке государственной политики и стратегий цифрового развития туристической отрасли.

Ключевые слова: *Цифровой туризм; многомерный статистический анализ; факторный анализ; метод главных компонент; кластерный анализ; композитный индекс; международные сопоставления.*

Введение

В условиях глобализации и цифровой трансформации экономики особую значимость приобретает развитие экспорта услуг, включая туризм, как одного из наиболее динамичных и доходных секторов мировой экономики. Туризм занимает ключевое место в формировании положительного имиджа страны, стимулирует развитие сопутствующих отраслей и способствует притоку иностранной валюты. В 2024 году туристическая отрасль обеспечила около 10% совокупного объема мировой экономики, внеся вклад в глобальный валовой внутренний продукт, оцениваемый в 10,9 трлн долларов США [1].

Мировая экономика в настоящее время находится на стадии широкого внедрения информационно-коммуникационных технологий, которые кардинально меняют методы и практики ведения бизнеса. Необходимость перехода на новый уровень развития обусловлена тем, что сегодня более половины населения мира онлайн. В 2023 году количество пользователей Интернета достигла около 5,6 миллиарда [2], что составляет более 68% от общего количество населения мира. В то же время повышение экспортного потенциала туристической отрасли требует внедрения инновационных подходов, среди которых цифровизация выступает в качестве одного из наиболее эффективных инструментов. Пандемия COVID-19 стала важным фактором, ускорившим цифровую трансформацию всех сфер экономики и активизировавшим применение цифровых платформ. Цифровизация открывает новые возможности для расширения рынков сбыта, повышения доступности туристических продуктов и улучшения качества обслуживания клиентов. Внедрение цифровых технологий позволяет не только оптимизировать внутренние бизнес-процессы, но и создать интегрированные платформы для взаимодействия участников туристического рынка на международном уровне. Ярким примером цифровизации индустрии туризма являются сети Airbnb и Booking Holdings, активы которых достигли в 2024 году примерно 21 млрд [3] и 27 млрд долларов США [4] соответственно. Данные компании предоставляют онлайн-площадку для туристов для бронирования гостиниц по всему миру.

Литературный обзор.

Цифровые технологии радикально изменили стиль жизни и работы людей, их подходы к развитию бизнеса и выбору маршрутов для путешествий. Через Интернет люди могут найти нужную информацию, выбрать пункт назначения или создать и разместить контент, который может повлиять на имидж места назначения. Это приводит к созданию совершенно новой экосистемы цифрового туризма – устойчивой и ответственной. При оцифровке аналоговые процессы и данные преобразуются в машиночитаемые форматы. Сегодня цифровизация преобладает в каждом сегменте туристической экосистемы. Например, полностью оцифрованные бизнес-процессы, цифровые продажи туристических услуг, предоставление услуг, поиск информации, онлайн-бронирование и запросы предложений, отзывы потребителей после путешествия и т.д.

Таблица 1. Факторы, определяющие на уровень развития цифрового туризма.

Классификация	Показатели	Авторы
Цифровизация	Подписка на мобильную сотовую связь (на 100 чел.)	Xinna Zhao [5], Rui Tang [6,7], Huyen Nguyen [7], Samira Borouji [8], Hao Wang [9], Qiang Ma [10], Gen Nian [11], Xiaoli Lu [12]
	Абоненты фиксированной широкополосной связи (на 100 чел.)	Xinna Zhao [5], Rui Tang [6,7], Huyen Nguyen [8], Samira Borouji [9], Hao Wang [10], Qiang Ma [11], Gen Nian [12]
	Лица, пользующиеся Интернетом (% от населения)	Rui Tang [6,7], Huyen Nguyen [8], Samira Borouji [9], Hao Wang [10], Gen Nian [12], Xiaoli Lu [13]
	Расходы на исследования и разработки (% от ВВП)	Xinna Zhao [5], Qiang Ma [11], Xiaoli Lu [13]
	Подписка на фиксированную телефонную связь (на 100 чел.)	Xinna Zhao [5], Huyen Nguyen [8], Qiang Ma [11], Gen Nian [12], Xiaoli Lu [13]
Человеческий капитал	Расходы на образование в % от ВВП	Xinna Zhao [5], Rui Tang [6,7], Haitao Wu [14], Qiang Ma [11], Xiaoli Lu [13]
	Число студентов колледжей на 10 000 человек	Xinna Zhao [5]
Конкуренто-способность экономики	Средства связи, компьютеры и т.д. в % от экспорта коммерческих услуг	Rui Tang [6,7], Haitao Wu [14], Hao Wang [10]
	Экспорт продукции ИКТ в процентах от общ его объема экспорта продукции	Rui Tang [6,7], Haitao Wu [14], Hao Wang [10]
	Экспорт продукции ИКТ в процентах от общ его объема экспорта продукции	Rui Tang [6,7], Haitao Wu [14], Hao Wang [10]
Туристическая инфраструктура	Общее количество гостиниц и анал. заведений на 1000 кв.км	Xinna Zhao [5], Anton O. [15], Ramos [16]
	Общее количество номеров в отелях и анал. на 100 человек	Xinna Zhao [5], Anton O. [15], Ramos [16], Kosnan [17]

Цифровой туризм предполагает применение цифровых технологий для интеграции цифрового контента в реальный мир, что способствует более глубокому и информативному восприятию путешествий. Многие исследователи определили это как цифровой туристический бизнес (DTB), где Интернет используется в качестве средства продаж и маркетинга. Для оценки уровня развития цифровизации туристической отрасли современными учеными применялись различные методы исследования. Китайские исследователи Синна Джао, Сюэхуэй Мэй и Чжэнцин Сяо (Xinna Zhao, Dr. Xuehui Mei, Zhengqing Xiao) [5] в своей работе «Влияние цифровой экономики на качественное развитие туризма – эмпирическое исследование Синьцзяна в Китае» использовали три метода исследования: метод взвешивания энтропии, модель с фиксированными эффектами и модель посреднического эффекта. В следующей таблице приведена система показателей развития цифрового туризма.

Согласно исследованиям ученых, цифровые показатели, приведённые в таблице 1 положительно влияют на развитие цифрового туризма. Цифровой туризм рассматривается в современной научной литературе как результат глубокой трансформации туристической отрасли под воздействием информационно-коммуникационных технологий. Исследователи отмечают, что переход от традиционного туризма к циф-

ровому туризму сопровождается изменением моделей поведения туристов, способов продвижения туристских услуг и механизмов управления дестинациями. Цифровые технологии обеспечивают доступность информации, снижение транзакционных издержек и персонализацию туристского опыта, что повышает конкурентоспособность туристских направлений.

Особое внимание в исследованиях уделяется концепции «умного туризма», основанной на интеграции цифровой инфраструктуры, больших данных, мобильных приложений и искусственного интеллекта. Умные туристские дестинации используют цифровые решения для управления туристскими потоками, повышения качества услуг и устойчивого развития туризма. Эмпирические исследования подтверждают, что развитие цифровой инфраструктуры и онлайн-сервисов положительно влияет на туристскую активность и экспорт туристских услуг, особенно в странах с открытой экономикой.

В то же время в литературе подчеркивается наличие институциональных и инфраструктурных ограничений цифрового туризма в развивающихся странах. Цифровое неравенство, ограниченный доступ к технологиям и недостаточная цифровая зрелость туристского сектора сдерживают реализацию потенциала цифровизации. Это определяет необходимость дальнейших исследований, направленных на выявление механизмов влияния цифровых технологий на развитие туризма и формирование эффективной государственной политики в данной сфере.

Методология исследования

Целью настоящего исследования является выявление ключевых латентных факторов, определяющих уровень развития цифрового туризма, а также разработка композитного индекса цифрового туризма и типологизация стран на основе методов многомерного статистического анализа. Для достижения поставленной цели в работе используется система показателей, отражающих уровень развития информационно-коммуникационной инфраструктуры, степень цифровой связанности населения, инновационную и научно-технологическую активность, внешнеэкономические аспекты в сфере информационно-коммуникационных технологий и высоких технологий, а также состояние туристической инфраструктуры. На первом этапе проводится предварительный статистический анализ данных, включающий оценку описательных характеристик показателей и анализ корреляционных связей между ними, а также проверку пригодности исходной совокупности данных для применения факторного анализа.

На следующем этапе применяется факторный анализ, позволяющий выявить скрытые (латентные) факторы, формирующие уровень развития цифрового туризма. Полученные факторы интерпретируются с экономической точки зрения и используются для обобщённого описания основных направлений цифрового развития туристического сектора, что способствует сокращению размерности исходного пространства признаков и устранению избыточности информации.

С целью количественной интеграции многомерных характеристик цифрового туризма в работе реализуется метод главных компонент (Principal Component Analysis, PCA). На основе результатов PCA формируется композитный индекс цифрового туризма, отражающий совокупный уровень цифрового развития туристического сектора стран. Значения главных компонент используются для определения весов от-

дельных показателей в структуре индекса, что обеспечивает его объективность и статистическую обоснованность. Рассчитанный композитный индекс позволяет провести сравнительный анализ стран и сформировать их рейтинг по уровню развития цифрового туризма.

Заключительным этапом исследования является проведение кластерного анализа, направленного на группировку стран по уровню и структуре развития цифрового туризма. Кластеризация позволяет выявить однородные группы стран с близкими характеристиками цифрово-туристического развития, определить типологические модели и различия в траекториях цифровой трансформации туристического сектора. Совместное использование факторного анализа, метода главных компонент и кластерного анализа обеспечивает комплексный и системный подход к исследованию цифрового туризма в международном разрезе.

Таблица 2. Система показателей для построения композитного индекса

Группа показателей	Показатель	Обозначение	Единица измерения
Цифровая инфраструктура	Мобильная сотовая связь	**MOB**	подписок на 100 человек
	Фиксированная широкополосная связь	**BB**	подписок на 100 человек
	Пользователи Интернета	**INT**	% населения
	Фиксированная телефонная связь	**TEL**	подписок на 100 человек
Человеческий капитал и инновации	Расходы на образование	**EDU**	% ВВП
	Расходы на НИОКР	**RD**	% ВВП
Конкурентно-способность страны	Экспорт ИКТ-услуг	**ICTS**	% экспорта услуг
	Экспорт высокотехнологичных товаров	**HTX**	% промышленного экспорта
	Экспорт ИКТ-товаров	**ICTX**	% экспорта товаров
Туристическая инфраструктура	Количество гостиниц	**HOT**	Единиц
	Количество гостиничных номеров	**ROOM**	Единиц

Статистическая база и показатели исследования

Эмпирическую основу исследования составляют официальные статистические данные, характеризующие уровень развития цифровой инфраструктуры, инновационного и научно-технологического потенциала, внешнеэкономической активности в сфере информационно-коммуникационных технологий, а также состояние туристической инфраструктуры. Выбор показателей обусловлен их способностью комплексно отражать ключевые аспекты цифрового развития, непосредственно влияющие на формирование и функционирование цифрового туризма.

Перед применением методов многомерного статистического анализа был проведён предварительный анализ исходных данных с целью оценки их общей структуры, уровня вариации и межстрановой неоднородности. Для этого рассчитаны основные описательные статистики показателей цифрового туризма, включающие меры центральной тенденции и разброса. Результаты данного анализа представлены в таблице 1 и позволяют получить первичное представление о характере распределения показателей, а также обосновать необходимость последующей нормализации данных и применения факторного анализа и метода главных компонент.

Таблица 3. Описательные статистики показателей цифрового туризма

Indicator	Mean	Median	Min	Q1	Q3	Max	SD
MOB	128,67	125,24	67,55	126,56	126,56	319,49	33,11
BB	25,04	26,91	0,06	25,11	25,11	55,88	15,81
INT	82,44	87,21	20,37	85,45	85,45	100	17,57
TEL	20,56	15,4	0	18,19	18,19	111	18,84
EDU	1,08	0,63	0,03	0,87	0,87	5	1,21
RD	4,14	4,1	0,38	4,16	4,16	7,2	1,33
ICTS	6,41	2,8	0	4,07	4,07	62,47	10,58
HTX	12,82	9,63	0	10,57	10,57	72,32	13,7
ICTX	35,57	37,9	0	35,4	35,4	78,24	22,4
HOT	15628,3	13929	13	4306,03	4306,03	405311	53241
ROOM	274458	159554	591	133728	133728	565000	639543

Описательные статистики показывают значительную межстрановую дифференциацию показателей цифрового туризма, особенно по параметрам туристической инфраструктуры и экспорта ИКТ и высокотехнологичной продукции. Показатели цифровой связанности (мобильная связь, доступ к Интернету) характеризуются высокими медианными значениями и относительно низкой вариативностью, что свидетельствует о сформированной базовой цифровой инфраструктуре в большинстве стран. В то же время высокая асимметрия и дисперсия инновационных и экспортных показателей указывают на концентрацию цифрово-туристического потенциала в ограниченном числе стран. Полученные результаты подтверждают целесообразность применения методов многомерного анализа для дальнейшей интегральной оценки цифрового туризма.

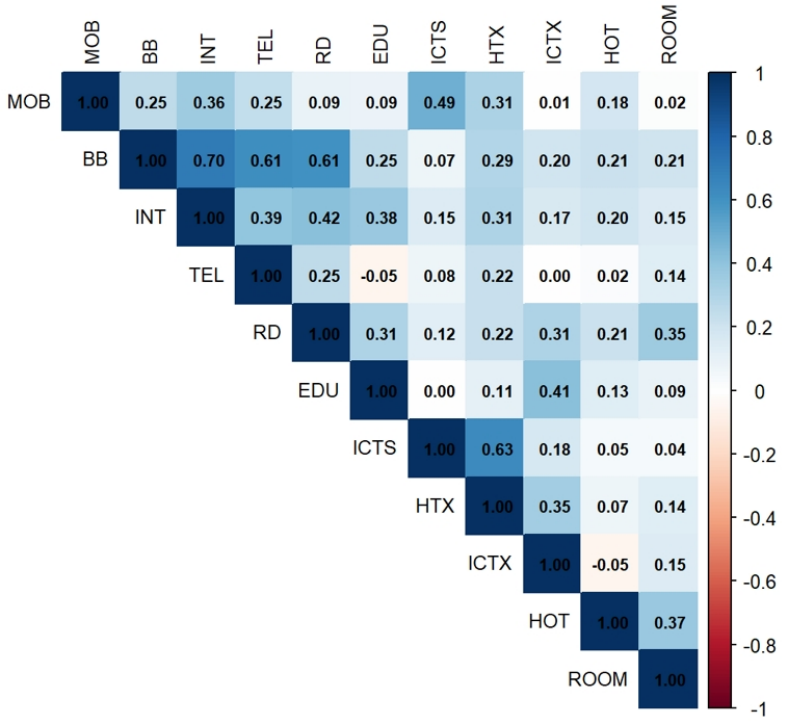

Рисунок 1. Корреляционная матрица показателей цифрового туризма

Перед применением факторного анализа и метода главных компонент необходимо выполнить корреляционный анализ, поскольку эти методы предполагают наличие статистически значимых взаимосвязей между исходными показателями. Корреляционная матрица позволяет оценить степень линейной зависимости между переменными, выявить группы тесно связанных индикаторов, а также предварительно проверить наличие мультиколлинеарности и избыточности информации. Наличие устойчивых корреляций служит эмпирическим обоснованием целесообразности снижения размерности данных и выделения латентных факторов, тогда как слабые связи могут указывать на ограниченную применимость факторных процедур к рассматриваемому набору показателей.

Корреляционный анализ выявил наличие устойчивых положительных взаимосвязей между показателями цифрового туризма, что подтверждает целесообразность применения факторного анализа и метода главных компонент. Наиболее тесные связи наблюдаются между показателями цифровой инфраструктуры: фиксированная широкополосная связь положительно коррелирует с использованием Интернета (r = 0,70) и фиксированной телефонной связью (r = 0,61). Существенная корреляция зафиксирована между экспортом ИКТ-услуг и экспортом высокотехнологичной продукции (r = 0,63), что отражает взаимосвязанность внешнеэкономических аспектов цифрового развития. Показатели туристической инфраструктуры демонстрируют умеренную взаимосвязь между собой (r = 0,37) и в целом слабые связи с цифровыми переменными, что указывает на относительную автономность масштабов туристического сектора. В целом структура корреляционной матрицы характеризуется наличием нескольких взаимосвязанных блоков показателей — цифровой инфраструктуры, инновационно-экспортного блока и туристической инфраструктуры. Данное распределение взаимосвязей подтверждает методологическую обоснованность использования факторного анализа для выявления латентных факторов цифрового туризма и

последующего построения композитного индекса.

Этап 1. Факторный анализ.

На первом этапе исследования для выявления ключевых латентных факторов, формирующих уровень развития цифрового туризма, применяется метод факторного анализа как один из базовых инструментов многомерного статистического анализа. Методологическая целесообразность его использования обусловлена наличием устойчивых корреляционных связей между исходными показателями, отражающими цифровую связанность, развитие ИКТ-инфраструктуры, человеческий капитал, инновационную активность и туристическую инфраструктуру. Факторный анализ позволяет представить наблюдаемые переменные в виде линейных комбинаций меньшего числа скрытых факторов, тем самым обеспечивая снижение размерности данных, устранение мультиколлинеарности и выявление структурных компонентов цифрового туризма. На данном этапе проводится проверка применимости метода к исходным данным, определяется количество значимых факторов и осуществляется их экономическая интерпретация, что создаёт методологическую основу для последующего построения композитного индекса цифрового туризма и проведения кластерного анализа стран. Оценка пригодности данных для факторного анализа. Для оценки пригодности исходных данных к применению факторного анализа проводится проверка наличия достаточных корреляционных связей между переменными. С этой целью используются критерий Кайзера–Мейера–Олкина (KMO), позволяющий оценить степень адекватности выборки, и тест сферичности Бартлетта, направленный на проверку гипотезы о единичности корреляционной матрицы.

Результаты критерия Кайзера–Мейера–Олкина свидетельствуют об удовлетворительной пригодности исходных данных для применения факторного анализа: общее значение KMO составляет 0,66, что превышает минимально допустимый порог. Индивидуальные показатели MSA для большинства переменных находятся в диапазоне 0,55–0,76, что указывает на приемлемый вклад отдельных показателей в формирование общей факторной структуры. Наименьшие значения MSA зафиксированы для показателей экспорта ИКТ-товаров и туристической инфраструктуры, однако они не опускаются ниже критического уровня и могут быть сохранены в модели на данном этапе анализа. Тест сферичности Бартлетта показал статистически значимый результат (
chi^2= 301,23, p $<$ 0,001), что позволяет отвергнуть нулевую гипотезу о единичности корреляционной матрицы. Это подтверждает наличие существенных корреляционных связей между переменными и обосновывает возможность выделения латентных факторов.

Выбор числа факторов. Для определения оптимального числа факторов используется анализ собственных значений корреляционной матрицы. В рамках данного этапа применяется график «каменистой осыпи» (scree plot), позволяющий визуально оценить вклад каждого фактора в объяснение общей дисперсии. Число факторов определяется на основе критерия Кайзера (собственные значения больше единицы) и анализа точки излома на scree plot, после которой вклад последующих факторов становится несущественным. Совместное использование указанных подходов обеспечивает обоснованный выбор факторной структуры для последующей интерпретации.

Собственные значения $\lambda_i = (3.4, 1.6, 1.4, 1.2, 0.95, 0.56, 0.53, 0.46, 0.39, 0.26, 0.17)$

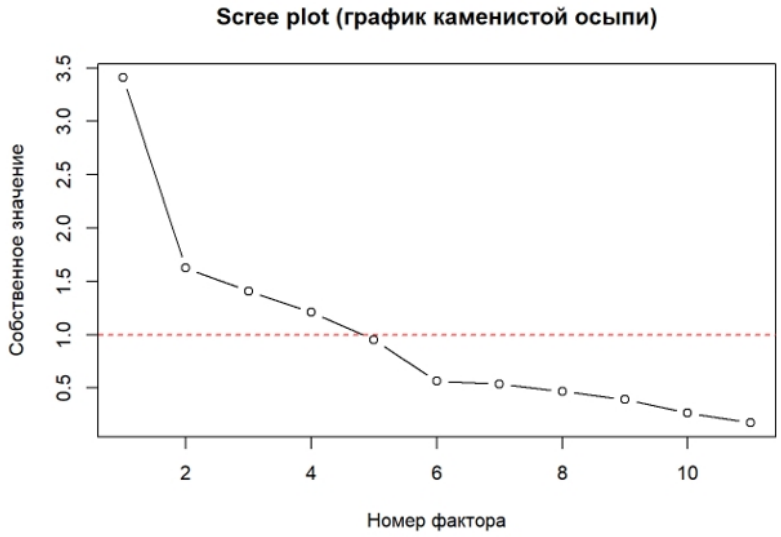

Рисунок 2. Scree plot собственных значений корреляционной матрицы

Анализ собственных значений корреляционной матрицы показал, что первые четыре фактора имеют собственные значения, превышающие единицу (3.41; 1.63; 1.41; 1.21), что соответствует критерию Кайзера. Начиная с пятого фактора собственные значения становятся меньше единицы, а их вклад в объяснение общей дисперсии существенно снижается. График каменистой осыпи демонстрирует отчётливую точку излома после четвёртого фактора, что подтверждает целесообразность выбора четырёхфакторной модели. Дальнейший факторный анализ и интерпретация результатов будут проводиться на основе четырёх латентных факторов, которые в совокупности отражают основную структуру взаимосвязей показателей цифрового туризма.

Интерпретация факторной структуры. Проведённый факторный анализ методом главных факторов с ортогональным вращением Varimax позволил выделить четыре латентных фактора, отражающих ключевые компоненты формирования цифрового туризма. Совокупно выделенные факторы объясняют 56% общей дисперсии исходных показателей, что является приемлемым результатом для социально-экономических исследований и свидетельствует о корректности выбранной факторной модели.

Первый фактор характеризуется высокими положительными нагрузками показателей фиксированной широкополосной связи (BB = 0.95), использования Интернета (INT = 0.63) и фиксированной телефонной связи (TEL = 0.65). Данный фактор отражает уровень развития телекоммуникационной и цифровой инфраструктуры и может быть интерпретирован как фактор «Цифровая инфраструктура и доступ к ИКТ», формирующий базовые условия для внедрения цифровых туристических платформ и сервисов.

Второй фактор формируется преимущественно за счёт показателя экспорта ИКТ-услуг (ICTS = 0.97) и экспорта высокотехнологичной продукции (HTX = 0.63). Высокие факторные нагрузки данных переменных указывают на внешнеэкономическую направленность данного фактора, который отражает степень интеграции стран в глобальные рынки цифровых и высокотехнологичных услуг. В связи с этим второй фактор может быть интерпретирован как «Внешнеэкономическая активность в

сфере ИКТ и высоких технологий».

Третий фактор объединяет показатели экспорта ИКТ-товаров (ICTX = 0.77), расходов на образование (EDU = 0.53) и расходов на исследования и разработки (RD = 0.42). Такая структура указывает на связь инновационного потенциала и человеческого капитала с производственно-технологическим аспектом цифровой экономики. Данный фактор интерпретируется как «Инновационно-производственный потенциал цифровой экономики», определяющий возможности создания и коммерциализации цифровых решений, включая туристические продукты.

Четвёртый фактор характеризуется высокими нагрузками показателей туристической инфраструктуры — числа гостиниц (HOT = 0.86) и количества гостиничных номеров (ROOM = 0.41). Он отражает масштаб и материальную обеспеченность туристического сектора и может быть обозначен как «Туристическая инфраструктура». Относительная независимость данного фактора от цифровых и инновационных показателей подчёркивает автономность инфраструктурного компонента в структуре цифрового туризма.

В целом полученная факторная структура подтверждает многокомпонентный характер цифрового туризма, включающий инфраструктурные, инновационные, внешнеэкономические и отраслевые элементы. Выделенные факторы создают методологическую основу для дальнейшего построения композитного индекса цифрового туризма и проведения кластерного анализа стран по уровню и структуре цифрового развития туристической отрасли.

Этап 2. Построение композитного индекса цифрового туризма на основе PCA.

На втором этапе исследования для получения интегральной количественной оценки уровня развития цифрового туризма применяется метод главных компонент (Principal Component Analysis, PCA). Использование PCA обусловлено необходимостью агрегирования многомерной системы взаимосвязанных показателей в единый композитный индекс, позволяющий осуществлять корректные межстрановые сопоставления. В отличие от факторного анализа, ориентированного на выявление латентной структуры данных, PCA направлен на максимальное сохранение общей дисперсии исходных переменных и обеспечивает объективное определение весов показателей на основе вкладов главных компонент.

Результаты анализа главных компонент показывают, что первая главная компонента (PC1) объясняет 30,99% общей дисперсии исходных показателей, что указывает на наличие доминирующего обобщающего фактора цифрового туризма. Вторая и третья компоненты (PC2 и PC3) объясняют соответственно 14,78% и 12,79% дисперсии, дополняя основную компоненту и отражая дополнительные аспекты цифрового и туристического развития. Совокупно первые четыре главные компоненты объясняют 69,56% общей дисперсии, что свидетельствует о высокой информативности сокращённого пространства признаков и обоснованности его использования для построения композитного индекса цифрового туризма. Вклад последующих компонент существенно ниже: начиная с пятой компоненты доля объяснённой дисперсии каждой из них не превышает 8,65%, а их предельный вклад носит маргинальный характер. Использование первых четырёх главных компонент позволяет сохранить основную информацию, содержащуюся в исходных данных, при одновременном су-

щественном снижении размерности. Полученные результаты подтверждают целесообразность применения PCA для формирования композитного индекса цифрового туризма и дальнейшего сравнительного анализа стран.

Формула композитного индекса цифрового туризма (PCA). Композитный индекс цифрового туризма рассчитывается как взвешенная сумма значений главных компонент:

$$\mathrm{DTI}_i = \sum_{k=1}^{m} w_k \cdot PC_{ik},$$

где DTI_i — значение композитного индекса цифрового туризма для i-й страны; PC_{ik} — значение k-й главной компоненты для i-й страны; w_k — вес k-й главной компоненты; m — число используемых компонент. Веса компонент определяются пропорционально доле объяснённой дисперсии каждой компоненты:

$$w_k = \frac{\lambda_k}{\sum_{j=1}^{m} \lambda_j},$$

где λ_k— собственное значение k-й главной компоненты.

Таблица 4. Определение весов компонент

Компонента	Собственное значение	Вес w_k
PC1	1.8463	**0.341**
PC2	1.2751	**0.236**
PC3	1.1861	**0.219**
PC4	1.1001	**0.204**
Сумма		**1.000**

Итоговая формула композитного индекса цифрового туризма представляет собой взвешенную сумму первых четырёх главных компонент, каждая из которых отражает отдельный аспект цифрового и туристического развития. Итоговая формула индекса в явном виде выглядет в следующем виде:

$$\mathrm{DTI}_i = 0.341 \cdot PC1_i + 0.236 \cdot PC2_i + 0.219 \cdot PC3_i + 0.204 \cdot PC4_i$$

Использование именно четырёх компонент обосновано результатами PCA, согласно которым они совместно объясняют 69,56% общей дисперсии исходных показателей (таблица 4), что является достаточным уровнем информативности для социально-экономических исследований.

Рейтинги стран по композитному индексу цифрового туризма. На основе рассчитанных значений композитного индекса цифрового туризма (DTI) был построен рейтинг стран. Полученный рейтинг отражает относительное положение стран по уровню развития цифрового туризма и позволяет провести сопоставимый межстрановой анализ. Более высокие значения индекса соответствуют более развитой цифровой инфраструктуре, инновационно-технологическому и внешнеэкономическому потенциалу, а также наличию развитой туристической инфраструктуры. Низкие значения индекса, напротив, указывают на структурные ограничения в цифровизации туристического сектора и недостаточную интеграцию цифровых технологий в туристическую деятельность. Рейтинг стран по композитному индексу цифрового туризма служит аналитической основой для выявления лидеров и аутсайдеров цифровой трансформации туризма, а также для последующей кластеризации стран по сходству цифрово-туристических характеристик.

Таблица 5. Лидеры и аутсайдеры по композитному индексу цифрового туризма

Country	DTI	Rank	Country	DTI	Rank
Monaco	1,941471	1	India	-1,10611	76
Liechtenstein	1,334419	2	Myanmar	-1,11434	77
Korea, Rep.	1,23455	3	Cambodia	-1,15955	78
France	1,201699	4	Turkmenistan	-1,27993	79
Germany	1,152069	5	Philippines	-1,34072	80
Andorra	1,039282	6	Zimbabwe	-1,40947	81
Japan	0,998095	7	Kenya	-1,44645	82
Portugal	0,996181	8	Angola	-1,47453	83
Belgium	0,94664	9	Mali	-1,53228	84
United Kingdom	0,940952	10	Madagascar	-1,78886	85

Этап 3. Кластерный анализ стран на основе исходных показателей цифрового туризма.

На третьем этапе исследования проводится кластерный анализ стран на основе исходной системы показателей, характеризующих цифровую инфраструктуру, инновационную активность, внешнеэкономические аспекты ИКТ и туристическую инфраструктуру. В отличие от предыдущих этапов, ориентированных на агрегирование информации и построение интегрального показателя, кластерный анализ направлен на выявление однородных групп стран с близкими многомерными характеристиками цифрового туризма. Кластеризация по исходным данным позволяет сохранить структурные различия между показателями и выявить типологические модели развития цифрового туризма, не сводя их к единой числовой оценке. Такой подход обеспечивает более детальное понимание различий между странами, позволяя идентифицировать группы с сопоставимыми уровнями цифровой связанности, инновационного потенциала и туристической инфраструктуры. Для определения числа кластеров применялась иерархическая кластеризация методом Уорда и анализ коэффициента силуэта. Дендрограмма используется для предварительного определения числа кластеров.

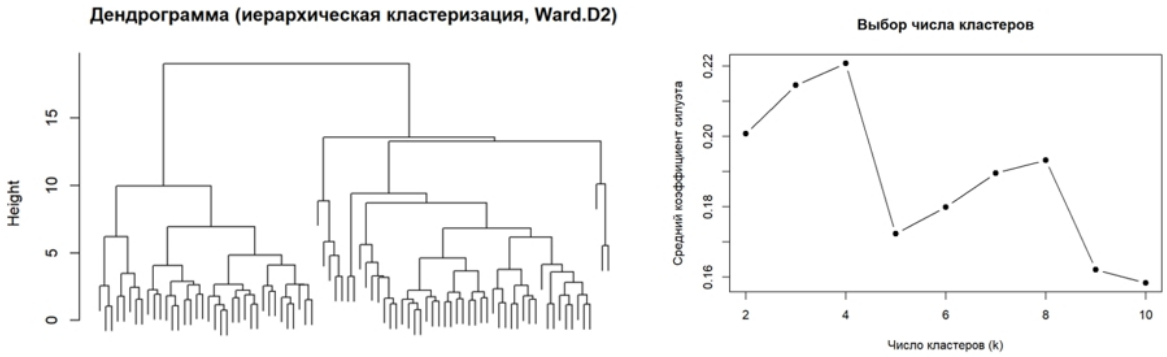

Рисунок 3. Определение оптимального числа кластеров

Анализ дендрограммы свидетельствует о наличии нескольких устойчивых кластеров стран, отражающих различия в уровне цифрового развития туристического сектора. Чёткое разделение на верхнем уровне иерархии указывает на существенную неоднородность цифровизации туризма. Визуальный анализ дендрограммы позволяет обоснованно выделить 3–4 кластера для последующего анализа. Максимальное значение среднего коэффициента силуэта наблюдается при $k = 4$, что указывает на наилучшее соотношение компактности и разделимости кластеров. Результаты представлены на рисунке 3. При увеличении числа кластеров значение коэффициента силуэта снижается, что свидетельствует об ухудшении качества кластеризации. Таким образом, выбор четырёх кластеров является статистически обоснованным и используется на следующем этапе анализа при применении метода k-means.

Таблица 6. Состав стран по кластерам

Кластер 1	Кластер 2	Кластер 3	Кластер 4
Ангола, Албания, Армения, Азербайджан, Бахрейн, Багамы, Босния и Герцеговина, Эквадор, Египет, Грузия, Гондурас, Гватемала, Ямайка, Иордания, Казахстан, Кения, Кыргызская республика, Камбоджа, Шри-Ланка, Марокко, Мадагаскар, Мальдивы, Мали, Мьянма, Монголия, Перу, Парагвай, Таджикистан, Туркменистан, Турция, Узбекистан, Сальвадор, Южная Африка, Иран, Зимбабве, Индия, Индонезия, Мексика	Объединённые Арабские Эмираты, Норвегия, Соединённые Штаты Америки	Андорра, Аргентина, Австралия, Австрия, Бельгия, Болгария, Китай, Кипр, Чехия, Германия, Дания, Испания, Эстония, Финляндия, Франция, Великобритания, Греция Хорватия, Венгрия, Италия, Япония, Республика Корея, Лихтенштейн, Люксембург, Латвия, Монако, Молдова, Маврикий, Нидерланды, Польша, Португалия, Румыния, Российская федерация, Саудовская Аравия, Словацкая республика, Словения, Швеция, Уругвай, Сербия	Гонконг, Кувейт, Мальта, Малайзия, Филиппины

Для экономической интерпретации результатов кластерного анализа были рассчитаны средние значения исходных числовых показателей цифрового развития туристического сектора по каждому кластеру.

Таблица 7. Кластерные центроиды по показателям цифрового развития туристического сектора

Кластер	MOB	BB	INT	TEL	RD	EDU	ICTS	HTX	ICTX	HOT	ROOM
1	-0,34	-0,86	-0,66	-0,55	-0,63	-0,37	-0,32	-0,46	-0,52	-0,23	-0,23
2	-0,38	0,96	0,85	-0,22	0,94	0,04	0,03	0,45	-0,52	4,74	2,49
3	0,12	0,8	0,5	0,51	0,59	0,32	-0,08	0,09	0,42	-0,11	0,06
4	1,48	-0,26	0,65	0,34	-0,4	-0,14	3,04	2,58	0,75	-0,24	-0,21

Анализ профилей кластерных центроидов показывает, что выделенные кластеры существенно различаются по уровню и структуре цифрового развития туристического сектора.

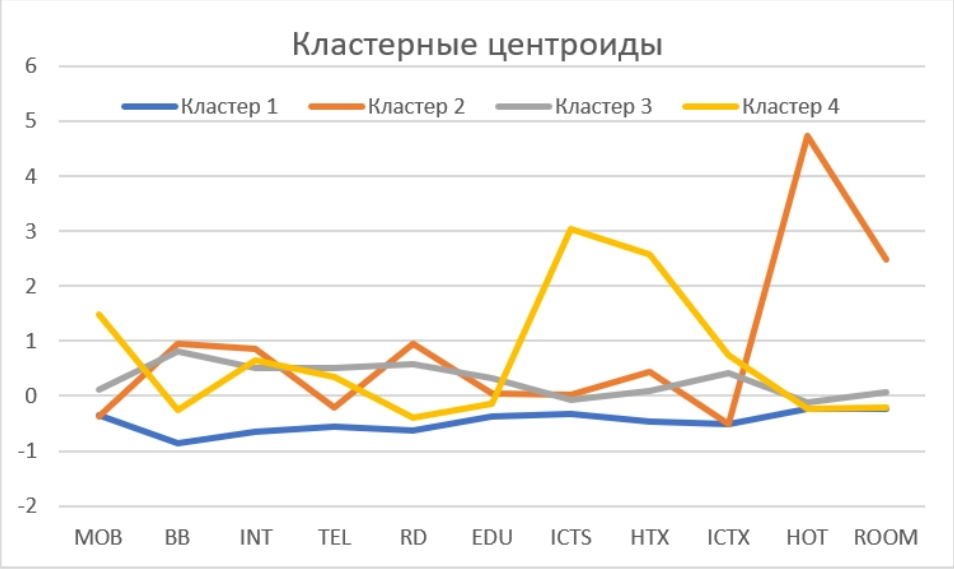

Рисунок 4. Профили кластерных центроидов

Кластер 1 (38 стран). Характеризуется показателей цифровой инфраструктуры и ИКТ-экспорта при сравнительно развитой гостиничной базе. Уровень использования интернета и широкополосного доступа ниже, чем в более развитых кластерах. Данный кластер отражает страны с ограниченной цифровизацией туризма, где традиционная инфраструктура преобладает над цифровыми сервисами.

Кластер 2 (3 страны). Отличается высокими значениями цифровых и инновационных показателей, включая расходы на НИОКР, экспорт ИКТ-услуг и высокотехнологичной продукции, а также максимальными значениями по числу гостиниц и номерного фонда. Этот кластер можно интерпретировать как группу цифровых и туристических лидеров.

Кластер 3 (39 стран).Представляет страны со средним уровнем цифрового развития, характеризующиеся сбалансированными значениями показателей цифровой инфраструктуры, интернет-доступа и гостиничного сектора. Данный кластер отражает переходную группу стран, находящихся на этапе активного развития цифровых технологий в туризме.

Кластер 4 (5 стран). Характеризуется очень высокими значениями показателей цифровой инфраструктуры (мобильная связь, интернет, широкополосный доступ), однако относительно более скромными масштабами гостиничного сектора по сравнению с кластером лидеров. Этот кластер отражает страны с высокой цифровой зрелостью, но меньшими масштабами туристической индустрии.

Проведённый кластерный анализ позволил выделить **четыре устойчивые типологические группы стран**, различающиеся по уровню и структуре цифрового развития туристического сектора. Полученные кластеры отражают **многоуровневый характер цифровизации туризма** — от стран с ограниченным внедрением цифровых технологий до цифровых лидеров. Результаты кластеризации создают основу для сравнительного анализа отдельных стран и выработки дифференцированных направлений цифровой трансформации туристического сектора.

Анализ позиции Республики Узбекистан в системе цифрового туризма и рекомендации по цифровой трансформации туристического сектора

Узбекистан отнесён к первому кластеру, который характеризуется преимущественно низкими и отрицательными стандартизированными значениями по большинству показателей цифрового развития туристического сектора. Это свидетельствует о сравнительно недостаточном уровне цифровой инфраструктуры, инновационной активности и экспорта ИКТ- и высокотехнологичных услуг по сравнению с более развитыми кластерами.

В то же время принадлежность Узбекистана к данному кластеру указывает на наличие значительного потенциала для цифровой трансформации туризма, прежде всего за счёт развития цифровых сервисов, расширения использования ИКТ в туристической деятельности и усиления связей между цифровым и традиционным сегментами отрасли. Полученные результаты могут быть использованы при формировании приоритетных направлений государственной политики и инвестиционных программ в сфере цифровизации туристического сектора Республики Узбекистан.

Рекомендации по направлениям цифрового развития туристического сектора Рес-

публики Узбекистан.

1. Развитие цифровой туристической инфраструктуры. Приоритетным направлением является расширение и модернизация цифровой инфраструктуры, включая доступность высокоскоростного интернета и мобильной связи в туристических регионах. Это создаёт базовые условия для внедрения цифровых сервисов, онлайн-бронирования и электронных платежей в туристической отрасли.

2. Внедрение цифровых платформ и сервисов в туризме. Целесообразно стимулировать развитие национальных и региональных цифровых туристических платформ, объединяющих информацию о размещении, транспорте, культурных объектах и услугах. Особое внимание следует уделить интеграции гостиниц, туроператоров и объектов культурного наследия в единые цифровые экосистемы. **3. Повышение цифровой грамотности и человеческого капитала.** Для устойчивого развития цифрового туризма необходимо усилить подготовку кадров в области ИКТ, цифрового маркетинга и аналитики данных в туризме. Рекомендуется внедрение специализированных образовательных программ и программ повышения квалификации для участников туристического бизнеса.

4. Поддержка цифровых инноваций и стартапов в туризме. Рекомендуется создание условий для развития стартапов и инновационных проектов в сфере цифрового туризма, включая применение технологий Big Data, искусственного интеллекта и интеллектуальных рекомендательных систем для персонализации туристических услуг.

5. Развитие цифрового маркетинга и международного продвижения. Для повышения международной конкурентоспособности туристического сектора важно активизировать использование цифровых каналов продвижения, онлайн-платформ и аналитических инструментов для анализа туристических потоков и предпочтений иностранных туристов.

6. Институциональная и нормативная поддержка цифровизации. Целесообразно совершенствовать нормативно-правовую базу, направленную на стимулирование цифровизации туристического сектора, включая меры государственной поддержки, цифровые стандарты и механизмы государственно-частного партнёрства.

Реализация указанных направлений позволит Узбекистану перейти от преимущественно традиционной модели туристического сектора к интегрированной цифровой модели развития, повысить эффективность управления туристической отраслью и укрепить её позиции на международном туристическом рынке.

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Системный анализ и моделирование международно-экономических процессов

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DOI 10.5281/zenodo.18224721

Аннотация В условиях углубления процессов глобализации, цифровизации и экономической нестабильности международно-экономические процессы приобретают всё более сложный и нелинейный характер, что снижает эффективность традиционных методов их анализа. В данной статье рассматриваются возможности применения системного анализа и моделирования для исследования международно-экономических процессов как сложных динамических систем. В качестве основной проблемы выделяется ограниченность классических экономических моделей, не учитывающих многоуровневые взаимосвязи, обратные связи и трансформационные процессы в мировой экономике. Для решения указанной проблемы предлагается использование системного подхода, включающего методы структурного анализа, системной динамики и моделирования. В результате исследования обоснована целесообразность применения системного анализа при изучении международно-экономических процессов, а также выявлены его преимущества для анализа, прогнозирования и поддержки принятия управленческих решений в сфере международной экономики. Полученные результаты могут быть использованы в научных исследованиях, аналитической деятельности и при разработке экономической политики в условиях глобальных трансформаций.

Annotatsiya Globallashuv, raqamlashtirish va iqtisodiy beqarorlik jarayonlarining chuqurlashuvi sharoitida xalqaro iqtisodiy jarayonlar tobora murakkab va nolinear xususiyat kasb etmoqda, bu esa ularni tahlil qilishda an'anaviy usullarning samaradorligini pasaytiradi. Mazkur maqolada xalqaro iqtisodiy jarayonlarni murakkab dinamik tizimlar sifatida o‘rganishda tizimli tahlil va modellashtirish usullarini qo‘llash imkoniyatlari ko‘rib chiqiladi. Tadqiqotning asosiy muammosi sifatida jahon iqtisodiyotidagi ko‘p darajali o‘zaro bog‘liqliklar, teskari aloqalar va transformatsion jarayonlarni yetarli darajada hisobga olmaydigan klassik iqtisodiy modellar cheklanganligi ajratib ko‘rsatiladi. Ushbu muammoni hal etish maqsadida tizimli yondashuvdan, jumladan, strukturaviy tahlil, tizimli dinamika va modellashtirish usullaridan foydalanish taklif etiladi. Tadqiqot natijasida xalqaro iqtisodiy jarayonlarni o‘rganishda tizimli tahlilni qo‘llashning maqsadga muvofiqligi asoslab berildi hamda uning tahlil, prognozlash va xalqaro iqtisodiyot sohasida boshqaruv qarorlarini qabul qilishni qo‘llab-quvvatlashdagi afzalliklari aniqlandi. Olingan natijalar ilmiy tadqiqotlar, tahliliy faoliyat hamda global transformatsiyalar sharoitida iqtisodiy siyosatni ishlab chiqishda qo‘llanilishi mumkin.

Abstract In the context of deepening globalization, digitalization, and increasing economic instability, international economic processes are becoming increasingly complex and nonlinear, which reduces the effectiveness of traditional methods of analysis. This article examines the possibilities of applying systems analysis and modeling to the study

of international economic processes as complex dynamic systems. The main problem addressed in the study is the limited capacity of classical economic models, which do not adequately account for multilevel interconnections, feedback mechanisms, and transformational processes in the global economy. To address this problem, a systems approach is proposed, incorporating methods of structural analysis, system dynamics, and modeling. As a result of the study, the expediency of using systems analysis in the examination of international economic processes is substantiated, and its advantages for analysis, forecasting, and decision-making support in the field of international economics are identified. The results obtained can be applied in scientific research, analytical activities, and in the development of economic policy under conditions of global economic transformation.

Ключевые слова: Системный анализ, моделирование, международно-экономические процессы, мировая экономика, системный подход, экономические системы, глобальные трансформации.

Введение:

Современное развитие мировой экономики характеризуется усилением процессов глобализации, цифровизации, интернационализации производства и финансовых потоков, а также ростом экономической нестабильности и неопределённости. [1;7] В этих условиях международно-экономические процессы приобретают сложный, многоуровневый и динамичный характер, что существенно осложняет их анализ и прогнозирование. Традиционные экономические подходы и модели, как правило, ориентированные на статическое равновесие и изолированное рассмотрение отдельных факторов, оказываются недостаточными для адекватного отражения реальных процессов, протекающих в мировой экономике. [6;7]

В связи с этим особую актуальность приобретает применение системного анализа и моделирования, позволяющих рассматривать международно-экономические процессы как целостные системы, состоящие из взаимосвязанных элементов и подсистем. [2;5] Системный подход обеспечивает возможность учёта нелинейных взаимосвязей, обратных связей, трансформационных процессов и влияния внешней среды, что особенно важно в условиях глобальных экономических изменений.

Основной проблемой исследования является ограниченность классических экономических моделей при анализе международно-экономических процессов, выражающаяся в неспособности в полной мере учитывать их комплексный и динамический характер. [6] Отсутствие системного взгляда на мировую экономику снижает эффективность аналитических выводов и управленческих решений, принимаемых на основе таких моделей.

В рамках данного исследования автором предпринята попытка обобщить и систематизировать существующие подходы к применению системного анализа и моделирования в изучении международно-экономических процессов. [2;4] Особое внимание уделяется рассмотрению мировой экономики как сложной динамической системы, а также выявлению возможностей использования системного инструментария для анализа и прогнозирования её развития.

Объектом исследования являются международно-экономические процессы в системе мировой экономики.

Предметом исследования выступают методы системного анализа и моделирова-

ния, применяемые для исследования международно-экономических процессов.

Цель исследования заключается в обосновании целесообразности и эффективности применения системного анализа и моделирования при изучении международно-экономических процессов. Для достижения поставленной цели в работе предполагается решение следующих задач:

• раскрыть сущность международно-экономических процессов как объекта системного анализа;

• обосновать применение системного подхода в их исследовании;

• рассмотреть основные методы моделирования международно-экономических процессов;

• определить преимущества системного анализа по сравнению с традиционными экономическими моделями.

В рамках исследования используются следующие базовые понятия. Системный анализ рассматривается как совокупность методов и подходов, направленных на изучение сложных систем с учётом структуры, функций и взаимосвязей элементов. Моделирование понимается как процесс построения формализованных моделей, отражающих ключевые характеристики и закономерности функционирования экономических систем. Международно-экономические процессы трактуются как совокупность экономических взаимодействий между субъектами мировой экономики, включающих торговые, финансовые, инвестиционные и интеграционные процессы.

Инструментарий исследования

Методологической основой данного исследования выступает системный анализ, который позволяет рассматривать международно-экономические процессы как сложные, многоуровневые и динамически развивающиеся системы. [2;5] В рамках системного подхода мировая экономика интерпретируется как совокупность взаимосвязанных элементов и подсистем, функционирование которых определяется структурными, функциональными и институциональными связями.

В качестве основных инструментов исследования используются методы структурного и функционального анализа, системной динамики, а также моделирования социально-экономических процессов. [5;9] Структурный анализ применяется для выявления ключевых элементов международно-экономической системы и характера их взаимодействия, в то время как функциональный анализ позволяет определить роль отдельных подсистем в обеспечении устойчивости и развития мировой экономики. Метод системной динамики используется для анализа нелинейных процессов, обратных связей и временных лагов, характерных для международных экономических взаимодействий. [1]

Моделирование в рамках исследования рассматривается как инструмент формализации международно-экономических процессов, позволяющий упростить их сложную структуру и выявить основные закономерности функционирования мировой экономики. Используемые модели носят концептуальный и аналитический характер и ориентированы на выявление причинно-следственных связей между ключевыми экономическими показателями.

Процесс исследования

Процесс исследования включает несколько последовательных этапов. На первом этапе проводится теоретический анализ научных источников, посвящённых системному анализу, моделированию и исследованию международно-экономических процессов. [2;8] Это позволяет сформировать целостное представление о сущности системного подхода и определить его место в современной экономической науке.

На втором этапе осуществляется идентификация международно-экономических процессов как объекта системного анализа. В рамках данного этапа мировая экономика рассматривается как открытая система, взаимодействующая с внешней средой и характеризующаяся высокой степенью неопределённости, адаптивности и самоорганизации. [6;10] Особое внимание уделяется выявлению ключевых факторов, влияющих на трансформацию международно-экономических процессов, включая глобализацию, технологическое развитие и изменение институциональной структуры мировой экономики.

На третьем этапе осуществляется построение обобщённой модели международно-экономических процессов, отражающей взаимосвязи между основными элементами мировой экономической системы. [4] Модель позволяет проследить влияние изменений в отдельных подсистемах на функционирование мировой экономики в целом, а также оценить возможные сценарии её развития.

Результаты исследования

В результате проведённого исследования установлено, что применение системного анализа и моделирования существенно расширяет возможности изучения международно-экономических процессов по сравнению с традиционными экономическими подходами. [2;5] Системный подход позволяет учитывать многоуровневую структуру мировой экономики, динамику взаимодействий между странами и регионами, а также влияние внешних факторов на развитие международно-экономических процессов.

Полученные результаты свидетельствуют о том, что моделирование на основе системного анализа способствует более глубокому пониманию причинно-следственных связей в мировой экономике и повышает качество прогнозирования её развития. Использование системного инструментария позволяет выявлять потенциальные риски и дисбалансы в международно-экономической системе, а также оценивать последствия принимаемых экономических решений. [1;10]

Таким образом, системный анализ и моделирование выступают эффективными инструментами исследования международно-экономических процессов, обеспечивая комплексный и динамичный подход к их изучению в условиях глобальных экономических трансформаций.

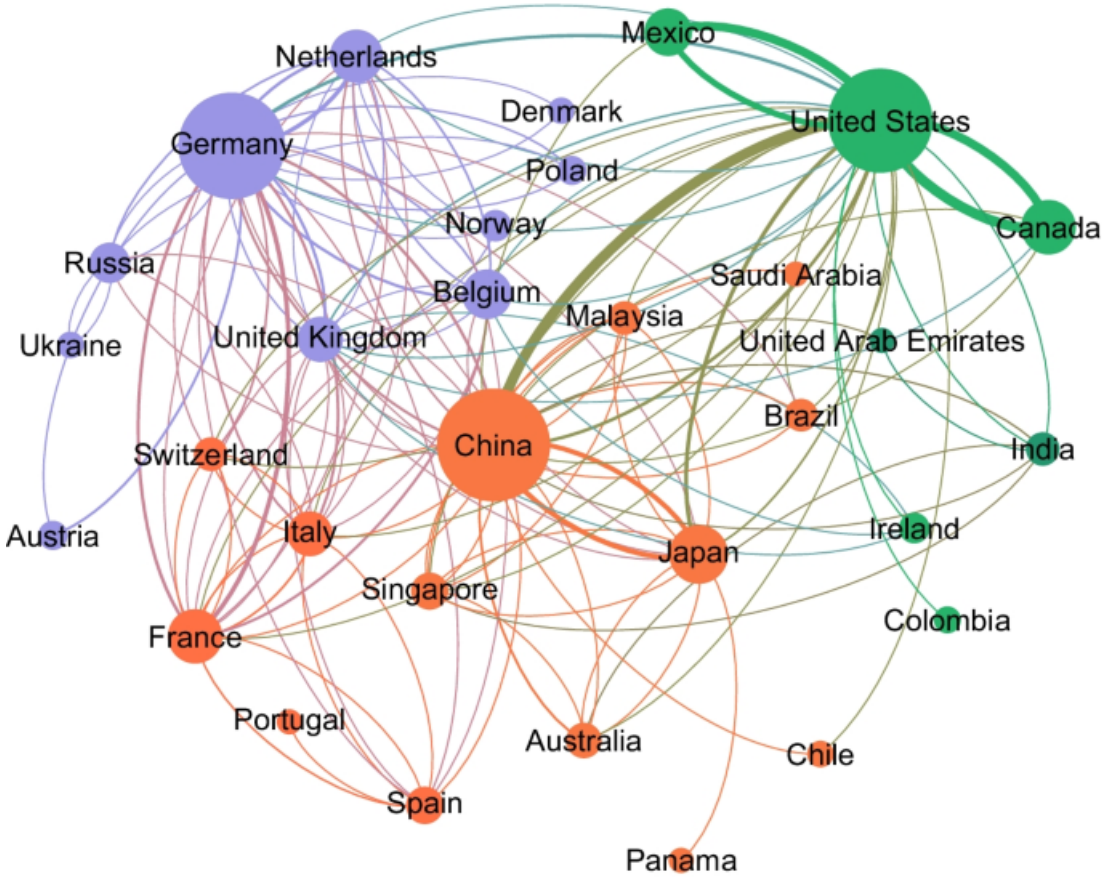

Рисунок 1 – Сетевая модель мировой экономики

Заключение Проведённое исследование показало, что международно-экономические процессы в современных условиях представляют собой сложные, многоуровневые и динамически развивающиеся системы, функционирование которых определяется совокупностью взаимосвязанных экономических, институциональных и технологических факторов. Возрастающая взаимозависимость национальных экономик, усиление глобальных трансформаций и нестабильность мировой экономической среды обусловливают необходимость применения комплексных методологических подходов к их изучению.

В ходе исследования обоснована целесообразность использования системного анализа в качестве универсального методологического инструмента исследования международно-экономических процессов. Системный подход позволяет преодолеть ограничения традиционных экономических моделей за счёт учёта нелинейных взаимосвязей, обратных связей, многоуровневой структуры мировой экономики и влияния внешней среды. Применение методов системного анализа обеспечивает более целостное и адекватное понимание закономерностей функционирования международной экономической системы. [2;6]

Установлено, что моделирование, основанное на принципах системного анализа, способствует выявлению причинно-следственных связей между ключевыми элементами мировой экономики и повышает возможности анализа и прогнозирования её развития. Использование системной динамики и структурных моделей позволяет оценивать последствия изменений в отдельных подсистемах и анализировать потен-

циальные сценарии трансформации международно-экономических процессов.

Таким образом, системный анализ и моделирование выступают эффективными инструментами исследования международно-экономических процессов и обладают значительным потенциалом для применения в научных исследованиях, аналитической деятельности и разработке экономической политики. Перспективы дальнейших исследований в данном направлении связаны с развитием формализованных моделей мировой экономики и расширением применения системного подхода в анализе глобальных экономических трансформаций.

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Влияние цифровой трансформации на динамику ВВП и благосостояние населения

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DOI 10.5281/zenodo.18224761

Аннотация. Статья посвящена исследованию влияния цифровой трансформации на социально-экономическое развитие государств в современных условиях. Раскрывается содержание термина «цифровая экономика» через призму определений ведущих международных организаций и теоретических подходов. Особое внимание уделено анализу факторов, влияющих на цифровую среду, и эффекту диффузии инноваций из цифрового сектора в традиционные отрасли промышленности. На основе эконометрических моделей доказано положительное влияние цифровой готовности на экспортно-импортные операции в сфере услуг и качество жизни населения. Статья содержит рекомендации по реализации национальных цифровых стратегий для обеспечения устойчивого экономического роста.

Annotation. The article is devoted to the study of the impact of digital transformation on the socio-economic development of states in modern conditions. The content of the term "digital economy"is revealed through the prism of definitions of leading international organizations and theoretical approaches. Special attention is paid to the analysis of factors affecting the digital environment and the effect of the diffusion of innovations from the digital sector into traditional industries. Based on econometric models, the positive impact of digital readiness on export-import operations in the service sector and the quality of life of the population has been proved. The article contains recommendations on the implementation of national digital strategies to ensure sustainable economic growth.

Annotatsiya. Maqola raqamli transformatsiyaning zamonaviy sharoitda davlatlarning ijtimoiy-iqtisodiy rivojlanishiga ta'sirini o'rganishga bag'ishlangan. "Raqamli iqtisodiyot"atamasining mazmuni etakchi xalqaro tashkilotlarning ta'riflari va nazariy yondashuvlar prizmasi orqali ochib beriladi.ulosa. Maqola raqamli transformatsiyaning zamonaviy sharoitda davlatlarning ijtimoiy- rivojlanishiga ta'sirini o'rganishga bag'ishlangan. "Raqamli iqtisodiyot"atamasining mazmuni etakchi xalqaro tashkilotlarning ta'riflari va nazariy yondashuvlar prizmasi orqali ochib beriladi. Raqamli muhitga ta'sir etuvchi omillarni tahlil qilish va innovatsiyalarning raqamli sektordan an'anaviy sanoatga tarqalish ta'siriga alohida e'tibor qaratiladi. Ekonometrik modellar asosida raqamli tayyorgarlikning xizmat ko'rsatish sohasidagi eksport-import operatsiyalariga va aholining hayot sifatiga ijobiy ta'siri isbotlandi. Maqolada barqaror iqtisodiy o'sishni ta'minlash uchun milliy raqamli strategiyalarni amalga oshirish bo'yicha tavsiyalar mavjud.

Цифровая экономика представляет собой комплекс видов экономической активности, тесно связанных с электронным бизнесом и торговлей, которые базируются на

передовых цифровых технологиях. В рамках этой системы создаются и реализуются цифровые продукты и услуги.

Основные преимущества цифровой экономики включают в себя высокий уровень автоматизации, электронный документооборот, интеграцию бухгалтерских и управленческих систем в цифровом формате, наличие электронных баз данных, а также использование CRM-систем для взаимодействия с клиентами и корпоративных сетей.

Практическая значимость и аспекты цифровой экономики, которая, в первую очередь, направлена на существенное повышение уровня жизни людей, представляют собой её фундаментальную задачу.

Во-вторых, цифровая экономика выступает в качестве основного инструмента борьбы с коррупцией и теневой экономикой, поскольку цифровые технологии позволяют фиксировать и хранить информацию, а также оперативно предоставлять доступ к ней при необходимости. В таких условиях становится невозможным утаивать информацию, заключать тайные сделки или оставаться в неведении относительно определённых процессов, поскольку компьютер способен выявить любые несоответствия.

В-третьих, выбор государством стратегии развития цифровой экономики открывает новые горизонты в области информационных технологий и электронного документооборота в целом. Таким образом, цифровая экономика – это, всемирная сеть экономической деятельности, коммерческих операций и профессиональных взаимодействий, которые поддерживаются информационно-коммуникационными технологиями (ИКТ). Значит, её можно кратко охарактеризовать как экономику, основанную на цифровых технологиях.

На протяжении всей истории, экономика трансформировалась благодаря революционным изобретениям (телеграф, железная дорога, автомобиль). Разница с Интернетом заключается в том, что он по своей сути глобален и используется как развитыми, так и развивающимися странами. Джон Свиокла сказал: “Интернет - одна из самых сложных вещей, когда-либо созданных. Это выводит человеческую организацию на другой уровень. Таким образом, цифровая экономика запустит третью волну капитализма, которая преобразит бизнес и правительство и приведет к созданию необычайного богатства по всему миру”.

Цифровая трансформация — это основополагающий инструмент, необходимый для современного мира, характеризующегося стремительными изменениями. Цифровые технологии играют ключевую роль в управлении экономическими и социальными процессами.

Цифровая экономика представляет собой сферу деловой активности, которая ежедневно доступна для миллионов людей, компаний, устройств, данных и процессов через онлайн-платформы. Разнообразие видов экономической деятельности служит основой для оцифрованной информации и знаний.

Цифровые технологии, такие как облачные вычисления, большие объёмы данных, финансовые технологии и другие, кардинально меняют подход к использованию данных, позволяя собирать, хранить и распространять их в цифровом формате.

Значение цифровой экономики для развития экономической системы определяется как с точки зрения повышения эффективности производства, так и с точки

зрения развития производственных отношений. Активное развитие цифровой экономики способствует повышению производительности и трансформации производственных отношений.

С точки зрения повышения эффективности производства, цифровая экономика раскрывается в двух аспектах: индустриализация и цифровизация промышленности. Индустриализация в контексте цифровой экономики относится к информационной индустрии, такой как интернет, большие данные, искусственный интеллект и т. д., которая является основой для развития цифровой экономики.



Рис 1. Важность цифровой экономики для развития экономической системы

Технологические инновации, технологическая интеграция и непрерывное появление информационных элементов приводят к тому, что доля стоимости продуктов, связанных с цифровой экономикой, в экономике всех стран продолжает увеличиваться и еще больше способствует устойчивому экономическому росту. По данным Бюро экономического анализа (BEA) и аналитических обзоров Forrester, США сохраняют статус крупнейшей цифровой экономики мира. В 2024 году объем цифровой экономики США оценивается в диапазоне 10% – 12% при строгом расчете (только ИКТ-сектор и цифровая торговля) и до 35% – 40%, если учитывать «цифровой охват» (все операции, совершаемые с помощью цифровых технологий), вклад только ИКТ-сектора (инфраструктура, ПО, облачные услуги) превысил $2,5 трлн. Согласно отчету OECD Digital Economy Outlook 2024, США лидируют по объему инвестиций в ИИ-технологии, что стало основным фактором роста цифрового сегмента за последний год. Мировой рынок стал более консолидированным.

Согласно прогнозу Global Digital Economy Forecast от Forrester, доля цифровой экономики в мировом ВВП составляет примерно 15,5% – 17%. При этом почти две трети (около 65%) всего объема мировой цифровой экономики приходится на две страны — США и Китай. Также в этот список входят Великобритания, Япония, Германия и Южная Корея. Цифровой сегмент растет в среднем на 7% в год, что в 2–3 раза быстрее роста мировой экономики в целом.

Одновременно с развитием технологических отраслей традиционные отрасли могут осваивать новые технологии цифровой экономики, тем самым способствуя тех-

нологическому инновации в традиционных секторах. В целом, цифровая экономика создала стимулы для инноваций в традиционных отраслях благодаря эффекту диффузии. С другой стороны, цифровая экономика оказывает демонстративное влияние на традиционную экономику, поскольку цифровая индустрия является высококонкурентной, способствуя трансформации предприятий традиционной отрасли и повышению качества обслуживания.

В целом можно выделить следующий перечень мер, реализуемых государствами и направленных на развитие цифровой экономики: развитие инфраструктуры, которая является основой для формирования новых бизнес-моделей и построения научных и социальных сетей; снижение барьеров в секторах цифровой экономики; повышение уровня знаний о цифровых технологиях, подготовке и переподготовке специалистов; обеспечение уверенности в надежности и безопасности цифровой инфраструктуры, оценка рисков; развитие цифрового сектора экономики.

В основе цифрового сектора экономики лежат инновационные технологии, разработанные в рамках электронной промышленности. Этот сектор включает в себя два основных компонента:

1. Электронная промышленность, которая занимается производством микрочипов, компьютеров, телекоммуникационных устройств и бытовой электроники.

2. Компании, предоставляющие услуги в области цифровых технологий и использующие цифровые средства для производства, хранения и управления данными.

Важность развития цифрового сектора для национальных экономик подтверждается тем, что ряд стран реализует комплексные и масштабные программы, направленные на развитие цифровых секторов своих экономик, создание новых рабочих мест и повышение конкурентоспособности электронной промышленности и IT-технологий.

Одним из ключевых факторов развития цифрового сектора экономики являются инвестиции.

Для более глубокого анализа воздействия цифровизации на экономические, социальные показатели был проведен анализ. Для модели были взяты 40 стран мира из разных континентов и с разными доходами страны. Так как, существует большое количество цифровых индексов, составляемые на основе разных факторов и стран, было решено выбрать из них несколько для проведения анализа. Так, для нашего анализа были выбраны Рейтинг цифровой конкурентоспособности (Digital Competitiveness Ranking), Глобальный инновационный индекс (Global Innovation Index), Индекс готовности сети (Network Readiness Index). Так как, исследование ведется для получения взаимосвязи цифровой трансформации на экономику страны, как результирующие величины были выбраны: показали ВВП на душу населения (в текущих долларах США) - , Безработица, общая (% от общей численности рабочей силы) -, Рабочая сила, всего -, ВВП (в текущих долларах США) - , Экспорт товаров (в текущих долларах США)- , Импорт товаров (в текущих долларах США)-, Экспорт услуг (ПбП, текущие доллары США) -, Импорт услуг (ПбП, текущие доллары США)-, Торговля (% от ВВП)-, Ожидаемая продолжительность жизни при рождении, всего (годы)- , Скорректированный чистый национальный доход (текущий в долларах США) - , Услуги, добавленная стоимость (% от ВВП) - .

Анализ проводился с каждым из результативных характеристик по трем индек-

сам. Так в ходе построения модели, в первую очередь был проведен анализ взаимосвязей и выявлен коэффициент корреляции:

Таблица. 1. Сила связи величин Y и DCR, GII, NRI. Как видно по таблице, с

Показатель	Коэффициент корреляции DCR	Коэффициент корреляции GII	Коэффициент корреляции NRI	Характеристика силы связи
	0,74	0,66	0,77	средняя сила связи
	-0,48	-0,43	-0,45	обратная слабая сила связи
	0,13	0,15	0,01	очень слабая сила связи
	0,36	0,38	0,26	очень слабая сила связи
	0,39	0,46	0,32	слабая сила связи
	0,43	0,49	0,36	слабая сила связи
	0,55	0,62	0,51	средняя сила связи
	0,56	0,64	0,53	средняя сила связи
	0,24	0,22	0,27	очень слабая сила связи
	0,76	0,77	0,85	высокая сила связи
	0,36	0,38	0,26	слабая сила связи
	0,51	0,60	0,61	средняя сила связи

Таблица 1. Сила связи величин Y и DCR, GII, NRI.

характеристиками Безработица, общая (% от общей численности рабочей силы) -, Рабочая сила, всего -, ВВП (в текущих долларах США) -, Экспорт товаров (в текущих долларах США)-, Импорт товаров (в текущих долларах США)-, Торговля (% от ВВП)-, Скорректированный чистый национальный доход (текущий в долларах США) -корреляция очень слабая, что означает, что внедрение цифровизации в экономику страны не так сильно повлияет на эти показатели, изменения будут не значительные, поэтому дальнейший анализ является не целесообразным.

Таким образом, ввиду наличия связи с остальными показателями представляется возможным построение частного вида модели влияния цифровизации на каждый из характеристик, но при построении модели выбор будет подать на тот индекс, с которым самый высокий коэффициент корреляции. Так, например, взаимосвязь ВВП на душу населения самый высокий с индексом готовности сети (NRI), что показывает R=0,77 (Рис. 2.12.).

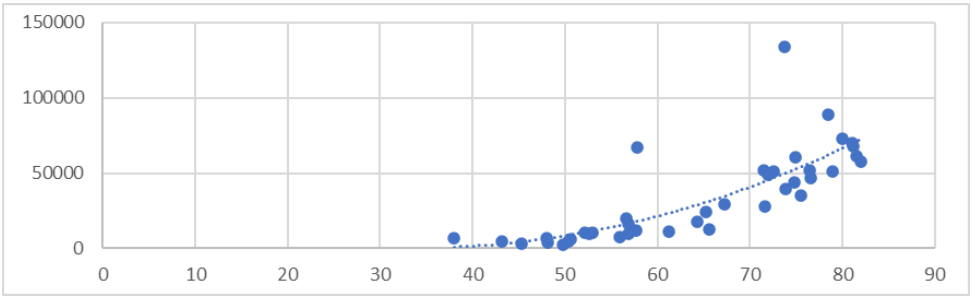

Рис 2. График взаимосвязи ВВП на душу населения и NRI

График описывает уравнение где можно удостовериться связи между двумя показателями.

Следующим показателем, имеющим среднюю связь с цифровой трансформацией, является экспорт услуг. Уравнение описывающие эту связь является а график

представлен на рис. 2.13.

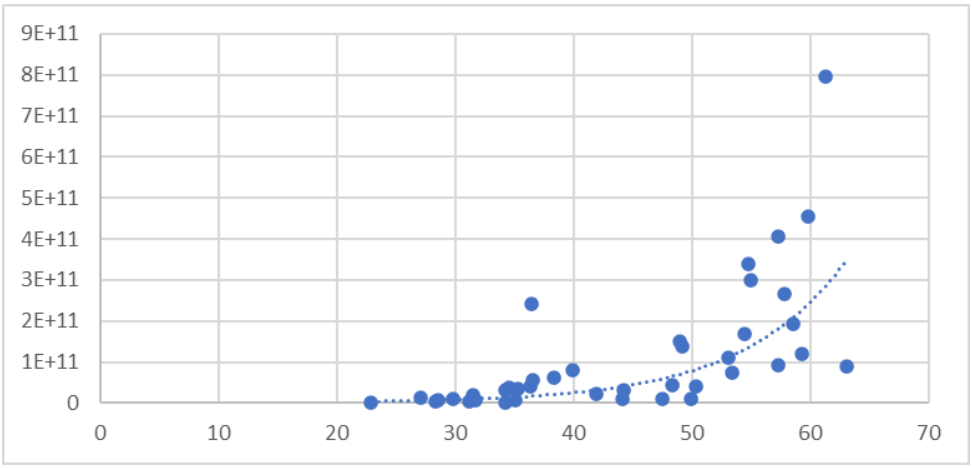

Рис 3. График взаимосвязи экспорта услуг и GII

Между экспортом услуг и Глобальным инновационным индексом существует хорошая связь, объясняемая тем, что цифровая трансформация приведет к росту экспорта услуг, которая в свою очередь со временем будет включать все больше ИТ услуг.

Аналогичным образом, импорт услуг имеет среднею корреляцию с Глобальным инновационным индексом, как и экспорт.

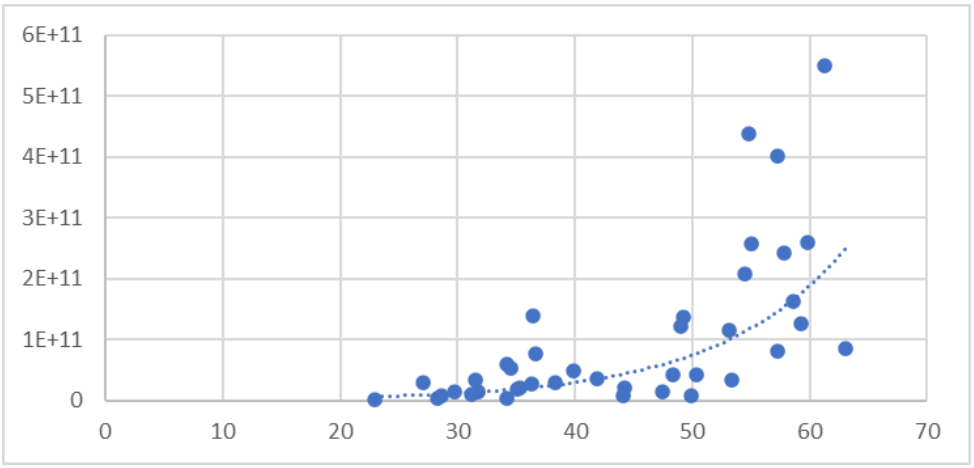

Рис. 4. График взаимосвязи импорт услуг и GII.

Уравнением объясняем этот график является

Еще одним интересным показателем, которая имеет высокую корреляцию является ожидаемая продолжительность жизни при рождении. Как объяснение можно предположить, что это может быть обусловлено широким использованием цифровых и высокотехнологичных решений в сфере здравоохранения и проведением национальных программ, направленных на поддержание здорового образа жизни.

Модель , самая высокая корреляция с индексом готовности сети (NRI).

Последний показатель со средней корреляцией с индексом готовности сети (NRI) - услуги.

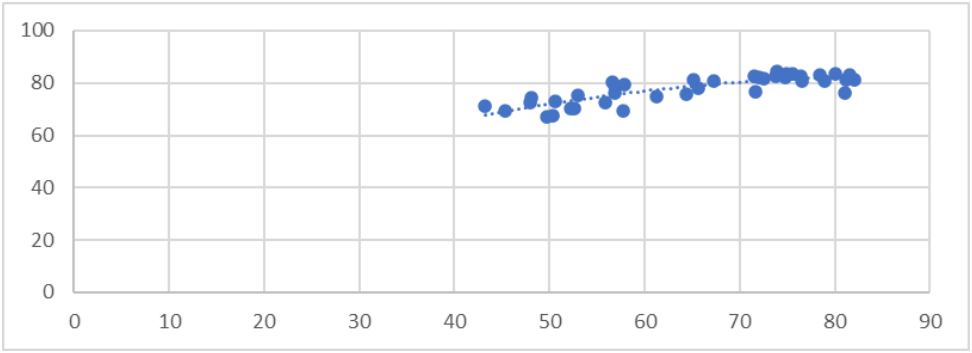

Рис 5. График взаимосвязи ожидаемой продолжительности жизни и NRI

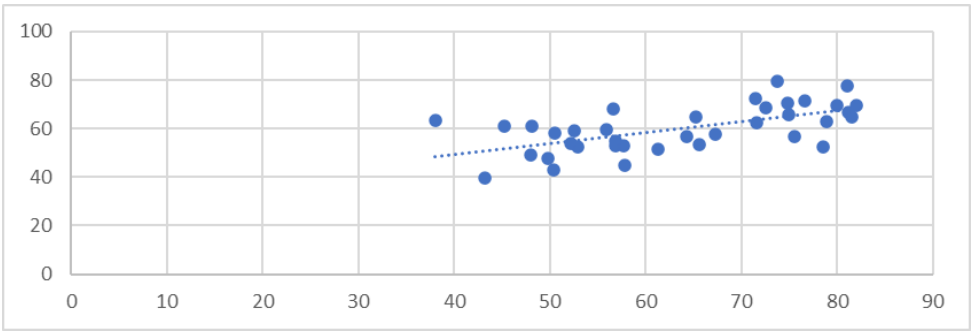

Рис 6. График взаимосвязи услуг и NRI

Модель .

Можно сделать вывод, что цифровая трансформация страны приведет к росту ВВП на душу населения, экспорта и импорта товара и услуг. Также прирост ВВП частично вызван таким последствием цифровизации, как импортозамещение в части технологий и сетевых сервисов. Это в целом является положительным моментом, способствующим развитию высокотехнологичных и наукоемких отраслей.

Цифровая трансформация оказывает существенное воздействие на конкурентоспособность различных отраслей мировой экономики, способствуя формированию новых рыночных ниш и ускоряя вывод инновационных продуктов на глобальный рынок. Более того, она открывает новые горизонты для социально-экономического развития.

В современном мире наблюдается ряд тенденций, таких как глобализация, изменение потребительских предпочтений, мобильность и доступность информации. Цифровые технологии радикально преобразуют мировую экономическую систему.

Создание эффективной цифровой экономики открывает широкие перспективы для развития бизнеса, что, в свою очередь, стимулирует приток инвестиций и накопление человеческих и финансовых ресурсов во всём мире.

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Математическое моделирование безработицы на основе системы дифференциальных уравнений дробного порядка

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DOI 10.5281/zenodo.18224784

Аннотация.

В данной работе предлагается простая и в то же время содержательная математическая модель динамики безработицы с учетом памяти рынка труда. Используется система дифференциальных уравнений дробного порядка в смысле Капуто. Дробный порядок позволяет учесть долговременные последствия экономических кризи-

сов, задержки восстановления занятости и инерционность демографических процессов. Получены аналитические решения через функцию Миттаг–Леффлера, исследована асимптотическая устойчивость равновесия.

Ключевые слова: *дробные уравнения, производная Капуто, рынок труда, безработица, функция Миттаг–Леффлера, устойчивость.*

Моделирование динамики безработицы является одной из ключевых задач экономической математики. Классические модели рынка труда обычно опираются на системы обыкновенных дифференциальных уравнений первого порядка. Однако в реальных экономических процессах часто наблюдается эффект памяти: последствия экономических шоков сохраняются длительное время, восстановление занятости происходит медленнее, чем предсказывают классические модели.

Дробное исчисление позволяет формально учитывать такие эффекты благодаря введению производных дробного порядка, что делает модель инерционной.

Цель настоящей работы – построить и исследовать простую фракционную модель безработицы с двумя классами: безработные $U(t)$ и занятые $E(t)$, используя производную Капуто порядка $0 < \alpha < 1$. Модель допускает полное аналитическое решение, что делает её удобной как для теоретического анализа, так и для численных экспериментов.

Основные результатом статьи является то, что получено аналитическое решение через функцию Миттаг-Леффлера, доказано существование и единственность решений, исследована устойчивость равновесия и асимптотика при $t \to \infty$.

Изучение динамики безработицы и процессов на рынке труда имеет длительную историю в экономико-математических исследованиях. Классические модели, как правило, основываются на системах обыкновенных дифференциальных уравнений, где переходы между состояниями занятости и безработицы задаются постоянными темпами. Первые математические формализации динамики занятости можно найти в работах таких ученных как А.К.Мисра и Арвинд К. Сингха [1],[2]. В своей работе они изучают процесс безработицы при помощи системы обыкновенных дифференциальных уравнений, находят точку равновесия и доказывают устойчивость системы. Дальнейшая развития данный темы нашли своё отражения в работах таких ученных как Гулбану Патхан и П.Х.Бхатхавала [3],[4], Сурeха Б. Маноли и Шанкреввa Р. Гани [9],[10]. В работе [3],[4] ученные предполагают ситуацию, при которой безработные также стремятся к самостоятельной занятости и создают возможности для самозанятости. Исследуется устойчивость точки равновесия модели также для проверки устойчивости найден неотрицательный точка равновесия системы. Также представлено численное моделирование для сравнения с аналитическими результатами. Отличительной чертой работ [9],[10] от работ [3],[4] является то что в работах [9],[10] исследование направлено на предложение модели безработицы и её оптимального управления и в работе [9] рассматривается пять классов таких как число безработных, временно занятых, регулярно занятых лиц в момент времени t, количество временных вакансий в момент времени t, количество постоянных вакансий в момент времени t, а во всех остальных работах рассматривалась только три класса число безработных, число занятых и число вакансии.

В последние десятилетия активно развивается направление применения дробного исчисления к социально-экономическим процессам. Дробные производные позволя-

ют описывать системы с памятью и наследственным эффектом, что особенно актуально для экономики, где последствия кризисов часто проявляются на протяжении длительного времени. Крупные монографии Подлубного, Килбаса и Дитхельма [5-7] сформировали математический фундамент для анализа линейных и нелинейных дробных уравнений.

В ряде работ дробные производные применялись к моделям экономического роста, инфляционной динамики, распределению капитала, моделям человеческого капитала. Показано, что дробный порядок позволяет корректнее описывать длительные тренды, инерционность и влияние прошлых состояний.

Однако модели рынка труда на основе дробных уравнений исследованы значительно меньше, и в большинстве случаев носят либо феноменологический, либо численный характер, без строгого математического анализа. Настоящая статья вносит вклад в литературу тем, что строит простую, но аналитически решаемую систему дробного порядка, полное аналитические решения получена через функцию Миттаг-Леффлера и доказана устойчивость системы и показана существования и единственность решения.

Таким образом, представленная модель объединяет строгий математический анализ и прикладную экономическую интерпретацию, заполняя разрыв между прикладными экономическими моделями и теорией дробных производных.

Рассмотрим систему дробного порядка:

$$\begin{cases} C_D^\alpha E(t) = \delta U(t) - \gamma E(t), \\ C_D^\alpha U(t) = -\delta U(t) + \gamma E(t), \qquad 0 < \alpha < 1,\ \delta > 0,\ \gamma > 0. \\ \quad U(0) = U_0, \quad E(0) = E_0, \end{cases} \tag{1}$$

Если обозначим, что

$$x(t) = \begin{pmatrix} U(t) \\ E(t) \end{pmatrix}, \qquad x_0 = \begin{pmatrix} U_0 \\ E_0 \end{pmatrix}, \qquad A = \begin{pmatrix} -\delta & \gamma \\ \delta & -\gamma \end{pmatrix}.$$

Тогда уравнения (1) примет компактный вид:

$$C_D^\alpha x(t) = Ax(t), \qquad x(0) = x_0. \tag{2}$$

Лемма 1 (о сохранения общего населения). Пусть $U(t), E(t)$ -решения системы (1). Тогда $\forall t \geq 0$

$$U(t) + E(t) = N(t) := U_0 + E_0.$$

Доказательство. Сложим уравнения системы (1):

$$C_D^\alpha E(t) + C_D^\alpha U(t) = \delta U(t) - \gamma E(t) - \delta U(t) + \gamma E(t) = 0$$

Таким образом, функция $S(t) = U(t) + E(t)$ удовлетворяет дробное уравнения Капуто $C_D^\alpha S(t) = 0, \quad S(0) = U_0 + E_0 = N$. Из общего вида решения уравнения $C_D^\alpha y(t) = 0$ [3] следует что $S(t) \equiv S(0) = N$. Следовательно $U(t) + E(t) \equiv N, \quad \forall\, t \geq 0$.

Теорема 1 (существование и единственность). Для любых начальных данных $x_0 \in E^2$ задачи Коши (2) имеет единственное глобальное решение $x(t) \in C([0,\infty); E^2)$.

Доказательство. Рассмотрим общую систему Капуто $C_D^\alpha x(t) = f(t, x(t))$, $x(0) = x_0$ где $f : [0,\infty) \times E^2 \to E^2$ непрерывна и удовлетворяет условию Липшица по на каждом ограниченном множестве. В нашем случае $f(t,x) = Ax$, не зависит от t и является линейной по x. То f глобально липщицева:

$$\|f(t,x) - f(t,y)\| = \|A(x-y)\| \le \|A\| \, \|x-y\|$$

Известно, что для таких систем дробного порядка [5][6] задача Коши эквивалентна интегральному уравнению Вольтера:

$$x(t) = x_0 + \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau, x(\tau)) \, d\tau. \tag{3}$$

В нашем случае:

$$x(t) = x_0 + \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} Ax(\tau) \, d\tau.$$

Рассмотрим пространство $X_T = C([0,T]; E^2)$ с нормой $\|x\|_{X_T} = \max_{t\in[0,T]} \|x(t)\|$ Определим оператор $\mathcal{T} : X_T \to X_T$ по формуле правой части (3):

$$(\mathcal{T}x)(t) = x_0 + \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} Ax(\tau) \, d\tau.$$

Оценим для

$$\|(\mathcal{T}x)(t) - (\mathcal{T}y)(t)\| \le \frac{\|A\|}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} \|x(\tau) - y(\tau)\| \, d\tau \le$$

$$\le \frac{\|A\|}{\Gamma(\alpha)} \|x-y\|_{X_T} \int_0^t (t-\tau)^{\alpha-1} \, d\tau = \frac{\|A\|}{\Gamma(\alpha)} \frac{T^\alpha}{\alpha} \|x-y\|_{X_T}.$$

Следовательно $\|\mathcal{T}x - \mathcal{T}y\|_{X_T} \le L_T \|x-y\|_{X_T}$, $\quad L_T = \dfrac{\|A\| \, T^\alpha}{\Gamma(\alpha) \, \alpha}$. Выберем T настолько малым, что $L_T < 1$. Тогда τ - сжимающиеся отображение на полном пространстве X_T . По теореме Банаха о сжимающем отображении существует и единственна неподвижная точка $x \in X_T$, является единственнным решением (3) на $[0,T]$, а значит задача Коши (2) на $[0,T]$. Продолжая шаг за шагом начиная с точки T получаем единственное решение на конечном отрезке $[0,T_1]$, а затем и на $[0,\infty)$.

Теорема 2 (о равновесии системы). Система (1) имеет единственную стационарную точку (U^*, E^*), заданную формулами

$$U^* = \frac{\delta}{\delta+\gamma} N, \qquad E^* = \frac{\gamma}{\delta+\gamma} N,$$

где $N = U_0 + E_0$.

Доказательство. Стационарная точка определяется системой

$$\begin{cases} 0 = -\delta U^* + \gamma E^*, \\ 0 = \delta U^* - \gamma E^*, \\ U^* + E^* = N. \end{cases}$$

Первые два уравнения эквивалентны одному: одному: $\delta U^* = \gamma E^*$. Из него $U^* = \frac{\gamma}{\delta} E^*$. Поставим в условие $U^* + E^* = N$, то $\frac{\gamma}{\delta} E^* + E^* = N \quad \Rightarrow \quad E^* = \frac{\delta}{\delta + \gamma} N$. Тогда $U^* = \frac{\gamma}{\delta} E^* = \frac{\gamma}{\delta} \frac{\delta}{\delta + \gamma} N = \frac{\gamma}{\cdot} \delta + \gamma N$.

Лемма 2. Положим $V(t) = U(t) - U^*$. Тогда $V(t)$ удовлетворяет одномерному уравнению дробного порядка

$$C_D^\alpha V(t) = -(\delta + \gamma) V(t), \qquad V(0) = U_0 - U^* \tag{4}$$

Доказательство. Из леммы 1 мы можем сказать, что $E(t) = N - U(t)$. Подставив это вырожения во второе уравнения системы (1):

$$C_D^\alpha U(t) = -\delta U(t) + \gamma E(t) = -\delta U(t) + (N - U(t)) = -(\delta + \gamma) U(t) + \gamma N.$$

Теперь подставим

$$U(t) = V(t) + U^*$$

то тогда $C_D^\alpha (V + U^*) = -(\delta + \gamma)(V + U^*) + \gamma N$. Так как U^* – стационарная точка, то из теоремы 2:

$$0 = -(\delta + \gamma) U^* + \gamma N.$$

Следовательно, $C_D^\alpha V(t) = -(\delta + \gamma) V(t)$ что и требовалась доказать.

Рассмотрим явное решение через функцию Миттаг-Леффлера и приведем теорему для явного решения.

Теорема 3. Решение уравнения (4) имеет вид $V(t) = (U_0 - U^*)\, E_\alpha(-(\delta + \gamma) t^\alpha)$ где $E_\alpha(z)$ - функция Миттаг-Леффлера порядка α : $E_\alpha(z) = \sum\limits_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + 1)}$. Соответственно решения исходной системы (1) запишутся как

$$U(t) = U^* + (U_0 - U^*)\, E_\alpha(-(\delta + \gamma) t^\alpha) \tag{5}$$

$$E(t) = N - U(t) \tag{6}$$

Доказательство. Общий вид скалярного уравнения Капуто $C_D^\alpha y(t) = \lambda y(t), \quad y(0) = y_0, \quad 0 < \alpha < 1$ известен из [7/1][4/2] и имеет вид $y(t) = y_0\, E_\alpha(\lambda t^\alpha)$. Уравнения (4) является частным случим с $y(t) = V(t), \quad y_0 = V(0) = U_0 - U^*, \quad \lambda = -(\delta + \gamma)$. Следовательно, $V(t) = (U_0 - U^*)\, E_\alpha(-(\delta + \gamma) t^\alpha)$. Поскольку $U(t) = U^* + V(t)$ получаем (5). Формула (6) следует из Леммы 1 $E(t) = N - U(t)$.

Рассмотрим асимптотическую устойчивость системы.

Теорема 4(асимптотическая устойчивость). Пусть $0 < \alpha < 1,\ \delta > 0,\ \gamma > 0$. Тогда стационарная точка (U^*, E^*) системы (1) асимптотически устойчива, то есть для любых начальных данных

$$\lim_{t \to \infty} U(t) = U^*, \qquad \lim_{t \to \infty} E(t) = E^*.$$

Доказательство. Рассмотрим матрицу . Её собственные значения находится из уравнения

$$\det(A - \lambda I) = \det \begin{pmatrix} -\delta - \lambda & \gamma \\ \delta & -\gamma - \lambda \end{pmatrix} = 0.$$

Нетрудно проверить, что $\lambda_1 = 0, \quad \lambda_2 = -(\delta + \gamma) < 0$. Для линейной системы Капуто типа $C_D^\alpha x = Ax$ известен критерий асимптотической устойчивости из [8], если для всех собственных значений λ_k матрицы A выполнено условие

$$|\arg(\lambda_k)| > \frac{\alpha\pi}{2}. \tag{7}$$

То нулевая траектория асимптотически устойчива. В нашем случи $\lambda_2 < 0$ поэтому $\arg(\lambda_2) = \pi$, и $|\arg(\lambda_2)| = \pi > \frac{\alpha\pi}{2}$. для любого $0 < \alpha < 1$. Второе собственное значение $\lambda_1 = 0$ связана с сохранением массы и отвечает нейтральному направлению вдоль гиперплоскости $U + E = N$, в координатах отклонения $V(t) = U(t) - U^*$ динамика определяется только λ_2. Из явной формулы (5) видно, что $\lim_{t\to\infty} E_\alpha(-(\delta + \gamma)t^\alpha) = 0$ и $\lim_{t\to\infty} U(t) = U^*, \quad \lim_{t\to\infty} E(t) = E^*$, следовательно . То есть равновесие асимптотически устойчиво.

С экономической точки зрения полученные результаты показывают, что рынок труда характеризуется устойчивым равновесием, к которому система стремится при неизменных параметрах переходов между состояниями занятости и безработицы. Однако в отличие от классических моделей, скорость сходимости к равновесию определяется порядком дробной производной и может быть существенно ниже, что соответствует эмпирически наблюдаемой затяжной безработице после кризисов. Аналитическое решение, выраженное через функцию Миттаг–Леффлера, наглядно демонстрирует долговременное влияние начальных условий, что подчёркивает важность ранних и целенаправленных мер экономической политики для смягчения последствий негативных шоков на рынке труда.

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Анализ финансового состояния предприятий: оценка эффективности собственного капитала

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DOI 10.5281/zenodo.18224847

Annotatsiya

Maqolada korxonaning moliyaviy holati iqtisodiy mazmuni va mohiyati ochib berilgan, shuningdek, uning tarkibini baholash masalalari yoritilgan. Shu munosabat bilan xususiy kapitalning samaradorligini, shu jumladan uning rentabellik darajasini baholash, aylanma koeffitsientlarining o'zgarishiga ta'sir etuvchi omillar tahlil qilinadi. Ilmiy maqola natijalari xususiy kapitaldan samarali foydalanish asosida korxonalarning moliyaviy barqarorligini ta'minlash imkonini beradi.
Kalit so'zlar: balans, moliyaviy holat, o'z mablag'lar manbalari, uzoq va qisqa muddatli majburiyatlar, ustav kapitali, zahira kapitali, qo'shimcha kapital, taqsimlanmagan foyda, xususiy kapital rentabelligi, xususiy kapital aylanmasi.

Аннотация

В статье раскрывается экономическое содержание и сущность финансового состояния предприятия, а также освещаются вопросы оценки его состава. В связи с этим проанализированы оценка эффективности собственного капитала, в том числе уровень его рентабельности, факторы, влияющие на изменение коэффициентов оборачиваемости. Результаты научной статьи позволяют обеспечить финансовую устойчивость предприятий на основе эффективного использования собственного капитала.
Ключевые слова: баланс, финансовое состояние, источники собственных средств, долгосрочные и краткосрочные обязательства, уставный капитал, резервный капитал, добавочный капитал, нераспределенная прибыль, рентабельность собственного капитала, оборачиваемость собственного капитала.

Annotation: The article reveals the economic content and essence of the financial condition of the enterprise, the issues of assessing its composition. In this regard, when assessing the efficiency of equity capital, including the level of its profitability, the factors influencing the change in turnover indicators are analyzed. The results of the scientific

article will ensure the financial stability of enterprises based on the effective use of equity capital.
Key words: balance sheet, financial condition, sources of equity, long-term and short-term liabilities, authorized capital, reserve capital, added capital, retained earnings, return on equity, equity turnover.

Введение.

Принятие новой редакции Конституции Республики Узбекистан всенародным голосованием на референдуме, состоявшемся 30 апреля 2023 года, послужило укреплению конституционных основ создания Нового Узбекистана. При этом он потребовал усовершенствовать основные направления развития нашей страны в обновленных конституционно-правовых условиях и вывести проводимые масштабные реформы на новый этап. Утверждена стратегия "Узбекистан – 2030 разработанная на основе опыта, полученного в ходе реализации новой стратегии развития Узбекистана и результатов общественного обсуждения. Согласно стратегии "УЗБЕКИСТАН – 2030 усиление прикладных исследований в наиболее быстро развивающихся сферах экономики, внедрение кластерной системы предприятие-вуз-научная организация, выделение из государственного бюджета на практические исследования 4 трлн сумов, производство 850 видов инновационной продукции по «драйверным» направлениям отраслей экономики, создание более 2,5 тысяч новых научных разработок по результатам практических исследований, транспорта и логистики, производительности труда в сельском хозяйстве, энергетике, биотехнологиях, геологии и металлообработке, машиностроении и создание 8 научно-производственных кластеров. [1] В условиях дальнейшего развития экономики особое внимание уделяется развитию субъектов предпринимательства в нашей стране. В настоящее время предприятиям необходимо обеспечить правильное управление движением имеющихся в их распоряжении финансовых ресурсов и эффективность их использования. Использование неэффективных методов управления может привести их к экономическому краху. Из этого следует, что анализ финансового состояния предприятия помогает улучшить его финансовую деятельность, укрепить учетную и финансовую дисциплину, обеспечить правильное соотношение собственных и заемных средств, найти и использовать внутренние возможности, проанализировать показателей эффективного использования финансовых ресурсов и улучшения их финансового положения, позволяет разработать необходимые рекомендации по анализу финансового состояния, что является важным условием повышения их финансовой устойчивости. С учетом этого в нашей стране принят ряд нормативных документов. В частности, принятие постановления Кабинета Министров Республики Узбекистан №207 "О введении критериев оценки эффективности деятельности акционерных обществ и других хозяйствующих субъектов с долей государства"[2] и полную оценку финансового состояния предприятий по основным и дополнительным критериям, исходя из текущего финансового состояния предприятий, можно даже прогнозировать их дальнейшую деятельность, а также принятие Постановления № 1013 Кабинета Министров Республики Узбекистан "О мерах по коренному совершенствованию системы финансовое оздоровление предприятий с государственным участием"[3] определяет актуальность оценки эффективности использования капитала.

Джо-Энн Сашар и Георг А. Шинкл [4] опубликовали в Австралийском журнале менеджмента статью под названием «Инновации в новых государственных фирмах: влияние государственных грантов, венчурного капитала и собственного капитала»,

в которой адекватно исследовался этот вопрос. В статье рассматривается влияние государственных грантов, венчурного капитала и фондов собственного капитала на инновации новых государственных фирм, вводимые ресурсы, инновационные результаты (патенты) и их качество (ссылки на патенты), механизм взаимодействия государственных грантов, венчурного капитала, собственного капитал и инновации. Это показывает, что гранты являются стимулом для финансирования прямых инвестиций, а не косвенных инвестиций. Кроме того, отмечается, что однородность венчурных и частных инвесторов оказывает существенное влияние на инновации в портфельных компаниях на уровне фирмы, и даются практические рекомендации.
Экономисты Сиви Ариантини, Сапто Джумонолар опубликовали статью [5] "Прибыльность и стоимость фирмы: пример обрабатывающей промышленности Индонезии"целью, которой является определение показателей рентабельности и влияющих на них факторов, а также их влияния на стоимость фирмы на основе анализа компании "DuPont". Детерминантная рентабельность, т.е. размер чистой прибыли, общий оборот активов и мультипликатор финансового рычага, оказывают положительное влияние на рентабельность собственного капитала, в то время как коэффициент продаж оказывает отрицательное существенное влияние. Было показано, что с точки зрения повышения стоимости компании прибыльность прямых инвестиций вносит значительный положительный вклад в отрасль. Это требует от руководства эффективного управления операционной деятельностью компании и снижения операционных и других затрат, увеличения рентабельности акционерного капитала и нормы прибыли, а также эффективного использования активов и обязательств. В данной статье эти вопросы достаточно исследованы. Исследования показали, что теоретические проблемы анализа финансового состояния являются одной стороной вопроса, а другой стороной является его практическая значимость для различных сфер. Это определяет актуальность темы научной статьи.
Обновление и модернизация нашей страны, обеспечение высоких и стабильных темпов экономического развития, эффективности и макроэкономической сбалансированности были признаны основными приоритетными задачами. В условиях либерализации экономики необходимо рационально управлять хозяйствующими субъектами в их экономически независимой и финансово устойчивой деятельности, адаптировать бухгалтерский учет к требованиям рыночной экономики, организовать производство или оказание услуг путем эффективного использования имеющихся материальных, финансовых и трудовых ресурсов. В результате при высоких прибылях своевременное выполнение государством обязательств перед бюджетом является одной из актуальных задач современности.

Обзор изученной литературы.

О финансовом состоянии до сегодняшнего дня написано большое количество учебников, монографий, диссертаций и статей, но при написании каждого нового издания были учтены особенности той страны финансовой состояние предприятий которой изучалось. Среди работ, имеются, как работы отечественных, так и зарубежных ученых, внесших свой вклад в развитие даной сферы. В частности отечественный ученый, доктор экономических наук, профессор Э.А.Акрамов в учебном пособии "Анализ финансового состояния предприятий рассматривает практически все аспекты прямо или косвенно касательные финансового состояния предприятия в целом, это и деловая и рыночная активность, анализ финансовых результатов,

оборачиваемость оборотных средств и т.д. В учебном пособии с учетом требований рыночных отношений последовательно излагаются теоретико-методологические и практические проблемы и этапы анализа финансового состояния предприятий. Рассмотрены основные принципы и источники финансового анализа, зарубежный его опыт, общая оценка финансового состояния предприятия, анализ финансовой устойчивости предприятия и ликвидности баланса, связь финансового состояния с оборачиваемостью оборотных средств и себестоимостью продукции, анализ деловой, рыночной активности, экономической несостоятельности и финансовых результатов деятельности предприятий. [Э.А.Акрамов,2004].
По мнению Семиколеновой М.Н. понятие финансового состояния определяется группой показателей, которые демонстрируют наличие, эффективность размещения и использования финансовых ресурсов, способность организации финансировать собственную деятельность на определенный момент времени. Состояние финансовых ресурсов предприятия позволяет ему распоряжаться денежными средствами, в результате их эффективного использования осуществлять обеспечение бесперебойности производственного процесса, а также процессов сбыта, расширения и обновления ассортимента. [Семиколенова М.Н.,2020]. Главной целью анализа финансового состояния является его своевременная оценка с помощью получения необходимого количества наиболее содержательных показателей, которые демонстрируют реальное и достоверное представление финансового состояния предприятия. К тому же интерес представляет, как правило, не только финансовое положение предприятия на текущий момент, а также его дальнейшая перспектива, ожидаемые величины финансового состояния, возможность увеличения эффективности.

Методология исследования.

При написании данной статьи были использованы традиционные и экономико-математические методы экономического анализа, с помощью которых было выведено влияние факторов на результативный показатель. В частности, табличные данные проделанного анализа могут быть использованы при анализе любого промышленного предприятия и этим сделаны выводы по оценке эффективности собственного капитала.

Анализ и обсуждение результатов.

В условиях либерализации экономики обеспечивается независимость каждого предприятия, а также повышается его ответственность. В таких условиях каждое предприятие должно иметь свои средства, свой метод управления имуществом, право собственности на имущество. Это основной источник всего существующего имущества, составляющего финансовый потенциал предприятия. Финансовые возможности могут быть достаточными у каждого экономического субъекта. Они состоят из двух источников: собственных средств и заемных средств.
Любое предприятие, фирма, организация должны иметь определенный уровень финансового состояния, чтобы обеспечить свою деятельность соответствующими средствами. Если предприятие не будет иметь финансовых возможностей оно не будет иметь возможности формировать свою материально-техническую базу и оборотный капитал. В результате становится трудно функционировать. Согласно аналитической научной статье: «При оценке финансового состояния предприятия необходимо

проанализировать структуру балансовых обязательств, то есть вклад собственного капитала в имущество предприятия и средств внешнего долга должен быть определенный. Наличие оптимального соотношения этих ресурсов определяет перспективы предприятия». При анализе динамики и структуры источника финансовых ресурсов, в частности, анализ структуры ресурсов осуществляется внутренними и внешними пользователями на основе данных бухгалтерского учета.

Собственный капитал предприятия состоит из уставного, добавленного и резервных инвестиций, нераспределенной прибыли, целевых доходов и фондов, резервов предстоящих расходов и платежей, доходов будущих периодов. Собственный капитал – это активы предприятия после вычета обязательств, то есть он представляет собой средства, принадлежащие только самим субъектам хозяйствования. [6] Собственный капитал – это показатель, составляющий основу собственных средств компании, и он является одним из важных показателей, характеризующих финансовое состояние компании. В зависимости от его положения экономическая зависимость или самостоятельность предприятия оценивается исходя из состава совокупных фондов. [7]

Анализ эффективности собственного капитала предприятия начинается прежде всего с определения структурной части и изменения собственного капитала на основе пассивного раздела 1 бухгалтерского баланса, для анализа обратим внимание на таблицу 1. Как видно из данных таблицы 1, собственный капитал АО «Махсусэнергогаз» в 2024 году по сравнению с 2019 годом уменьшился на 40731566,00 тыс.сум. Уставный капитал увеличился до 7847000,00 тыс.сум в 2019 году 744441,00 и 8591441,00 тыс.сум в 2024 году. В зависимости от его положения экономическая зависимость или самостоятельность предприятия оценивается исходя из состава совокупных фондов. Данную ситуацию можно оценить положительно. Капитал, добавленный к предприятию за годы анализа, составил 2 610,00 тыс.сум без изменений. В 2024 году резервный капитал уменьшился на 8373718,00 тыс.сум по сравнению с базовым годом. Нераспределенная прибыль составила в 2019 году 15664219,00 тыс.сум, в 2020 году 14360305,00 тыс.сум, в 2021 году 3606,00 тыс.сум, в 2022 году 24876397,00 тыс.сум. В 2023 году непокрытый убыток общества составил -22824216,00 тыс.сум, а в 2024 году – 24540592,00 тыс.сум. Таким образом, показатели финансового результата снизились.

Таблица 8: Динамический анализ состава собственного капитала АО «Махсусэнергогаз» за 2019–2024 годы

Показатели	**2019**	**2020**	**2021**	**2022**	**2023**	**2024**	**Изменение 2024 к 2019 (+,–)**
Состав собственного капитала	104 757 204	99 909 823	24 275 963	119 965 378	72 348 385	64 025 638	–40 731 566
Уставный капитал	744 441	744 441	19 157 277	8 724 441	8 724 441	8 591 441	+7 847 000
Добавленный капитал	2 610	2 610	–	2 610	2 610	2 610	–
Резервный капитал	88 345 934	84 802 467	5 115 080	86 361 930	86 445 550	79 972 216	–8 373 718
Нераспределенная прибыль (непокрытый убыток)	15 664 219	14 360 305	3 606	24 876 397	–22 824 216	–24 540 592	–8 876 373
Выкупленные собственные акции	–	–	–	–	–	37	+37

За период анализа стоимость приобретенных предприятием собственных акций составила 37 000 сумов. Таким образом, результаты анализа показывают, что собствен-

ный капитал АО «Махсусэнергогаз» изменился в результате воздействия внешних и внутренних факторов. Теперь рассмотрим горизонтальный и вертикальный анализ собственного капитала за 2022-2024 отчетные годы АО «Махсусэнергогаз» в таблице 2.

Согласно данным таблицы 2, источники собственных средств АО «Махсусэнергогаз» в 2023 году уменьшились на 47616993,00 тыс.сум. В отчетном году основное место в его структуре занимал резервный капитал, его доля на начало года составляла 71,99 процента, на конец года 56,40 процента. В 2022 году доля уставного капитала в источниках собственных средств составила 7,27 процента, а к 2023 году – 12,06 процента. Удельный вес добавленного капитала в отчетном периоде был незначительным и составлял 0,002 и 0,004 процента. В 2022 году доля нераспределенной прибыли в структуре собственного капитала компании составила 20,74 процента. И в 2023 году в обществе возникло 22824216,00 тыс.сум непокрытых убытков, что составило 31,53

Таблица-2

Горизонтальный и вертикальный анализ собственного капитала АО «Махсусэнергогаз»

Показатели	2021 год		2022 год		2023 год		Изменение в 2023году по отношению к2022 году (+, -)		Темп роста в 2023 году по сравнению с 2022 годом, %
	Сумма, т.с.	Удельный вес, %	Сумма, т.с.	Удельный вес, %	Сумма, т.с.	Удельный вес, %	Сумма, т.с.	Удельный вес, %	
Состав собственного капитала:	24275963,0	100	119965378,0	100	72348385,0	100	-47616993,0	-	60,31
Уставный капитал	19157277.0	78,91	8724441,0	7,27	8724441,0	12,06	-	+4,79	100
Добавленный капитал	-	-	2610,00	0,002	2610,0	0,004	-	+0,002	100
Резервный капитал	5115080,0	21,07	86361930,0	71,99	86445550,0	56,40	+83620,0	-15,59	100,10
Нераспределенная прибыль (непокрытый убыток)	3606,0	0,0001	24876397,0	20,74	(-) 22824216,0	31,53	-2052181,0	+10,79	91,75

Прибыль занимает особое место в структуре источников приумножения денежных средств. Увеличение объема прибыли приводит прежде всего к пополнению фонда и созданию других резервных фондов. Итак, как видно из результатов анализа, основная часть совокупных ресурсов общества в отчетных 2022 и 2023 годах соответствует доле резервного капитала. В 2021 году основное место в структуре собственного капитала занимала доля уставного капитала, которая составила 78,91 процента. В анализируемые годы произошло существенное изменение доли уставного капитала в структуре собственного капитала: она составляет 78,91% в 2021 году, 7,27% в 2022 году и 12,06% в 2023 году. Доля резервного капитала в 2021 году считается низкой по сравнению с другими годами и составляет 21,07 процента. Доля нераспределенной прибыли в прямых инвестициях в 2021 году составила 0,0001 процента, что значительно ниже. Как видно из таблицы, сумма нераспределенной прибыли в обществе за отчетный период уменьшилась, и на конец периода получено 2052181,00 тыс.сум невозмещенных убытков.

В процессе анализа важно определить соотношение заемных средств и собственных средств (собственного капитала). Данный коэффициент указывает на сумму заемных средств, соответствующую каждой сумме собственных средств общества, и определяется как отношение суммы заемных средств к сумме собственных средств (Форма 1, строка 770/строка 480). В АО «Махсусэнергогаз» на начало года это соот-

ношение составляло 152615295,00/119965378,00= 1,27, или на один сум собственных средств предприятия приходилось 1,27 тийина заемных средств. На конец года коэффициент 317697361,00/72348385,00=4,39, то есть одному суму собственных средств соответствовало 4,39 тийина заемных средств. Поэтому зависимость предприятия от заемных средств существенно возросла. Такая ситуация снижает уровень устойчивости его финансового положения. В начале года ресурсы объекта анализа формировались за счет собственных средств, а в конце года увеличились за счет заемных ресурсов. Такая ситуация приводит к снижению финансовой независимости предприятия, ухудшению платежеспособности. Рентабельность является одним из основных показателей при проверке деятельности акционерных обществ, а их финансовая устойчивость связана с правильным формированием получаемой прибыли.

Таблица-3

Анализ доходности собственного капитала АО "Махсусэнергогаз"

№	Показатели	2022 год	2023 год	Изменения в 2023 году по сравнению с 2022 годом, (+,-)	
				В абсолютной сумме,т.с.	В процентах, %
1.	Чистая прибыль, т.с	20154192,00	23259690,00	+3105498,00	+15,41
2.	Среднегодовая стоимость собственного капитала, т.с.	118112382,00	96156881,5	-21955500,5	-18,59
3.	Рентабельность собственного капитала (1x100:2), %	17,06	24,19	+7,13	+41,79

Известно, что основным источником собственности предприятий является собственный капитал. Поэтому анализ уровня рентабельности собственного капитала является основной задачей в процессе анализа. Рентабельность собственного капитала характеризует рентабельность собственных средств компании. Данный коэффициент рентабельности показывает количество прямых инвестиций на сум чистой прибыли. Для анализа этого показателя составим следующую таблицу (табл. 3).

Таблица-4

Расчет факторного анализа уровня доходности собственного капитала

Порядковый номер	Изменение порядкового номера	Факторы, входящие в формулу расчета		Обобщенный показатель – доходность собственного капитала,%	Разница с предыдущим расчетом (+.-)%
		Чистая прибыль	Среднегодовая стоимость собственного капитала		
A	B	1	2	3(1x100:2)	4
1	-	20154192,00	118112382,00	17,06	-
2	1	23259690,00	118112382,00	19,69	+2,63
3	2	23259690,00	96156881,5	24,19	+4,5

Факторный баланс: +7,13 %

Согласно данным таблицы 3, объем собственного капитала АО «Махсусэнерго-

газ» в 2023 году по сравнению с 2022 годом уменьшился на 21955500,5 тыс.сум. Рентабельность собственного капитала выросла на 7,13% до 24,19% в 2023 году. Этому изменению способствовали два фактора: 1. Изменение чистой прибыли. 2. Изменение среднегодовой стоимости собственного капитала. Влияние этих факторов мы определяем методом цепных подстановок в таблице 4 (табл. 4).

По данным таблицы 4, в результате увеличения чистой прибыли в акционерном обществе «Махсусэнергогаз» на 3105498,00 тыс.сум по итогам года по сравнению с началом года, рентабельность собственного капитала увеличился на 2,63%. Уменьшение среднегодовой стоимости собственного капитала на 21955500,5 тысяч сумов увеличило его доходность на 4,5 процента. Таким образом, рассчитанное выше влияние двух факторов составляет: 2,63+4,5=+7,13 процента. По итогам года чистая прибыль АО "Махсусэнергогаз"составила 3105498,00 тыс.сум и увеличилась до 396011,8 тыс.сум. Этому изменению способствовали два фактора: 1. Изменение среднегодового объема собственного капитала: 282214,5х 27,90/100= +78737,8 тыс.сум. 2. Изменение уровня рентабельности собственного капитала: 42,4 х 748405,5/100= +317323,9 тыс.сум. Равенство коэффициентов: 78737,8 +317323,9= +396061,7 тыс.сум. Из-за округления цифр (396011,8-396061,7) образовалась разница в 49900 сумов. Применение метода расчета факторов, влияющих на уровень рентабельности собственного капитала, в практической деятельности предприятий позволяет повысить их экономическую эффективность и обеспечить финансовую устойчивость. При анализе эффективности собственного капитала важно оценить коэффициенты его оборачиваемости. Оборачиваемость собственного капитала относится к уровню и сроку их обоснованности. Ниже представляется таблица 5 для анализа коэффициентов оборачиваемости.

Таблица-5

Анализ коэффициентов оборачиваемости собственного капитала АО «Махсусэнергогаз»

Показатели	2022 год	2023 год	Разница между 2023 и 2022 годом (+,-)
1. Среднегодовая стоимость собственного капитала, т.с.	118112382,0	96156881,5	-21955500,5
2. Чистый доход от реализации продукции, т.с.	89577498,0	249943612,0	+160366114,0
3. Коэффициент оборачиваемости капитала (2:1).	0,76	2,60	+1,84
4. Период оборота капитала, в днях (1х360:2)	473,68	138,50	-335,18
5. Коэффициент оборачиваемости собственного капитала (1/2)	1,32	0,38	-0,94

Как видно из таблицы 5, коэффициент оборачиваемости собственного капитала АО "Махсусэнергогаз"на начало года составил 0,76, на конец 2,60 и увеличился на 1,84. Коэффициент веса оборота собственного капитала снизился на 0,94. Срок оборота собственного капитала в обществе составил на конец года 138,50 дней и снизился до 335,18 дней. Теперь на основании данных табл. 5 определимся с расчетом факторов, влияющих на изменение срока оборота собственного капитала, (табл. 6). По данным таблицы 6, оборот собственного капитала в АО "Махсусэнергогаз"ускорился до 335,18 дней. Данное изменение было вызвано следующими факторами: 1. Увеличение объема чистого дохода от реализации продукции на 160366114,00 тыс. сум ускорило

оборот дня собственного капитала на 303,56 дня. 2. В результате уменьшения среднегодовой стоимости собственного капитала на 21955500,5 тыс.сум оборотный день ускорился до 31,62 дня. Таким образом, действие всех факторов равно суммарному изменению результата: (-303,56)+(-31,62)=-335,18 дней.

Таблица-6

Расчет факторов, влияющих на изменение даты оборота собственного капитала

Порядковый номер	Изменение порядкового номера	Факторы, входящие в формулу расчета		Обобщенный показатель – День оборота собственного капитала	Разница с предыдущим расчетом (+.-)
		Чистый доход от реализации продукции	Среднегодовой остаток собственного капитала		
A	B	1	2	3 (2x360:1)	4
1	-	89577498,00	118112382,00	473,68	-
2	1	249943612,00	118112382,00	170,12	-303,56
3	2	249943612,00	96156881,5	138,50	-31,62

Факторный баланс: -335,18 день

На основе применения метода корреляции можно рассчитать, что изменения факторов, влияющих на изменение срока оборота собственного капитала, связаны между собой. Если коэффициент корреляции равен 0, то это свидетельствует об отсутствии корреляции между изучаемыми показателями. Если коэффициент корреляции равен 1, то связь в изучаемых показателях является высокополной, то есть функциональной. Коэффициент корреляции от 0,1 до 0,3 – низкий, от 0,3 до 0,5 – средний, от 0,5 до 0,7 – значимый, от 0,7 до 0,9 – высокий, от 0,9 до 1 – очень высокий, то есть считается полностью функциональным. На основе данных следующей таблицы, используя метод корреляции, мы можем увидеть связь между объемом чистого дохода от реализации продукции и стоимостью собственного капитала (табл. 7).

Таблица-7

Расчет зависимости факторов, влияющих на изменение срока оборота собственного капитала, корреляционным методом

Годы	Чистый выручка от реализации продукции, (тыс сум) X	Стоимость собственного капитала (тыс сум) Y	X^2 квадрат ряда X	Y^2 квадрат ряда Y	Множители факторов X*Y
A	1	2	3	4	5
2019	11720	10476	13736	10975	12279
2020	89790	99910	80622	99820	89709
2021	76778	24276	58949	58932	18639
2022	89577	11996	80240	14390	10746
2023	24994	72348	62470	52342	18083
Сумма	**292859**	**219006**	**296017**	**236459**	**149456**

Коэффициент корреляции между чистым доходом от реализации продукции и

остатком собственного капитала можно выразить следующей формулой:

$$R_{xy} = \frac{\sum X \cdot Y}{\sqrt{\sum X^2 \cdot \sum Y^2}}$$

Расчет по этой формуле:149456/296017х 236459 =0,2 Таким образом, связь между чистым доходом от реализации продукции и собственным капиталом весьма функциональна. По результатам расчета коэффициент корреляции составил 0,2 балла. 20% чистого дохода от реализации продукции зависит от размера собственного капитала, а остальные 80% зависят от других факторов.

Согласно результатам анализа, собственный капитал АО "Махсусэнергогаз"за отчетный период снизился. Уменьшение собственного капитала предприятия произошло в основном за счет уменьшения нераспределенной прибыли. Потому что, если собственный капитал впервые предоставляется через уставный фонд, сформированный за счет средств учредителей, то он должен увеличиваться за счет отчислений из прибыли, полученной в ходе деятельности предприятия. Поэтому для увеличения собственного капитала прежде всего необходимо увеличить прибыль предприятия. Прибыль предприятия – это не результат какой-либо экономической деятельности, а результат эффективной деятельности.

Оценка финансового состояния является одним из основных вопросов обеспечения финансовой устойчивости предприятий и повышения показателей эффективности и всегда должна находиться в центре внимания руководителя предприятия, аналитиков и аудиторов.

Выводы и предложения.

В данной научной статье с использованием практической информации изучено финансовое состояние акционерного общества "Махсусэнергогаз"и даны практические рекомендации. На предприятии в отчетном году доля обязательств в пассивах баланса занимала основное место. Эту ситуацию можно считать негативной. Итак, в обществе существует потребность в кредитных средствах. Поэтому в ходе анализа был рассчитан коэффициент соотношения собственных и заемных средств. По итогам года АО "Махсусэнергогаз"привлекло заемные средства в размере 4,39 копейки на каждый сум собственных средств. Такая ситуация снизила уровень устойчивости его материального положения, привела к ухудшению его платежеспособности.

Собственный капитал составляет основу источника собственных средств компании и является одним из важных показателей, характеризующих финансовое состояние компании. [8] В зависимости от его положения экономическая зависимость или самостоятельность предприятия оценивается исходя из состава совокупных фондов. Таким образом, были проанализированы коэффициенты оборачиваемости, относящиеся к собственному капиталу акционерного общества "Махсусэнергогаз даны практические рекомендации по анализу факторов, влияющих на изменение периода оборота, взаимосвязь между чистым доходом от реализации продукции и стоимость собственного капитала рассчитывалась корреляционным методом. При этом был рекомендован и метод факторного анализа уровня доходности собственного капитала.

В результате оценки финансового состояния предприятий можно будет наладить целенаправленное управление собственным капиталом и пассивами, ускорить обращение денег и ликвидных средств, определить практические меры по обеспечению

их финансовой стабильности. Ведь анализ финансового состояния важен при определении финансовой независимости, устойчивости и кредитоспособности предприятия.

По результатам данной научной статьи можно дать следующие практические рекомендации:
- для обеспечения стабильности финансового состояния компании необходимо увеличивать объем собственного капитала и эффективно его использовать. Для увеличения собственного капитала прежде всего необходимо увеличить прибыль предприятия;
- большую часть расходов предприятий составляют проценты по кредиту в банках и расходы на транспорт. В связи с этим предприятие должно максимально увеличить долю собственного капитала в оборотном капитале и, наконец, покрыть весь оборотный капитал за свой счет;
- одним из основных способов обеспечения финансовой устойчивости является принятие мер по недопущению увеличения дебиторской задолженности предприятия и текущих кредиторских обязательств. Большой размер обязательств приводит к ухудшению финансового состояния предприятия. Невыполнение платежных обязательств может привести к экономической неплатежеспособности.

Мы считаем, что применение данных рекомендаций в практической деятельности предприятий позволит точно диагностировать их финансовое состояние, повысить эффективность собственного капитала, обеспечить их конкурентоспособность, предотвратить банкротство, повысить их финансовый потенциал и обеспечить финансовую устойчивость. Отечественные и зарубежные публикации о финансовой ситуации и ее анализе опубликованы авторами настолько обширно , что можно прийти к одному выводу, то есть эффективность функционирования экономики действующих предприятий и оценка ее эффективности всегда осуществляется широким кругом заинтересованных сторон (менеджеров, собственников недвижимости и т.п.) контрагентов был одним из важных вопросов, находящихся в их внимании. [9] Финансовый не только путем анализа ситуации и изменений в ней, но и эффективности и результативности процессов, но и к ним прогнозируются будущие ожидания. И это конкурентоспособно становится все более важным в экономических условиях. Эффективность анализа финансового состояния заключается в его информационном обеспечении и определяется качеством источников. информационное обеспечение основным источником является финансовый отчет. Финансовая отчетность активы, капитал и обязательства, доходы и расходы известны в совокупности, сбалансированный, отсортированный по ликвидности, из форм периодической отчетности и прилагаемый к ним источников [10].

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Информационная безопасность в мире цифровых технологий

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DOI 10.5281/zenodo.18224869

Аннотация: В статье рассмотрены основные понятие безопасности. А также показаны основные угрозы и меры защиты от этих угроз. Выявлены основные меры защиты информации.

Ключевые слова: безопасность, технологии, решения, угрозы, Интернет.

Наш мир сегодня это - мир технологий. Если раньше мы не так активно пользовались технологиями, то сейчас они прочно вошли в нашу жизнь. Где бы мы не были нас окружают современные технологии. Мир стал цифровым, если до пандемии это было не заметно, то после неё использование технологий возросло. Технологии помогаю нам учиться, получать ответы, разговаривать на расстояние, совершат покупки и т.д.

В наши дни цифровые технологии глубоко проникли в жизнь общества. Интернет, мобильные гаджеты, облачные хранилища и соцсети стали для многих привычны-

ми вещами. Но с ростом объёмов и скорости передачи информации растёт и число рисков неразрешенного доступа к личным данным. По этой причине вопросы информационной безопасности становятся всё более важными.

Современный мир сталкивается с беспрецедентным ростом количества данных, создаваемых ежедневно - от личных фотографий и сообщений в социальных сетях, до медицинских записей и финансовой информации - вся эта информация циркулирует в цифровом пространстве.

Появление таких технологий, как Интернет вещей (IoT), облачные хранилища и аналитика больших данных (Big Data), еще больше ускоряют этот процесс. Но чем больше данных генерируется и хранится, тем сильнее увеличивается риск их компрометации. [1]

В нашей статье мы бы хотели рассмотреть вопрос о безопасности информации. А также, дать рекомендации по обеспечению безопасности информации.

Если сказать коротко, то информационная безопасность это совокупность мер, которые помогают защитить информацию от не санкционного доступа, взлома данных и других мер для получения нашей личной информации.

Можно сказать, что безопасность информации — это обеспечения мер защиты трёх свойств информации:

Конфиденциальности, целостности и доступности. Самое первое, что нужно сделать, это обеспечить конфиденциальность информации. То есть, нужно чётко понять какую информацию можно сообщат и что нельзя раскрывать, когда спрашивают. Сейчас, появились много способов обманным путём получения конфиденциальной информации.

Второе это сохранения целостности информации. Информация должна быть полной для принятия решений, если нарушит целостность информации, то у нас не будет полной картины для принятий решений.

Третий шаг при безопасности информации это её доступность. То есть, информация должна быть открытой для использования, кроме той которая по закону считается конфиденциальной. (личные данные, номера карт, пароли и т.п.)

В нашем мире информационных технологий сохранить безопасность информации очень актуально. Если, рассмотреть то безопасность информации это не только защита данных и устройств, но и защита материальных ценностей тоже. То есть, уже крадут не только с устройства, а с банковских карт, счетов.

Давайте рассмотрим основные угрозы, которые могут нанести вред нашей информации:

1. Цифровая неграмотность то есть, человек не обучен работать с цифровыми носителями.

2. Разные приложения, которые, могут навредить нашем устройствам.

3. Хакеры, киберпреступники которые могут нанести вред нашей информации.

4. Сотрудники, которые не соблюдают информационную безопасность.

Любую информацию необходимо охранять, вне зависимости от того, каким образом она хранится или используется. В наше время существует большое количество информационных проблем, поэтому грамотный руководитель должен оценивать состояние собственной информационной сети. Для этого руководитель организации может проводить повышение квалификации ответственных лиц и пользователей, которые работают с важной информацией, основам и особенностям информационной безопасности. В дополнение к этому устанавливаются и регулярно обновляются антивирусные защитные программы, производится шифрование данных. [2]

Следующие меры могут помочь нам сохранит и обезопасить себя

1. Повысит цифровую грамотность. Нужно проводит семинары, тренинги, где можно будет научиться, как защитить информацию.
2. Ставит надёжные пароли, если есть возможность, то использовать двухэтапную аутентификация.
3. Всегда обновлять программное обеспечения и антивирусы, не устанавливать сомнительные приложения.
4. Не сообщат коды, смс сообщения незнакомым людям. Проверят сомнительные звонки.
5. Шифровать данные.

Можно сказать это меры безопасности могут помочь, при защити своей информации. Кроме этих методов можно и нужно использовать современные технические средства и принят законы, которые помогут нам организовать защиту информации. Однако, нужно учесть самое главное человек сам отвечает за свою защиту. Мы всегда должны учитывать человеческий фактор. Если человек, сам не будет принимать меры защиты, то его невнимательность может свести все усилия на нет.

Ещё один фактор в безопасности — это появления искусственного интеллекта. Многие уже используют искусственный интеллект в корыстных целях. Поэтому любые подозрительные действия и сообщения нужно тщательно проверят.

Развитие искусственного интеллекта, интернета вещей и больших данных ставит перед нами новые проблемы в сфере информационной безопасности. Растет потребность в создании умных систем защиты, которые могут сами находить и останавливать угрозы. Информационная безопасность станет очень важной для устойчивого развития цифрового общества. Будущее цифровой безопасности зависит от сотрудничества правительств, бизнеса и граждан для создания прозрачной и защищенной среды. Чтобы технологии служили на благо, необходимо постоянно улучшать правовые и этические нормы использования данных. В мире, где информация наиболее ценна, надежная защита – гарантия прогресса и суверенитета в цифровом пространстве.

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Оптимальная квалратупная формула с производными

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DOI 10.5281/zenodo.18224877

Keywords: функционал погрешности, оптимальная квадратурная формула, оптимальные коэффициенты.

1.**Введение.** В настоящее время во всем мире все большее значение приобретает построение оптимальных квадратурных формул при приближенном вычислении сильно осциллирующих интегралов.

Одной из важнейших задач вычислительной математики является построение оптимальных квадратурных формул дифференцируемых функций в различных пространствах для приближенного вычисления сильно осциллирующих интегралов.

Построение оптимальных (в определенном смысле) методов приближенного вычисления интегралов сильно осциллирующих функций — интересная и важная область компьютерной технологии и математики, поскольку много важных задач, решаемых на практике (например, цифровая обработка сигналов, обработка изображений, радиолокация, медицинская электроника, компьютерная томография и т. д.), требуют вычисления таких интегралов. В нашей работе мы также рассматриваем построение оптимальных квадратурных формул для приближенного вычисления сильно осциллирующих интегралов.

2.**Постановка задач.** Предположим, что производная первого порядка функции $\varphi(x)$ абсолютно непрерывна, интегрируется по квадрату производной второго порядка и является комплекснозначной функцией с действительными аргументами.

Набор таких функций образует гильбертово пространство $L_2^{(2)}(0,1)$. Скалярное

произведение двух функций φ и ψ в этом пространстве определяется равенством

$$\langle \varphi, \psi \rangle = \int_0^1 \varphi''(x)\overline{\psi''(x)}dx, \tag{1}$$

где $\overline{\psi}$ является сопряженной функцией к функции ψ. Норма функции φ соответственно определяется формулой

$$\|\varphi|L_2^{(2)}(0,1)\|^2 = \langle \varphi, \varphi \rangle = \int_0^1 \varphi''(x)\overline{\varphi''(x)}dx, \tag{2}$$

В этом пространстве рассмотрим квадратурную формулу с производными следующего вида

$$\int_0^1 e^{2\pi i\omega x}\varphi(x)dx \cong \sum_{\beta=0}^{N} d_0[\beta]\varphi(h\beta) + \sum_{\beta=0}^{N} d_1[\beta]\varphi'(h\beta), \tag{3}$$

где $e^{2\pi i\omega x}$ весовая функция, $[\beta] = (h\beta), \quad h = \frac{1}{N}$, N- натуральное число, $i^2 = -1$, $\omega h \in \mathbb{Z}$, $\omega \neq 0$. $d_0[\beta], d_1[\beta]$ - коэффициенты квадратурной формулы (3). Здесь $d_0[\beta]$-оптимальные коэффициенты квадратурной формулы

$$\int_0^1 e^{2\pi i\omega x} f(x)dx \cong \sum_{\beta=0}^{N} d_0[\beta]\varphi(h\beta),$$

при $\omega h \in \mathbb{Z}$, $\omega \neq 0$ в пространстве $L_2^{(1)}(0,1)$ и имеют вид

$$\begin{aligned} &d_0[0] = -\tfrac{1}{2\pi i\omega}, \\ &d_0[\beta] = 0, \qquad\qquad \beta = \overline{1, N-1}, \\ &d_0[N] = \tfrac{1}{2\pi i\omega}, \end{aligned} \tag{4}$$

а коэффициенты $d_1[\beta]$ в (3) - неизвестные коэффициенты квадратурной формулы.

Разность между суммой иинтегралов

$$(\ell, \varphi) = \int_0^1 e^{2\pi i\omega x}\varphi(x)dx - \sum_{\beta=0}^{N} d_0[\beta]\varphi(h\beta) - \sum_{\beta=0}^{N} d_1[\beta]\varphi'(h\beta) = \int_{-\infty}^{\infty} \ell(x)\varphi(x)dx \tag{5}$$

называется погрешностью квадратурной формулы (3). Этой погрешности соответствует следующий функционал погрешности:

$$\ell(x) = \varepsilon_{[0,1]}(x)e^{2\pi i\omega x} - \sum_{\beta=0}^{N} d_0[\beta]\delta(x - h\beta) - \sum_{\beta=0}^{N} d_1[\beta]\delta'(x - h\beta), \tag{6}$$

где $\varepsilon_{[0,1]}(x)$ характеристическая функция отрезка $[0,1]$, $\delta(x)$ - дельта-функция Дирака.

Функционал погрешности ℓ квадратурной формулы (3) является линейным функционалом, принадлежащим сопряженному пространству $L_2^{(2)^*}(0,1)$. Так как функционал погрешности ℓ определен в пространстве $L_2^{(2)}(0,1)$, должны быть выполнены следующие условия [1,2]

$$(\ell, x^\alpha) = 0, \quad \alpha = 0, 1.$$

Нетрудно проверить, что коэффициенты $d_0[\beta]$ в квадратурной формуле (3) обеспечивают точность формулы до постоянного числа, т. е. ортогональность функционал ортогонален ℓ произвольному постоянному числу

$$(\ell, 1) = 0.$$

Действительно, согласно (4),

$$(\ell,1) = \int\limits_{-\infty}^{\infty} \left(\varepsilon_{[0,1]}(x) e^{2\pi i\omega x} - \sum_{\beta=0}^{N} d_0[\beta]\delta(x-h\beta) - \sum_{\beta=0}^{N} d_1[\beta]\delta'(x-h\beta) \right) \cdot 1 dx =$$

$$= \int\limits_{0}^{1} e^{2\pi i\omega x} dx - \sum_{\beta=0}^{N} d_0[\beta] \int\limits_{-\infty}^{\infty} \delta(x-h\beta)dx - \sum_{\beta=0}^{N} d_1[\beta] \int\limits_{-\infty}^{\infty} \delta'(x-h\beta)dx =$$

$$= \int\limits_{0}^{1} e^{2\pi i\omega x} dx - \sum_{\beta=0}^{N} d_0[\beta] = 0.$$

Отметим, что ℓ будет также ортогонален функциональному многочлену первой степени.

Условие $(\ell, x) = 0$ равносильно условию $\sum\limits_{\beta=0}^{N} d_1[\beta] = 0.$

Действительно, вычисляя $(\ell, x) = 0$ имеем

$$0 = \int\limits_{-\infty}^{\infty} \left(\varepsilon_{[0,1]}(x) e^{2\pi i\omega x} - \sum_{\beta=0}^{N} d_0[\beta]\delta(x-h\beta) - \sum_{\beta=0}^{N} d_1[\beta]\delta'(x-h\beta) \right) \cdot x dx =$$

$$= \int\limits_{0}^{1} e^{2\pi i\omega x} x dx - \sum_{\beta=0}^{N} d_0[\beta] \int\limits_{-\infty}^{\infty} \delta(x-h\beta)x dx - \sum_{\beta=0}^{N} d_1[\beta] \int\limits_{-\infty}^{\infty} \delta'(x-h\beta)x dx =$$

$$= \int\limits_{0}^{1} x e^{2\pi i\omega x} dx - \sum_{\beta=0}^{N} d_0[\beta](h\beta) - \sum_{\beta=0}^{N} d_1[\beta][\beta] = -\sum_{\beta=0}^{N} d_1[\beta],$$

от при $\sum\limits_{\beta=0}^{N} d_1[\beta] = 0.$

Нам известно, что на основе неравенства Коши-Шварца (3) абсолютное значение погрешности квадратурной формулы (3) оценивается сверху с помощью произведения норм функции и функционала погрешности ℓ следующим образом

$$|(\ell, \varphi)| \le \|\ell | L_2^{(2)^*}(0,1)\| \cdot \|\varphi | L_2^{(2)}(0,1)\|.$$

Из неравенства Коши-Шварца видно, что абсолютное значение погрешности квадратурной формулы (3) для элемента φ, принадлежащего пространству $L_2^{(2)}(0,1)$, оценивается нормой функционала погрешности ℓ в сопряженном пространстве $L_2^{(2)^*}(0,1)$.

Норма функционала погрешности ℓ зависит от коэффициентов $d_0[\beta]$, $d_1[\beta]$. Задача нахождения минимума коэффициентов при фиксированных узлах нормы функционала погрешности ℓ называется *задачей Сарда*, а полученная формула — *оптимальной квадратурной формулой в смысле Сарда.*

Основной целью данной работы является решение задачи Сарда для квадратурных формул вида (3) в пространстве $L_2^{(2)}(0,1)$ методом С.Л. Соболева, т.е. нахождение коэффициентов $d_1[\beta]$ (если таковые имеются) при

$$\|\mathring{\ell}|L_2^{(2)^*}(0,1)\| := \inf_{d_1[\beta]} \|\ell|L_2^{(2)}(0,1)\|$$

для заданных коэффициентов $d_0[\beta]$.

Поэтому для построения оптимальных квадратурных формул вида (3) в смысле Сарда в пространстве $L_2^{(2)}(0,1)$ необходимо решить следующие задачи.

Задача 1. Найти вид нормы функционала погрешности ℓ квадратурной формулы вида (3) в пространстве $L_2^{(2)^*}(0,1)$.

Задача 2. Найти коэффициенты $d_1[\beta]$, дающие минимальное значение найденной норме.

Задача 3. Вычислить квадрат нормы функционала погрешности ℓ с найденными коэффициентами $d_1[\beta]$.

Найденные коэффициенты $d_1[\beta]$ называются *оптимальными коэффициентами* квадратурной формулы (3) и обозначаются $\mathring{d}_1[\beta]$. Соответствующий коэффициенту $\mathring{d}_1[\beta]$ функционал погрешности $\mathring{\ell}$ называется *функционалом погрешности, соответствующим оптимальной квадратурной формуле*, а квадратурная формула называется *оптимальной квадратурной формулой.*

Аналогичные задачи рассмотрены в работах [3-6].

3. **Основные результаты.**

Теорема 1. При $\omega h \in \mathbb{Z}$ и $\omega \neq 0$ коэффициенты оптимальной квадратурной формулы вида (3) в смысле Сарда пространства $L_2^{(2)}(0,1)$ имеют следующий вид:

$$\begin{array}{llll} d_0[0] = -\frac{1}{2\pi i\omega}, & & d_1[0] = -\frac{1}{(2\pi\omega)^2}, & \\ d_0[\beta] = 0, & \beta = \overline{1, N-1}, & d_1[\beta] = 0, & \beta = \overline{1, N-1} \\ d_0[N] = \frac{1}{2\pi i\omega}, & & d_1[N] = \frac{1}{(2\pi\omega)^2}. & \end{array}$$

Теорема 2. В пространстве $L_2^{(2)}(0,1)$ квадрат нормы функционала погрешности (6) оптимальной квадратурной формулы (3) имеет следующий вид:

$$\|\ell\|^2_{L_2^{(2)^*}} = \frac{h^4}{16\pi^4 k^4},$$

здесь $h = \frac{1}{N}, N \in \mathbb{N}, \omega h \in \mathbb{Z}, \omega \neq 0$ и $\omega = kN$.

Из результатов теоремы 1 очевидно следует, что $(\ell, 1) = 0$ и $(\ell, x) = 0$.

Заключение

В данной работе для приближенного вычисления интегралов от сильно осциллирующих функций построена оптимальная квадратурная формула с производными в пространстве комплекснозначных функций с действительными аргументами, интегрируемая с квадратом обобщенной производной второго порядка.

То есть, с помощью экстремальной функции находится квадрат нормы функционала погрешности рассматриваемой квадратурной формулы. Для минимизации нормы этого функционала погрешности по коэффициентам была построена функция Лагранжа и получена система типа Винера-Хопфа.

Путем решения этой системы с помощью дискретного аналога дифференциального оператора было найдено аналитическое представление оптимальных коэффициентов. Далее с помощью аналитического представления этих коэффициентов вычислен квадрат нормы функционала погрешности.

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Оптимальные формулы приближённого интегрирования

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DOI 10.5281/zenodo.18225351

Keywords: Метод Файлона, функционал погрешности, оптимальная квадратурная формула, оптимальные коэффициенты.

1.**Введение.** При решении таких классов задач, как статистическая обработка экспериментальных данных, цифровая фильтрация, распознавание образов, моделирование оптических систем и синтезированных голограмм, краевые задачи для уравнений частных производных и другие, возникает необходимость в вычислении интегралов вида [1]

$$I_0(\omega) = \int_0^1 f(x)e^{2\pi i\omega x}dx, \tag{1}$$

$$I_1(\omega) = \int_0^1 f(x)\sin\omega x dx, \tag{1}$$

$$I_2(\omega) = \int_0^1 f(x)\cos\omega x dx. \tag{1}$$

где $f(x) \in F$ (F - некоторый заданный класс функций), ω - произвольное вещественное число, информация о $f(x)$ задана не более чем в N точках. Хорошо известно, что численн?й счет таких интегралов наталкивается на определение трудности при больших значениях ω из-за того, что подинтегральная функция сильно осцилирует.

Вычисление интеграла (1) при $\omega \in \mathbb{Z}$($\mathbb{Z}$ - множество целых чисел) часто выполняется методом Файлона [2]. Метод Файлона напоминает квадратурную формулу Симпсона. Однако в то время, как в методе Симпсона вся подинтегральная функция заменяется параболой, в методе Файлона параболой заменяется только функция $f(x)$. Таким путём Файлон получил квадратурую формулу с коэффициентами, зависящими от ω. Для приближенного вычисления интегралов (1)-(3) в работе [3] Н.П. Еручиным и С.Л. Соболеваым предложен метод, который также сохраняет равномерную точность относительно ω. В работе [4] В.И.Крылов развивал метод Файлона. В [5] при построении квадратурной формулы для вычисления коэффициентов Фурье была использована сплайн интерполяции и многочлены Лагранжа. Работы [6,7] посвящены нахождению оптимальных оценок и построению оптимальных по точности квадратурных формул вычисления преобразования Фурье финитных функций вида (2) и (3) в предположении, что $f(x)$ принадлежит в интерполяционный класс функций, удовлетворяющих условию Липщица. В работах [8,9] с помощью функциональных методов построены оптимальные квадратурные формулы в пространствах Гильберта.

В настоящей работе мы будем построить оптимальные квадратурные формулы для приближенного вычисления интеграла (1) в пространстве $\tilde{H}_2^{(m)}(0,1)$.

2.**Постановка задач.** Пусть $H_2^{(m)}(0,1)$ - пространство Соболева 1- периодических комплекснозначных функций $\varphi(x)$, $-\infty < x < \infty$, отличающихся более, чем на константу, m- е производные которых (в обобщенном смысле) квадратично интегри-

руемы со скалярным произведением

$$(f,\varphi)=\int\limits_0^1 \frac{d^m f\overline{d^m\varphi}}{dx^m dx^m}dx, \tag{4}$$

и нормой

$$\|\varphi|\tilde{H}_2^{(m)}\|=\left(\int\limits_0^1 \varphi^{(m)}(x)\overline{\varphi^{(m)}(x)}dx\right)^{\frac{1}{2}},$$

Здесь обозначение $\overline{\varphi(x)}$ сопряженное к $\varphi(x)$.

Для $\varphi(x)\in\tilde{H}_2^{(m)}(0,1)$ рассмотрим следующую квадратурную формулу

$$\int\limits_0^1 \exp(2\pi i\omega x)\varphi(x)dx\cong\sum_{k=1}^N C_k\varphi(x_k), \tag{5}$$

где C_k и x_k соответственно коэффициенты и узлы квадратурной формулы, $N=2,3,...,\omega\in\mathbb{R}$ ($\mathbb{R}$- множество действительных чисел). Погрешность квадратурной формулы (5) определяется равенством

$$(\ell,\varphi)=\int\limits_0^1 \exp(2\pi i\omega x)\varphi(x)dx-\sum_{k=1}^N C_k\varphi(x_k)=$$

$$=\int\limits_0^1\left(\exp(2\pi i\omega x)-\sum_{k=1}^N C_k\delta(x-x_k)*\phi_0(x)\right)\varphi(x)dx. \tag{6}$$

Здесь $\delta(x)$ - дельта функция Дирака,

$$\ell(x)=\varepsilon_{[0,1]}(x)\exp(2\pi i\omega x)-\sum_{k=1}^N C_k\delta(x-x_k)*\phi_0(x) \tag{7}$$

функционал погрешности квадратурной формулы (5),

$$\phi_0(x)=\sum_{\beta=-\infty}^{\infty}\delta(x-\beta).$$

Наша задача о построении оптимальной квадратурной формулы для приближенного вычисления интеграла

$$I_0(\omega)=\int\limits_0^1 \exp(2\pi i\omega x)\varphi(x)dx$$

при фиксированном x_k заключается в следующем.

Требуется найти величину

$$\inf_{C_k}\left(\sup_{\varphi\in\tilde{H}_2^{(m)}}\frac{|(\ell,\varphi)|}{\|\varphi|\tilde{H}_2^{(m)}\|}\right)=\left\|\mathring{\ell}|\tilde{H}_2^{(m)*}\right\| \tag{8}$$

Это означает, что найти функцию $\psi_\ell(x)$ из $\tilde{H}_2^{(m)}(0,1)$ для которой достигается точная верхняя грань, далее найти коэффициенты $\mathring{C}_k$ для которых достигается точная нижняя грань в (6). Это означает, что выполняется следующее равенство

$$(\ell,\psi_\ell)=\left\|\ell|\tilde{H}_2^{(m)*}\right\|\left\|\psi_\ell|\tilde{H}_2^{(m)}\right\|.$$

Коэффициенты $\mathring{C}_k$ называется оптимальными коэффициентами а $\left\|\mathring{\ell}|\tilde{H}_2^{(m)*}\right\|$ нормой функционала погрешности оптимальной квадратурной формулы. В следующем пункте мы приводим оптимальные квадратурные формулы при $x_k=hk, h=\frac{1}{N}$.

3.**Основные результаты.** Основные результаты в этой работе приводятся в следующих теоремах.

Теорема 1. Среди квадратурных формул вида (1) в пространстве $\tilde{H}_2^{(1)}(0,1)$ при $\omega\in\mathbb{R}$ существует единственная оптимальная квадратурная формула коэффициенты которой определяется формулой

$$\mathring{C}[\beta]=h\frac{e^{2\pi i\omega}-1}{2\pi i\omega}-h\sum_{\gamma\neq 0}\left(\frac{\sin\pi h\gamma}{\pi h\gamma}\right)^2\frac{e^{2\pi i(\omega-\gamma)}-1}{2\pi i(\omega-\gamma)}e^{2\pi i\gamma h\beta},\beta=1,2,...,N,h=\frac{1}{N}.$$

При $\omega\in\mathbb{Z}$ оптимальные коэффициенты $\mathring{C}[\beta]$ принимает следующий вид:

$$\mathring{C}[\beta]=h\left(\frac{\sin\pi\omega h}{\pi\omega h}\right)^2e^{2\pi i\omega h\beta},\beta=1,2,...,N.$$

А при $\omega=0$, оптимальные коэффициенты принимает следующий вид:

$$\mathring{C}[\beta]=h,\qquad \beta=1,2,...,N.$$

Теорема 2. В пространстве $\tilde{H}_2^{(2)}(0,1)$ Соболева периодических функций следующая квадратурная формула является оптимальной

$$\int_0^1 e^{2\pi i\omega x}\varphi(x)dx\cong\sum_{\beta=1}^N\mathring{C}_\beta\varphi(h\beta).$$

Здесь оптимальные коэффициенты $\mathring{C}[\beta]$ задается равенством

$$\mathring{C}[\beta]=h\frac{e^{2\pi i\omega}-1}{2\pi i\omega}-h\sum_{\gamma\neq 0}\frac{e^{2\pi i(\omega-\gamma)}-1}{2\pi i(\omega-\gamma)}3\left(\frac{\sin\pi h\gamma}{\pi h\gamma}\right)^4\frac{e^{2\pi ih\gamma\beta}}{\cos 2\pi h\gamma+2},\quad \omega\in\mathbb{R},\quad \omega\overline{\in}\mathbb{Z},\quad \beta=1,2,...,N.$$

А при $\omega\in\mathbb{R}/0$ оптимальные коэффициенты $\mathring{C}[\beta]$ принимает следующий вид:

$$\mathring{C}[\beta]=3h\left(\frac{\sin\pi h\gamma}{\pi h\gamma}\right)^4\frac{e^{2\pi i\omega h\beta}}{\cos 2\pi h\omega+2},\qquad \beta=1,2,...,N.$$

В случае $\omega=0$ оптимальные коэффициенты $\mathring{C}[\beta]$ определяется равенством

$$\mathring{C}[\beta]=h,\qquad \beta=1,2,...,N.$$

Теорема 3. Оптимальные коэффициенты квадратурной формулы вида (4) в пространстве $H_2^{(3)}(0,1)$ определяется формулой

$$\mathring{C}[\beta] = h\frac{e^{2\pi i\omega}-1}{2\pi i\omega} - 3h\sum_{\gamma\neq 0}\frac{e^{2\pi i(\omega-\gamma)}-1}{2\pi i(\omega-\gamma)}\times$$

$$\times\left(\frac{\sin\pi h\gamma}{\pi h\gamma}\right)^6\frac{e^{2\pi ih\gamma\beta}}{\cos 4\pi h\gamma + 26\cos 2\pi\omega h + 33}, \quad \omega\in\mathbb{R}, \quad \omega\overline{\in}\mathbb{Z}, \quad \beta = 1,2,...,N.$$

При $\omega\in\mathbb{R}/0$ оптимальные коэффициенты принимают следующий вид:

$$\mathring{C}[\beta] = 3h\left(\frac{\sin\pi h\omega}{\pi h\omega}\right)^6\frac{e^{2\pi i\omega h\beta}}{\cos 4\pi\omega h + 26\cos 2\pi\omega h + 33}, \qquad \beta = 1,2,...,N.$$

А при $\omega = 0$ оптимальные коэффициенты $\mathring{C}[\beta]$ принимает вид: $\mathring{C}[\beta] = h$, $\beta = 1,2,...,N$.

Заключение. В данной работе построены оптимальные квадратурные формулы в пространстве Соболева комплекснозначных периодических функций для приближенного вычисления быстроосцилирующих интегралов. В пространствах $\tilde{H}_2^{(1)}(0,1)$, $\tilde{H}_2^{(2)}(0,1)$ и $\tilde{H}_2^{(3)}(0,1)$ найдены аналитический вид оптимальных коэффициентов квадратурных формул.

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Норма функционала погрешности оптимальной квадратурной формулы

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DOI 10.5281/zenodo.18225415

Keywords: Квадратурные формулы, функционал погрешности, оптимальные коэффициенты.

В настоящей работе вариационным методом будут построены оптимальные квадратурные формулы в пространстве Соболева комплекснозначных функций.

Рассмотрим квадратурные формулы вида

$$\int_0^1 p(x)\varphi(x)dx \cong \sum_{\beta=0}^{N} C[\beta]\varphi[\beta], \tag{1}$$

с функционалом погрешности

$$\ell_N(x) = \varepsilon_{[0,1]}(x)p(x) - \sum_{\beta=0}^{N} C[\beta]\delta(x-\beta). \tag{2}$$

Здесь $p(x)$ весовая функция, от этой функции. Требуется, чтобы интеграл от этой функции существовал в каком то смысле, т.е.

$$\int_0^1 p(x)dx < \infty,$$

$\varphi(x) \in H_2^{(m)}(0,1)$- пространство Соболева комплекснозначных функций со скалярным произведением и нормой, соответственно.

$$(\psi,\varphi)_{H_2^{(m)}} = \sum_{n=0}^{m} \binom{m}{k} \frac{1}{\omega^{2k}} \int_0^1 \frac{d^k}{dx^k}\overline{\psi}(x)\frac{d^k}{dx^k}\varphi(x)dx, \tag{3}$$

$$\|\varphi\|_{H_2^{(m)}} = \left(\sum_{n=0}^{m} \binom{m}{k} \frac{1}{\omega^{2k}} \int_0^1 \frac{d^k}{dx^k}\overline{\varphi}(x)\frac{d^k}{dx^k}\varphi(x)dx\right)^{\frac{1}{2}}, \tag{4}$$

где $\omega \in \mathbb{R}$- множество действительных чисел и $\omega \neq 0$, $\overline{\psi}(x)$ сопряженная функция к функцию $\psi(x)$, $C[\beta], \quad \beta = 0,1,...,N$ коэффициенты квадратурной формулы (1),

$\varepsilon_{[0,1]}(x)$- характеристическая функция отрезка $[0,1]$, $\delta(x)$- дельта функция Дирака, $\binom{m}{k} = \frac{m!}{k!(m-k)!}$, $[\beta] = h\beta, \quad h = \frac{1}{N}, \quad N = 1, 2, ...$

Следующая разность

$$(\ell_N, \varphi) = \int_0^1 p(x)\varphi(x)dx - \sum_{\beta=0}^{N} C[\beta]\varphi[\beta] = \int_R \ell_N(x)\varphi(x)dx \tag{5}$$

называется погрешностью квадратурной формулы (1).

Из неравенства Коши–Шварца

$$|(\ell_N, \varphi)| \le \|\ell_N|H_2^{(m)^*}\| \cdot \|\varphi|H_2^{(m)}\|$$

видно, что абсолютное значение погрешности квадратурной формулы (1) оценивается с помощью нормы

$$\|\ell_N|H_2^{(m)^*}\| = \sup_{\varphi, \|\varphi\| \neq 0} \frac{|(\ell_N, \varphi)|}{\|\varphi|H_2^{(m)}\|} \tag{6}$$

функционала погрешности (2) в сопряжённом пространстве $H_2^{(m)^*}$.

Отсюда следует, что оценка погрешности квадратурной формулы (1) на комплекснозначных функциях пространства Соболева $H_2^{(m)}(0,1)$ сводится к нахождению нормы функционала погрешности (2) в сопряжённом пространстве $H_2^{(m)^*}(0,1)$. Видно, что норма функционала погрешности (2) зависит от коэффициентов $C[\beta]$ квадратурной формулы (1). Минимизируя нормы функционала погрешности $\ell_N(x)$ по коэффициентам $C[\beta]$ мы получаем оптимальные коэффициенты квадратурной формулы. И полученная формула называется оптимальной квадратурной формулой.

Таким образом, для того чтобы построить оптимальную квадратурную формулу в пространстве Соболева $H_2^{(m)}(0,1)$ требуется найти следующую величину

$$\inf_{C[\beta]} \|\ell_N|H_2^{(m)^*}\| \tag{7}$$

Эту величину обозначим через $\|\mathring{\ell}_N|H_2^{(m)^*}\|$, которую назовём нормой функционала погрешности оптимальной квадратурной формулы, т.е.

$$\|\mathring{\ell}_N|H_2^{(m)^*}\| = \inf_{C[\beta]} \|\ell_N|H_2^{(m)^*}\|. \tag{8}$$

Здесь коэффициенты $C[\beta]$ для которых достигается точная нижняя грань в (8) называются оптимальными коэффициентами квадратурной формулы и обозначаются $\mathring{C}[\beta], \quad \beta = 0, 1, ..., N.$

Впервые построение оптимальных квадратурных и кубатурных формул рассмотрены в работах С.М. Никольского [1], А. Сард [2], С.Л. Соболева [3,4].

В работах М.Д. Рамазанова [5,6,7] создан новый алгоритм построения кубатурных формул с пограничным слоем. В работе М.Д. Рамазанова и Х.М.Шадиметова [8] построены весовые оптимальные кубатурные формулы в пространстве Соболева периодических функций.

Итак, нам требуется последовательно решать следующие задачи.

Задача 1. Найти норму функционала погрешности $\ell_N(x)$ квадратурной формулы (1) над пространством $H_2^{(m)}(0,1)$.

Задача 2. Найти такие коэффициенты $\mathring{C}[\beta], \quad \beta = 0,1,...,N$ чтобы получить равенство (8).

Для нахождения в явном виде нормы функционала погрешности $\ell_N(x)$ квадратурных формул в пространстве $H_2^{(m)^*}(0,1)$ используется так называемая экстремальная функция данного функционала [1,2].

Определение 1. (С.Л. Соболева) функция $\psi_{\ell,H}(x) \in H_2^{(m)}(0,1)$ называется экстремальном функцией данного функционала $\ell_N(x)$, если выполняется

$$(\ell_N, \psi_{\ell,H}(x)) = \|\ell_N|H_2^{(m)^*}\| \cdot \|\psi_{\ell,H}|H_2^{(m)}\|. \tag{9}$$

Справедлива следующие теоремы.

Теорема 1. В пространстве Соболева $H_2^{(m)}(0,)$ экстремальная функция функционала погрешности $\ell_N(x)$ задается формулой

$$\psi_{\ell,H}(x) = \ell_N(x) * \varepsilon_m(x) = \int_0^1 \overline{p}(y)\varepsilon_m(x-y)dy - \sum_{\beta=0}^{N} \overline{C}[\beta]\varepsilon_m(x-h\beta), \tag{10}$$

где

$$\varepsilon_{m,\omega}(x) = \frac{\omega e^{-\omega|x|}}{2^{2m-1}(m-1)!} \sum_{k=0}^{m} \frac{(2m-k-2)!(2\omega)^k|x|^k}{k!(2m-k-1)!}. \tag{11}$$

$\overline{\ell}_N(x)$ - сопряженный функционал к функционалу $\ell_N(x)$, $\overline{p}(y)$ - сопряжённая функция к функции $p(y)$, $\overline{C}[\beta]$ - сопряжённый коэффициент к $C[\beta]$.

Теорема 2. Квадрат нормы функционала погрешности квадратурных формул вида (1) в пространстве Соболева $H_2^{(m)^*}(0,1)$ выражается следующей формулой

$$\left\|\ell_N|H_2^{(m)^*}\right\|^2 = \sum_{\beta=0}^{N}\sum_{\gamma=0}^{N} \overline{C}[\beta]C[\gamma]\varepsilon_{m,\omega}(h\gamma - h\beta) - \sum_{\gamma=0}^{N} C[\gamma]\int_0^1 \overline{p}(y)\varepsilon_{m,\omega}(h\gamma - y)dy -$$

$$- \sum_{\beta=0}^{N} \overline{C}[\beta]\int_0^1 p(x)\varepsilon_{m,\omega}(x-h\beta)dx + \int_0^1\int_0^1 p(x)\overline{p}(y)\varepsilon_{m,\omega}(x-y)dxdy. \tag{12}$$

Здесь $C[\gamma]$ коэффициенты квадратурной формулы а $\overline{C}[\beta]$ сопряженное значение к ним, $\varepsilon_{m,\omega}(x)$ определяется равенством (11), $\overline{p}(y)$ сопряженная функция к функцию $p(y)$.

Теорема 3. Квадрат нормы функционала погрешности оптимальных квадратурных формул в пространстве $H_2^{(m)^*}(0,1)$ определяется формулой

$$\left\|\mathring{\ell}_N|H_2^{(m)^*}\right\|^2 = \int_0^1\int_0^1 p(x)\overline{p}(y)\varepsilon_{m,\omega}(x-y)dxdy - \sum_{\beta=0}^{N} \mathring{C}[\beta]\int_0^1 \overline{p}(x)\varepsilon_{m,\omega}(x-h\beta)dx.$$

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Минимизация погрешности весовых квадратурных формул

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DOI 10.5281/zenodo.18225449

Keywords: Квадратурные формулы, функционал погрешности, точная верхняя оценка погрешности, оптимальные коэффициенты.

1.**Введение.** Приближенные вычисления интегралов лежат в основе численного решения интегральных уравнений, а следовательно и значительной долее вычислений математической физики. В конце сороковых годов прошлого века для вычислений интегралов возник вероятностно-статистический метод Монте-Карло использующий идею, где интеграл интерпретируется как среднее значение случайной величины

с распределением, заданным подинтегральной функцией. Он и сейчас основной, позволяя получить приближенные значения интегралов до кратностей несколько сотен.

Другой подход - формулы приближенного интегрирования, параметры которых определяются условиями точного интегрирования многочленов возможно болеевысоких степеней. Этот алгебраический подход позволяет считать интегралы функций одной, двух, трёх переменных по областям простых форм: отрезкам, шарам, параллелепипедам.

Альтернативный указанным направлениям стали формулы, приближащие интегралы линейными комбинациями значений интегралов в узлах заданной решеткирешетчатые квадратурные и кубатурные формулы. Исследования в этом направлении были заложены в работах академика С.Л. Соболева [1,2], в терминах и понятиях функционального анализа. В монографиях [3,4,5] дается описание достижений теории этого функционального аналитического направления, послуживший основной создания оптимальных алгоритмов и программ квадратурных, кубатурных формул высокой точности. Связь между функциональным анализом и вычислительной математикой, уже вполне осознанная в последние полвека, слишком широка, чтобы чтобы говорить о ней в целом. Поэтому мы ограничимся в настоящей работе лишь одним ее аспектом – теории приближенного интегрирования. Здесь применением функционально аналитических методов удается получить ряд важных результатов.

2. Функциональный подход построения оптимальных формул приближенного интегрирования от быстроосцилирующих функций. Пусть $L_2^{(m)}(0,1)$ - пространство Соболева комплекснозначных функций $f(x)$, отличающихся более, чем на многочлен степени $m-1$, m - е производные которых (в обобщенном смысле) квадратично интегрируемы со скалярным произведением

$$(f,\varphi)=\int\limits_0^1 \frac{d^m f\overline{d^m\varphi}}{dx^m dx^m}dx,$$

и нормой

$$\|\varphi|L_2^{(m)}\|=\left(\int\limits_0^1 \varphi^{(m)}(x)\overline{\varphi^{(m)}(x)}dx\right)^{\frac{1}{2}}.$$

Всякой формуле механических квадратур, пригодной для интегрирования функции $\varphi(x)$ из $L_2^{(m)}(0,1)$ с весовой функцией $x^\alpha e^{2\pi i\omega x}$, α и ω - действительные числа, отвечает линейный функционал погрешности

$$(\ell,\varphi)=\int\limits_0^1 x^\alpha e^{2\pi i\omega x}\varphi(x)dx-\sum_{k=1}^N C_k\varphi(x_k), \tag{1}$$

$$\ell(x)=\varepsilon_{[0,1]}(x)x^\alpha e^{2\pi i\omega x}-\sum_{k=1}^N C_k\delta(x-x_k). \tag{2}$$

Предметом нашего изучения будет норма этого функционала погрешности:

$$\|\ell|L_2^{(m)*}\|. \tag{3}$$

Различным x_k - узлам и C_k – коэффициентам отвечают разные формулы. Важная задача – их исследование и минимизация нормы

$$\|\ell|L_2^{(m)*}\|.$$

В практических вопросах важна, разумеется, не норма функционала погрешности $\ell(x)$, а величина погрешности рассматриваемой формулы, т.е. (ℓ, φ) для каждой конкретной функции. Известно что для любой непрерывной функции эта погрешность стремиться к нулю [1,2]. Норма функционала $\ell(x)$ характеризует степень приближения к функционалу $\varepsilon_{[0,1]}(x)x^\alpha e^{2\pi i\omega x}$ - функционалов специального вида

$$\sum_{k=1}^{N} C_k\delta(x-x_k) = R_N(x), \tag{4}$$

При увеличении N. Здесь $\varepsilon_{[0,1]}(x)$ - характеристическая функция отрезка $[0,1]$, $\delta(x)$ - дельта функция Дирака.

Задача изучения $\ell(x)$ сводится к исследованию различных функционалов вида (4). Здесь мы можем при этом изменить:

1. Коэффициенты C_k при заданных x_k и N;

2. Расположение узлов x_k при заданных N;

3. Число узлов N.

Эти три задачи, составляющие три этапа отыскания оптимальных формул приближенного интегрирования быстроосцилирующих функций, являются частными задачами одной общей проблемы функционального анализа – проблемы приближений в пространстве функционалов. И так, задача построениям оптимальной квадратурной формулы

$$\int_0^1 x^\alpha e^{2\pi i\omega x}\varphi(x)dx \cong \sum_{k=1}^{N} C_k\varphi(x_k), \tag{5}$$

в функциональной постановке состоит в нахождении такого функционала (2) норма которого в пространстве $L_2^{(m)*}(0,1)$ минимальна. По другому задача о построении оптимальной квадратурной формулы для приближенного вычисления интеграла

$$\int_0^1 x^\alpha e^{2\pi i\omega x}\varphi(x)dx,$$

при фиксированном узлов x_k квадратурной формулы (4) заключается в следующем.

Требуется найти величину

$$\inf_{C_k}\left(\sup_{\varphi\in L_2^{(m)}(0,1)}\frac{|(\ell,\varphi)|}{\|\varphi|L_2^{(m)}\|}\right). \tag{6}$$

Это означает, что найти функцию $\psi_\ell(x)$ из $L_2^{(m)}(0,1)$ для которой достигается точная верхняя грань, далее найти коэффициенты $\mathring{C}_k$ для которых достигается точная нижняя грань в (6). При этом $\mathring{C}_k$ является оптимальным коэффициентом квадратурной

формулы вида (5), а соответствующая формула называется оптимальной квадратурной формулой в пространстве Соболева $L_2^{(m)}(0,1)$.

3. Основные результаты. Основными результатами этой работы являются оптимальные квадратурные формулы с равноотстоящими узламы в пространстве $L_2^{(1)}(0,1)$.

Справедливы следующие теоремы.

Теорема 1. В пространстве $L_2^{(1)}(0,1)$ существует единственная оптимальная квадратурная формула вида

$$\int_0^1 xe^{2\pi i\omega x}\varphi(x)dx \cong \sum_{\beta=1}^{N} \mathring{C}[\beta]\varphi(h\beta),$$

Оптимальные коэффициенты $\mathring{C}[\beta]$ определяются формулой

$$\mathring{C}[\beta] = \frac{h^{-1}}{(2\pi\omega)^2}\left[\triangle^2[\beta] * \left(e^{2\pi ih\beta}\left(\frac{1}{\pi i\omega} - h\beta\right)\right)\right],$$

где $\beta = 1, 2, ..., N$, $h = \frac{1}{N}$, $\omega \in \mathbb{R}$ а оператор свёрки $\triangle^2[\beta]$ задается

$$\triangle^2[\beta] * f[\beta] = f[\beta+1] - 2f[\beta] + f[\beta-1]. \tag{7}$$

Теорема 2. Среди квадратурных формул вида (5) в пространстве $L_2^{(1)}(0,1)$ следующая квадратурная формула с равноотстоящими узлами является оптимальной

$$\int_0^1 x^\alpha e^{2\pi i\omega x}\varphi(x)dx \cong \sum_{\beta=0}^{N} \mathring{C}[\beta]\varphi(h\beta).$$

При этом оптимальные коэффициенты $\mathring{C}[\beta]$ определяется равенством

$$\mathring{C}[\beta] = h^{-1}\sum_{n=0}^{\alpha}\frac{(-1)^n\alpha!}{(\alpha-n)!(2\pi i\omega)^{n+1}}\triangle^2[\beta] * \left((h\beta)^{\alpha+1-n}e^{2\pi i\omega h\beta}\right) -$$

$$-h^{-1}\sum_{n=0}^{\alpha+1}\frac{(-1)^n(\alpha+1)!}{(\alpha+1-n)!(2\pi i\omega)^{n+1}}\triangle^2[\beta] * \left((h\beta)^{\alpha+1-n}e^{2\pi i\omega h\beta}\right),$$

Здесь $h = \frac{1}{N}$, α - натуральное число,

$$\triangle^2[\beta] * \left((h\beta)^{\alpha+1-n}e^{2\pi i\omega h\beta}\right) = (h(\beta+1))^{\alpha+1-n}e^{2\pi i\omega h(\beta+1)} -$$

$$-2(h\beta)^{\alpha+1-n}e^{2\pi i\omega h\beta} + \left((h(\beta-1))^{\alpha+1-n}e^{2\pi i\omega h(\beta-1)}\right).$$

Заключение. В настоящей работе построены весовые оптимальные формулы с равноотстоящими узлами в пространстве Соболева комплекснозначных функций. Здесь найдены явные аналитические формулы для оптимальных коэффициентов построенных квадратурных формул для приближенного вычисления весовых быстроосцилирующих интегралов.

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SAMEIR-2026

International Scientific and Practical Conference

SYSTEM ANALYSIS AND MODELING IN ECONOMY AND INTERNATIONAL RELATIONS

5632

Bosishga ruxsat etildi: 10.01.2026-yil.
Bichimi 70x100 1/16, “Times New Roman” garniturada raqamli bosma usulida bosildi.
Shartli bosma tabog‘i: 29. Adadi: 100.

“Noble Print” MChJ bosmaxonasida chop etildi.
100194, Toshkent shahri, Yunusobod-11, 62-uy.